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Availability and Economic Analysis of a Repairable Redundant System

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Abstract

Availability and profit of an industrial system are becoming an increasingly important issue. Where the availability of a system increases, the related profit will also increase. This paper dealt with the evaluation of availability, busy period and profit of redundant system. In this paper, the system consists of four subsystems arranged in series-parallel. Performance evaluation model for the availability, busy period and profit has been developed with the help of mathematical formulation based on Markov Birth-Death process using probabilistic approach. A transition diagram representing the operational behavior of the system has been developed. Failure and repair rates of subsystems are constant. The effect of failure and repair rates of each subsystem on generated profit has been determined.

Keywords: Performance evaluation, availability, busy period. availability matrices, Profit

1.0 Introduction

Failure is an unavoidable phenomenon which can be dangerous and costly and bring about less production and profit. Proper maintenance planning plays a role in achieving high system reliability, availability and production output. It is therefore important to keep the equipments/systems always available and to lay emphasis on system availability at the highest order. System availability represents the percentage of time the system is available to users.

A large volume of literature exists on the issue of predicting performance evaluation of various systems. Kumar *et al* [1] discussed the reliability analysis of the Feeding system in the paper industry. Kumar *et al* [2] discussed the availability analysis of the washing system in the paper industry. Kumar *et al* [3] dealt with reliability, availability and operational behavior analysis for different systems in paper plant. Kumar *et al* [4] discussed the behavior analysis of Urea decomposition in the fertilizer industry under the general repair policy. Kumar *et al* [5] studied the design and cost analysis of a refining system in a Sugar industry. Srinath [6] has explained a Markov model to determine the availability expression for a simple system consisting of only one component. Gupta *et al* [7] has evaluated the reliability parameters of butter manufacturing system in a diary plant considering exponentially distributed failure rates of various components. Gupta et al. [8] studied the behavior of Cement manufacturing plant. Arora and Kumar [9] studied the availability analysis of the cool handling system in paper plant by dividing it into three subsystems. Singh and Garg [10] perform the availability analysis of the core veneer manufacturing system in a plywood manufacturing system under the assumption of constant failure and repair rates. Yusuf [11] examined the performance evaluation of a series-parallel system.

However, availability/profit of an industrial system may be enhancing using highly reliable structural design of the system or subsystem of higher reliability. Improving the reliability and availability of system/subsystem, the production and associated profit will also increase. Increase in production lead to the increase of profit. This can be achieve be maintaining reliability and availability at highest order. To achieve high production and profit, the system should remain operative for maximum possible duration. It is important to consider profit as well as the quality requirement.

Little literature can be found dealing with evaluation of availability, busy period of repairman as well as overall profit generated. This paper intends to use profit as a means of measuring the effectiveness of the system. The objectives of this paper are three fold. The first is to obtain explicit expressions for availability, busy period and profit for the system under study. The second is to capture the effect of both failure and repair rates on the profit based on assumed numerical values given to the system parameters. The third is to construct profit matrices for each subsystem for the maximum profit and optimum values of failure and repair rates.

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2.0 Methodology

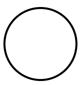
A typical system consists of a number of subsystems connected to each other logically either in series or in parallel in most cases. The performance of the system depends on the configuration and performance of its subsystems. Before analyzing the failure data, it is better to describe the configuration of the system and classify it into various subsystems so that the failures can be categorized. The present system consists of four subsystems, A,B,C and D. The system's model formulation is carried based on Markovian birth-death process using probabilistic approach.

2.1 System structure

The System consists of four dissimilar subsystems which are:

1. Subsystem A: Single unit in series whose failure cause complete failure of the entire system.
2. Subsystem B: Single unit in series whose failure cause complete failure of the entire system.
3. Subsystem C: consisting of three units in cold standby. Failure of the system occurs when all the three units have failed.
4. Subsystem D: Single units in series whose failure cause complete failure of the entire system.

2.3 Notations

 Indicate the system is in full working state

 Indicate the system is in failed state

A, B, C,D, represent full working state of subsystem

C1,C2,C3 denote subsystem is working on standby unit

a, b, c,d, represent failed state of subsystem

$\beta_1, \beta_2, \beta_3, \beta_4$ represent failure rates of subsystems A, B,C

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$: represent repair rates of subsystems A,B,C

$P_0(t)$, $P_4(t)$, $P_8(t)$: Probability of the system working with full capacity at time t

$P_J(t)$: Probability of the system in failed state, $J = 1, 2, 3, 5, 6, 7, 9, 10, 11, 12$

$P_i'(t), i = 0, 1, 2, ..., 12$: represents the derivatives with respect to time t

P_R : Profit

B_P : Busy period of the repairman

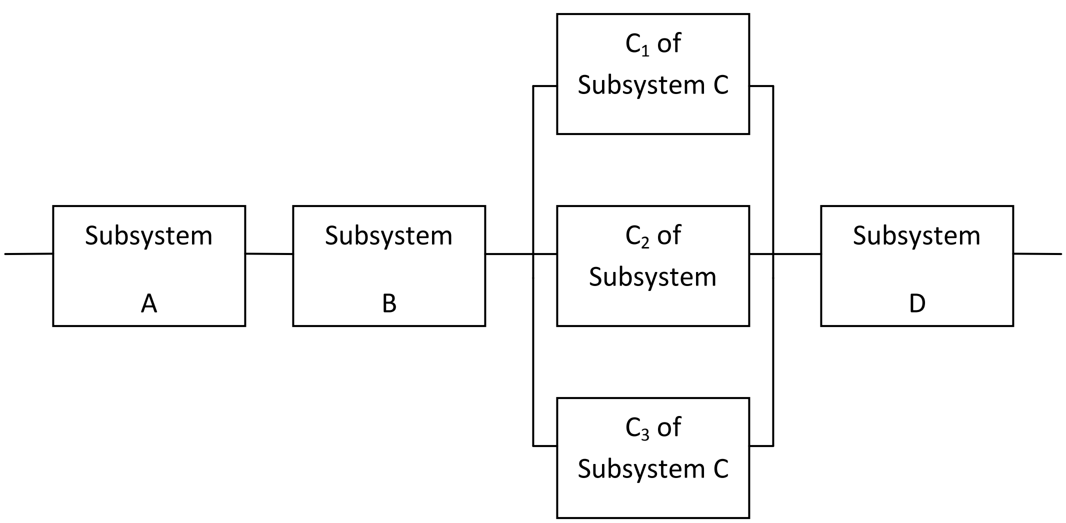


Fig.1 Reliability block diagram of the system

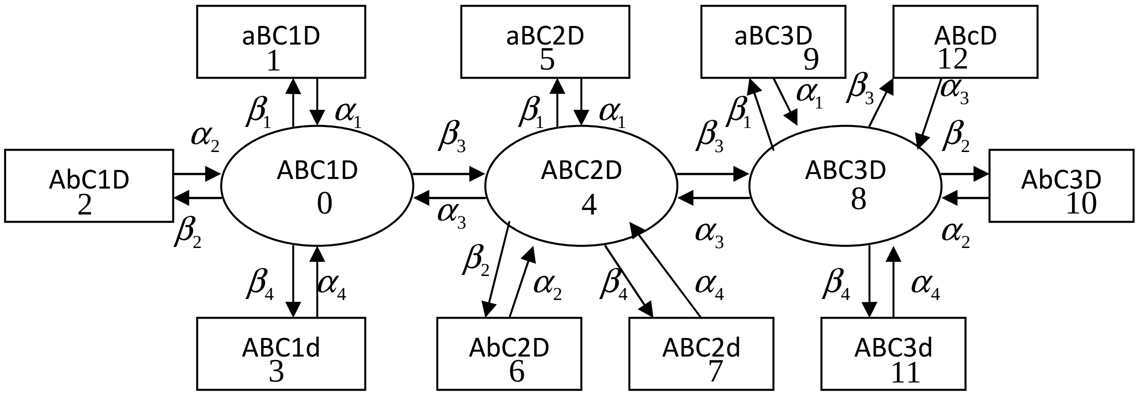


Fig.2 Transition diagram of the system

2.4 Simulation Modeling

The following system linear differential equations associated with the transition diagram (Fig. 2) are derived:

$$\frac{dP_0(t)}{dt}+\sum_{i=1}^{4}\beta_i P_0(t)=\alpha_1 P_1(t)+\alpha_2 P_2(t)+\alpha_3 P_4(t)+\alpha_4 P_3(t) \quad (1)$$

$$\frac{dP_1(t)}{dt}+\alpha_1 P_1(t)=\beta_1 P_0(t) \quad (2)$$

$$\frac{dP_2(t)}{dt}+\alpha_2 P_2(t)=\beta_2 P_0(t) \quad (3)$$

$$\frac{dP_3(t)}{dt}+\alpha_4 P_4(t)=\beta_4 P_0(t) \quad (4)$$

$$\frac{dP_4(t)}{dt}+\sum_{i=1}^{4}\beta_i P_4(t)+\alpha_3 P_4(t)=\beta_3 P_0(t)+\alpha_1 P_5(t)+\alpha_2 P_6(t)+\alpha_3 P_8(t)+\alpha_4 P_7(t) \quad (5)$$

$$\frac{dP_5(t)}{dt}+\alpha_1 P_5(t)=\beta_1 P_4(t) \quad (6)$$

$$\frac{dP_6(t)}{dt}+\alpha_2 P_6(t)=\beta_2 P_4(t) \quad (7)$$

$$\frac{dP_7(t)}{dt}+\alpha_4 P_7(t)=\beta_4 P_4(t) \quad (8)$$

$$\frac{dP_8(t)}{dt}+\sum_{i=1}^{4}\beta_i P_8(t)+\alpha_3 P_8(t)=\beta_3 P_4(t)+\alpha_1 P_9(t)+\alpha_2 P_{10}(t)+\alpha_3 P_{12}(t)+\alpha_4 P_{11}(t) \quad (9)$$

$$\frac{dP_9(t)}{dt}+\alpha_1 P_9(t)=\beta_1 P_8(t) \quad (10)$$

$$\frac{dP_{10}(t)}{dt}+\alpha_2 P_{10}(t)=\beta_2 P_8(t) \quad (11)$$

$$\frac{dP_{11}(t)}{dt}+\alpha_4 P_{11}(t)=\beta_4 P_8(t) \quad (12)$$

$$\frac{dP_{12}(t)}{dt}+\alpha_3 P_{12}(t)=\beta_3 P_8(t) \quad (13)$$

Setting $\frac{d}{dt}=0$ as $t\to\infty$ in equations (1) to (13) in steady-state and solving them recursively we obtained the steady state probabilities given below:

$$P_1(t)=X_1P_0(t)$$
$$P_2(t)=X_2P_0(t)$$
$$P_3(t)=X_4P_0(t)$$
$$P_4(t)=X_3P_0(t)$$
$$P_5(t)=X_1X_3P_0(t)$$
$$P_6(t)=X_2X_3P_0(t)$$
$$P_7(t)=X_4X_3P_0(t)$$
$$P_8(t)=X_3^{\,2}P_0(t)$$
$$P_9(t)=X_1X_3^{\,2}P_0(t)$$
$$P_{10}(t)=X_2X_3^{\,2}P_0(t)$$
$$P_{11}(t)=X_4X_3^{\,2}P_0(t)$$
$$P_{12}(t)=X_3^{\,3}P_0(t)$$

Where $X_1=\frac{\beta_1}{\alpha_1},\ X_2=\frac{\beta_2}{\alpha_2},\ X_3=\frac{\beta_3}{\alpha_3},\ X_4=\frac{\beta_4}{\alpha_4}$

P_0 (the probability of full working state) is determine using the condition normalizing below:

$$P_0(t)+P_1(t)+P_2(t)+P_3(t)+P_4(t)+\ldots+P_{12}(t)=1 \qquad (14)$$

Thus,

$$P_0(t)=\frac{1}{1+\left(1+X_3+X_3^{\,2}\right)\left(X_1+X_2+X_3+X_4\right)}$$

2.5 Steady-state availability, busy period and profit of the system

Availability = Summation of all working states

$$A_V=P_0(t)+P_4(t)+P_8(t) \qquad (15)$$
$$=P_0(t)\left(1+X_3+X_3^{\,2}\right)$$

Busy period of repairman = Summation of all states involving failure

$$B_P=\sum_{K=1}^{12}P_K(t) \qquad (16)$$
$$=P_o(t)\left(1+X_3+X_3^{\,2}\right)\left(X_1+X_2+X_3+X_4\right)$$

The units/subsystems are subjected to corrective maintenance as can be observed in Fig. 2. The repairman performed corrective maintenance to failed units/subsystems in states 1,2,3,...,12. Let C_0 and C_1 be the revenue generated when the system is in working state and no income when in failed state and cost of each repair (corrective maintenance) respectively. Following El-said [12] and Haggag [13], the expected total profit per unit time incurred to the system in the steady-state is

Profit=total revenue generated – cost incurred when repairing the failed units.

$$P_R = C_0 A_V - C_1 B_P \tag{17}$$

3. Results and Discussions

Table 1: Profit matrix of the subsystem A of the system

β_1 \ α_1	0.02	0.04	0.06	0.08	0.1	$\beta_2 = 0.00076$ $\alpha_2 = 0.3$ $\beta_3 = 0.06$ $\alpha_3 = 0.3$ $\beta_4 = 0.007$ $\alpha_4 = 0.125$
0.0006	886.98	900.63	905.29	907.64	909.05	
0.0007	882.45	898.32	903.75	906.46	908.11	
0.0008	878.01	896.03	902.17	905.29	907.17	
0.0009	873.60	893.74	900.65	904.12	906.23	
0.01	869.23	891.46	899.11	902.97	905.29	

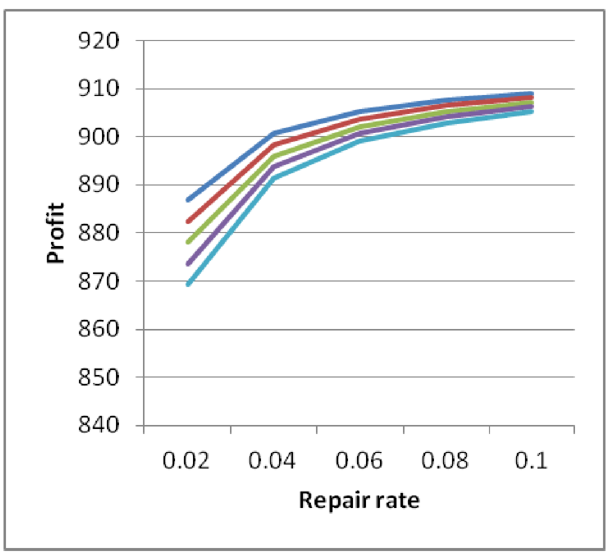


Fig.3 effect of α_1 on Profit

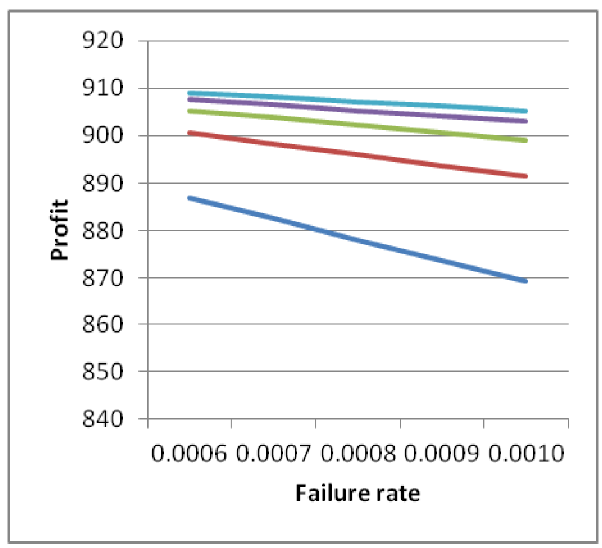


Fig.4 effect of β_1 on

Table 2. Profit Matrix of subsystem C of the system

β_2 \ α_2	0.1	0.2	0.3	0.4	0.5	$\beta_1 = 0.0008$ $\alpha_1 = 0.06$ $\beta_3 = 0.06$ $\alpha_3 = 0.3$ $\beta_4 = 0.007$ $\alpha_4 = 0.125$
0.0050	860.19	882	889.51	893.29	895.38	
0.0063	849.21	876.23	885.85	890.32	893.18	
0.0076	838.85	870.53	881.68	887.40	890.83	
0.0089	828.05	864.93	877.89	884.42	888.49	
0.0102	415.71	581.58	654.84	712.22	744.62	

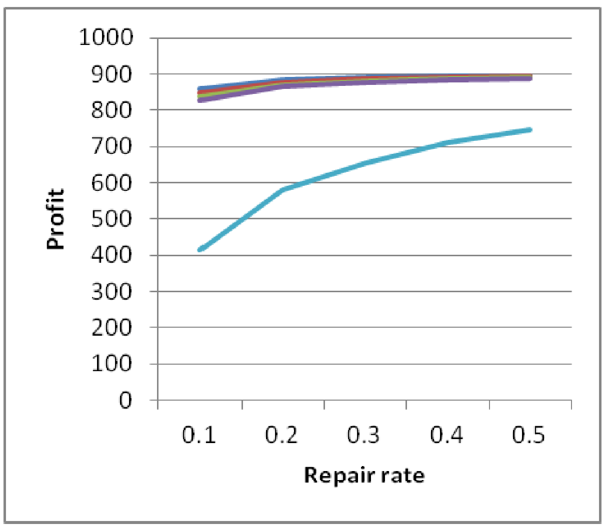


Fig. 5 effect of α_2 on Profit

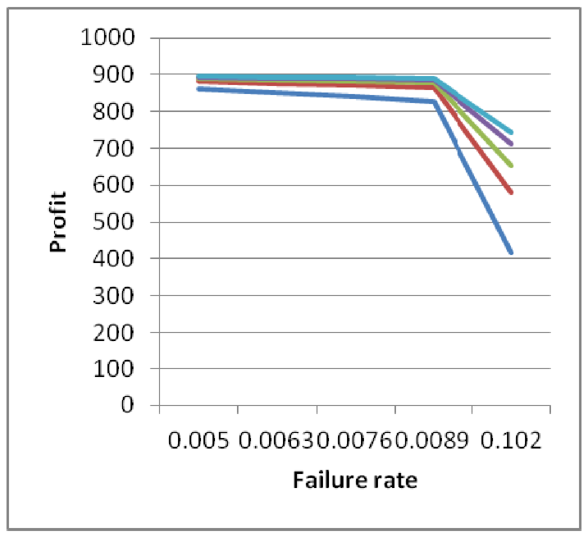


Fig.6 effect of β_2 on Profit

Table 3. Profit Matrix of subsystem C of the system

β_3 \ α_3	0.1	0.2	0.3	0.4	0.5	
						$\beta_1 = 0.0008$ $\alpha_1 = 0.06$ $\beta_2 = 0.0076$ $\alpha_2 = 0.3$ $\beta_4 = 0.007$ $\alpha_4 = 0.15$
0.02	890.10	903.82	907.39	909.11	910.09	
0.04	845.01	890.10	900.24	903.82	906.02	
0.06	779.37	870.33	890.10	897.65	901.46	
0.08	703.45	845.03	877.11	890.83	896.20	
0.1	628.55	814.14	863.56	881.11	890.10	

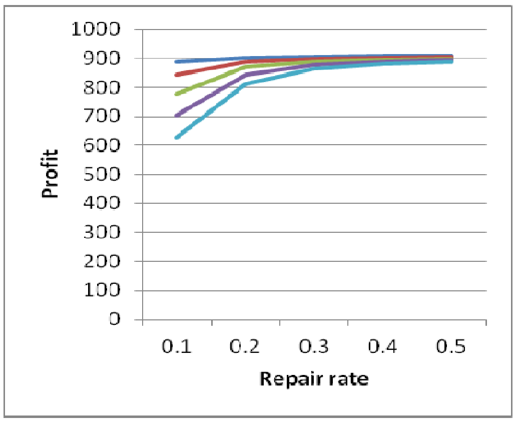


Fig.7 effect of α_3 on Profit

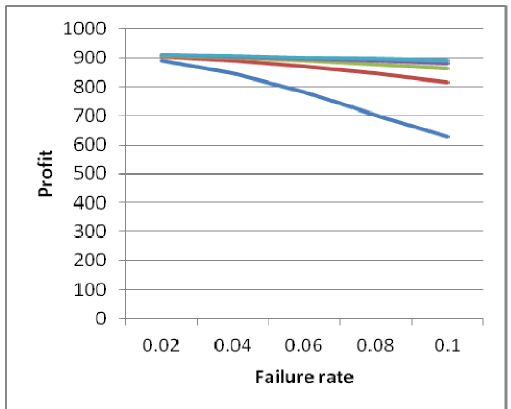


Fig. 8 effect of β_3 on Profit

Table 4. Profit matrix of the subsystem D of the system

β_4 \ α_4	0.05	0.087	0.125	0.162	0.2	
						$\beta_1 = 0.0008$ $\alpha_1 = 0.06$ $\beta_2 = 0.00076$ $\alpha_2 = 0.3$ $\beta_3 = 0.06$ $\alpha_3 = 0.3$
0.004	880.66	911.85	925.33	932.51	937.15	
0.0055	854.67	895.84	913.75	923.47	929.73	
0.007	830.02	880.25	902.41	914.53	922.42	
0.0085	806.62	865.14	891.46	905.78	915.21	
0.01	784.36	850.56	880.66	897.18	908.11	

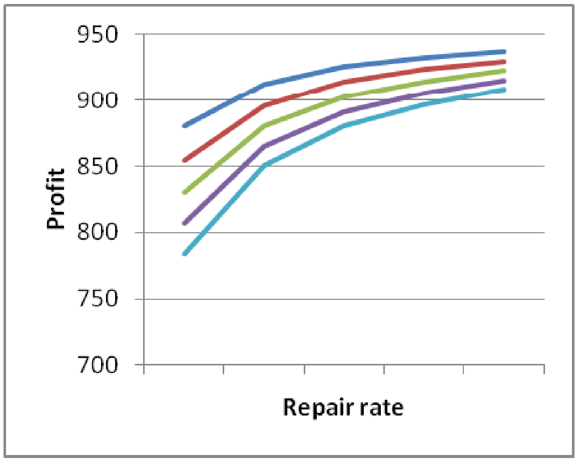


Fig.9 effect of α_4 on Profit

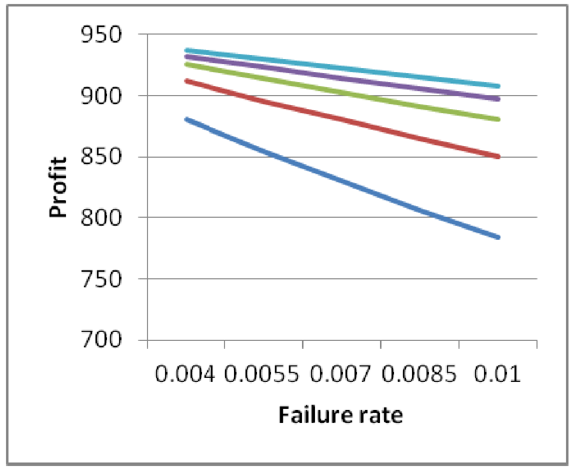


Fig. 10 effect of β_4 on Profit

Table 5 Optimum values of Failure/Repair rates of Subsystems of Series-Parallel system

S/N	Subsystem	Failure rate β_i	Repair rate α_i	Maximun Profit Level
1	A	0.0006	0.1	909.05
2	B	0.0050	0.5	895.38
3	C	0.02	0.5	910.09
4	D	0.004	0.2	937.15

Table 1 and Figure 3 and 4 reveal the effect of failure and repair rates of subsystem A on the profit of the system. It is observed that for some known values of failure / repair rates of other three subsystems, as failure rate of first subsystem increases from 0.0006 to 0.01, the subsystem profit decreases. Similarly as repair rate of the subsystem A increases from 0.02 to 0.1 (once in 2.5 hrs), the subsystem profit increases.

Table 2 and Figure 5 and 6 reveal the effect of failure and repair rates of subsystem A on the profit of the system. It is observed that for some known values of failure / repair rates of other three subsystems, as failure rate of first subsystem increases from 0.0006 to 0.01, the subsystem profit decreases. Similarly as repair rate of the subsystem A increases from 0.02 to 0.1 (once in 2.5 hrs), the subsystem profit increases.

Table 3 and Figure 7 and 8 reveal the effect of failure and repair rates of subsystem A on the profit of the system. It is observed that for some known values of failure / repair rates of other three subsystems, as failure rate of first subsystem increases from 0.0006 to 0.01, the subsystem profit decreases. Similarly as repair rate of the subsystem A increases from 0.02 to 0.1 (once in 2.5 hrs), the subsystem profit increases.

Table 4 and Figure 9 and 10 reveal the effect of failure and repair rates of subsystem A on the profit of the system. It is observed that for some known values of failure / repair rates of other three subsystems, as failure rate of first subsystem increases from 0.0006 to 0.01, the subsystem profit decreases. Similarly as repair rate of the subsystem A increases from 0.02 to 0.1 (once in 2.5 hrs), the subsystem profit increases.

Table 5 helps in determining the subsystem with maximum profit. It is observed that subsystem D is having maximum profit. Shown in the Table 5 are the optimum values of failure and repair rates for maximum profit for each subsystem. From Table 5, it is observed that the most critical subsystem as far as maintenance is concerned and required immediate attention is subsystem B, as the effect of failure rates on system profit is lower than that of subsystems A,C and D. Therefore, on the basis of repair rates, the maintenance priority should be given as per the following order:

Subsystem B, Subsystem A, Subsystem C, Subsystem D.

4.0 Conclusion

Explicit expression for the profit model is developed and used for the evaluation of performance of different subsystems of the series-parallel system in this study. Using the model, tables 1-4 are constructed to show the relationship between failure and repair rates on system availability. The profit decreases as the failure rate increases. Similarly as profit increases so also the repair rates. The model will assist maintenance engineers and managers for proper maintenance utilization. The results of this study will be beneficial to the plant management for the availability, busy period of the repairman and profit analysis of series-parallel system and hence to decide about the maintenance priorities of various subsystems of the system concerned.

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