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Appropriate Choice of Scale in Graphical and Computational Analysis

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Abstract

The need for a scale to be 'acceptable' and 'reasonable' in statistical and computational analysis have been stressed in recent times by researchers and examination bodies such as West Africa Examination Council (WAEC), National Examination Council (NECO) etc. In this work we proved the condition necessary for the scale factor K which is an element of set of real numbers $\mathbb{R}$ ($\mathrm{K} \in \mathbb{R}$) ***to be 'acceptable' and 'reasonable' as scale. We further developed a scale choosingformula that is dependent on the data in question.***

1.0 Introduction

In graphical and computational analysis, scale$[1-3]$ is normally employed to obtain a scatter plot$[2]$. In choice of scale, prime numbers such as 3,7,11 etc. and their multiples or submultiples are not accepted $[4,5]$. Hence a number that can be accepted as scale factor must be an element of the set Q=$\{\ldots 1,2,4,5,8,10,16,20 \ldots\}$. Analysis of scatter points is much convenient if the plots are well varied on the graph paper. And it is true that not every element of Q can give a good variability of plots. The elements of Q for which good variability is obtained are said to be 'reasonable'$[4-6]$.

Graph paper; It is any paper that have vertical and horizontal grid lines equally spaced thereby forming squares. The grid lines are classified into major and minor[7][8].

Division; In mathematics, especially in elementary arithmetic, **division** (÷) is an arithmetic operation. Specifically, if *b*times*c* equals *a*, written:

$$a = b \times c \qquad (1)$$

Where*b* is not zero, then *a* divided by *b* equals *c*, written:

$$a \div b = c \qquad (2)$$

In the expression a ÷ b = c, *a* is called the **dividend** or **numerator**, *b* the **divisor** or **denominator** and the result *c* is called the quotient. Division is often shown in algebra and science by placing the *dividend* over the *divisor* with a horizontal line, also called a vinculum or fraction bar, between them. For example, *a* divided by *b* is written

$$\frac{a}{b}$$

This can be read out loud as "a divided by b", "a by b" or "a over b". A way to express division all on one line is to write the *dividend* (or numerator), then a slash, then the *divisor* (or denominator), like this:

$$^{a}/_{b}$$

Division of any number by zero (where the divisor is zero) is undefined. This is because zero multiplied by any finite number always results in a product of zero. Entry of such an expression into most calculators produces an error message[9].

Sequence; Is an ordered list or an ordered set of numbers, the idea of ordering is important[10]. The individual objects in the list are called 'terms' e.g.0,1,2,3,4,…a_n is a sequence with a_n as the n^{th}

Mappings; The term mapping, function and transformation will be used synonymously. The symbol $f: X \rightarrow Y$ will mean that f is a single valued function whose domain is X and whose range is contained in Y such that for every $x \in X$, the function f assigns a uniquely determined element $(x) = y \in Y[11]$. The range of f will generally be smaller than Y, but if the range is Y then we say that f is a function onto Y or f is ***surjective***$[12]$**.**

The Axiom of choice; let $\mathbb{C}$ be any collection of nonempty sets, then there is a function f defined on $\mathbb{C}$ which assigns to each set $A \in \mathbb{C}$ an element $f(A)$ in A. The function f is called a choice function, and its existence may be thought of as the result of choosing for each of the sets A in $\mathbb{C}$ an element in$A[12][11]$.

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2.0 Methodology

Physics concepts when reduced to symbols allow the powerful techniques of logical and mathematical manipulations as exemplified in: Boyle's law, Coulomb's law etc.[13]

Definition; Let $m, n, P, \in \mathbb{R}$be dividend, divisor and quotient of division operation, we define a successive division operation in which the quotient becomes the next dividend and the divisor remain constant as;

$$m \gg/n = P_{\varepsilon\geq 0} \quad (3)$$

$where\ the\ simbol\ \gg/\ is\ read$ **sucdiv**. $to\ mean\ successive\ division$
$and\ \varepsilon\ is\ the\ number\ of\ successive\ divisions\ carried\ out.$

Lemma; if $m, n \in \mathbb{R}$, then $m \gg/n$ is closed over the set of real numbers $\forall n \neq 0$.

Proof;

Since division operation is closed on the set of real numbers with the exception of zero[13]then, the proof follows.

Axiom; division of the elements of set of real number $A = \{a_i\}$ by a constant c $\in \mathbb{R}$ i.e.' /c' can be regarded as a function $f(/c)$ which maps elements of a real set $A = \{a_i\}\ to\ B = \{b_i\}$.e.g.$A = \{1,2,3,4\}$ if c=2 then $B = \{0.5,1,1.5,2\}$

Definition; Let $X = \{x_i\}, Y = \{y_i\}, n \in \mathbb{R}$, then $\gg/n$ is a function mapping $X\ to\ Y$ with ε not necessarily being the same$\forall\ y_i$.

Definition: Precision of plots; in scaling an axis, the scale is usually written against the major grid of the paper. But the precision of plots is determined by the minor grid[4][5]. In most graph papers, the minor grids are5[8].

Now, choosing $n = 5$,

Let the function $f = \gg/5$ be considered as a choice function which will transform set Q={... 1,2,4,5,8,10,16 ...} into a set S={... 1,2,4,8,16 ...} for ε not necessarily the same $\forall q_i$.

Lemma; if P_ε is the quotient of the $m \gg/n$ then $m = n^\varepsilon P_\varepsilon \quad \forall\ m, n, P \in \mathbb{R} : n \neq 0$

Proof;

Let $a, b, c \in \mathbb{R}$ since $a = bc$ if $^a/_b = c$[9]then the above lemma also follows.

Consider set S above, it has a sequence whose n^{th}term $S_n = 2^{n-1} \quad \forall\ n \in \mathbb{Z}$

Theorem: let K $\in \mathbb{R}$: K $\neq 0$, if K is a scale then there exist $n \in \mathbb{Z}$: $|\mathrm{K}| = 10^{-x}2^{n-1}5^\varepsilon$

$where: \varepsilon\ is\ the\ number\ of\ division\ carried\ out\ on\ |K \times 10^x|\ for\ which\ P_\varepsilon < 5$
$x\ is\ the\ decimal\ place\ for |K| < 1\ \ and\ x = 0\ for\ |K| \geq 1.$

Proof;

First we choose arbitrary number from set of real which is a prime such as 3, 7,11,etc. or their multiples and submultiples, and in turn we choose any element of Q.

We prove by contradiction.

Let K $= 45$,

Suppose K $\in Q$

Then we find $n \in \mathbb{Z} : |\mathrm{K}| = 10^{-x}2^{n-1}5^\varepsilon$

$\therefore\ 45 \gg/5 = 1.8_2 \quad hence\ \varepsilon = 2, \quad x = 0$

Solving for n yields $n = 1.8480\ \notin \mathbb{Z}$ which contradicts our assumption

Hence K $\notin Q$

Let K $= 50$

Suppose K $\notin Q \quad then\ n \notin \mathbb{Z} : |\mathrm{K}| = 10^{-x}2^{n-1}5^\varepsilon$

$\therefore 50 \gg/5 = 2_2 \quad hence\ \varepsilon = 2, \quad x = 0$

Solving for n yield $n = 2 \in \mathbb{Z}$ which contradicts our assumption

Hence K $\in Q$ end of proof.

From the theorem, the n^{th} term of set$V \supset Q$ can be determined for given a ε, x.

Definition; Furthermore, any element of$Q$ is considered 'reasonable' for a given data only if it allows good variability of plots[6]. To find such Kthe space provided for graph plotting, lower and the upper bound of the data are considered.

Let: N be the number of major grid on the graph sheet,

U, Lbethe upper and lower bound of the data respectively,

We define:

$$\left.\begin{aligned} R &= U - L \\ Z &= |U| + |L|. \end{aligned}\right\} \quad (4)$$

If K is reasonable, then;

$$KN \geq Z \qquad \therefore K \geq {^Z/_N},$$

But $\quad K = 10^{-x}2^{n-1}5^\varepsilon \quad (5)$

$$\therefore 10^{-x}2^{n-1}5^{\varepsilon} \geq Z/_{N}$$
$$n-1 \geq 3.3219 \log\left(Z10^{x}/_{N5^{\varepsilon}}\right), \text{ setting}$$
$$n-1 \geq m \geq 3.3219 \log\left(Z10^{x}/_{N5^{\varepsilon}}\right)$$
$$\therefore K = 10^{-x}2^{m}5^{\varepsilon}$$
$$= 2^{m-x}5^{\varepsilon-x} \qquad (6)$$
$$where\ \varepsilon\ is\ determine\ from\ Z10^{x}/_{N} \gg /5 = P_{\varepsilon}\ \ for\ which\ P_{\varepsilon} < 5$$

The scale can be expressed as $\lambda : K$

$$where\ \lambda\ is\ number\ of\ major\ grid\ line\ and\ it\ is = 1\ if\ origin\ of\ axis\ is\ zero\ (0)$$
$$and\ \lambda \approx \frac{N}{(R/_{K})}\ if\ 0 <\ origin\ \leq L \qquad (7)$$

3.0 Results and Application

Consider the table below, we use the above formula to determine the scale factors, plot the graph and compare to what has been plotted by the author of the data[15].

Table 1: Table of values for the determination of focal length of a converging lens.

h_o=20.0cm

i	x(cm)	h(cm)	m=h/h_o	m^{-1}	x^{-1}(cm^{-1})
1	42.0	40.0	2.00	0.500	0.0238
2	28.0	27.0	1.35	0.740	0.0357
3	21.0	20.0	1.00	1.000	0.0476
4	17.0	16.0	0.80	1.250	0.0588
5	14.0	14.0	0.75	1.429	0.0714

Plotting m^{-1} on the vertical and x^{-1} on the horizontal axis;

Using $K = 2^{m-x}5^{\varepsilon-x}$, $\quad m \approx 3.3219 \log\left(\frac{Z10^{x}}{N5^{\varepsilon}}\right), \frac{Z10^{x}}{N} \gg /5,\ \lambda \approx \frac{N}{R/_{K}}$

In the column m^{-1},

L=0.500, U=1.429

∴ Z=0.500+1.429=1.929,

R=1.429-0.500=0.929

Since Z>1, x=0, N=12

To determine ε, we do; $\frac{1.929\times10^{0}}{12} \gg /5 = 0.16075_{0} \quad \therefore \varepsilon = 0$

To determine m, we do; $m \approx 3.3219 \log\left(\frac{1.929\times10^{0}}{12\times5^{0}}\right) \approx -3$

$\therefore K = 2^{-3-0}5^{0-0} = 2^{-3} = 0.125$

With axis starting from zero, the scale is given as $1: 0.125$.

In the column x^{-1},

L=0.0238, U=0.0714

∴ Z=0.0238+0.0714=0.0952,

R=0.0714-0.0238=0.0476

Since Z<1, x=4, N=10

To determine ε, we do; $\frac{0.0952\times10^{4}}{10} \gg /5 = 3.808_{2} \quad \therefore \varepsilon = 2$

To determine m, we do; $m \approx 3.3219 \log\left(\frac{0.0952\times10^{4}}{10\times5^{2}}\right) \approx 2$

$\therefore K = 2^{2-4}5^{2-4} = 2^{-2}5^{-2} = 0.01$

With axis starting from zero, the scale is given as $1: 0.01$

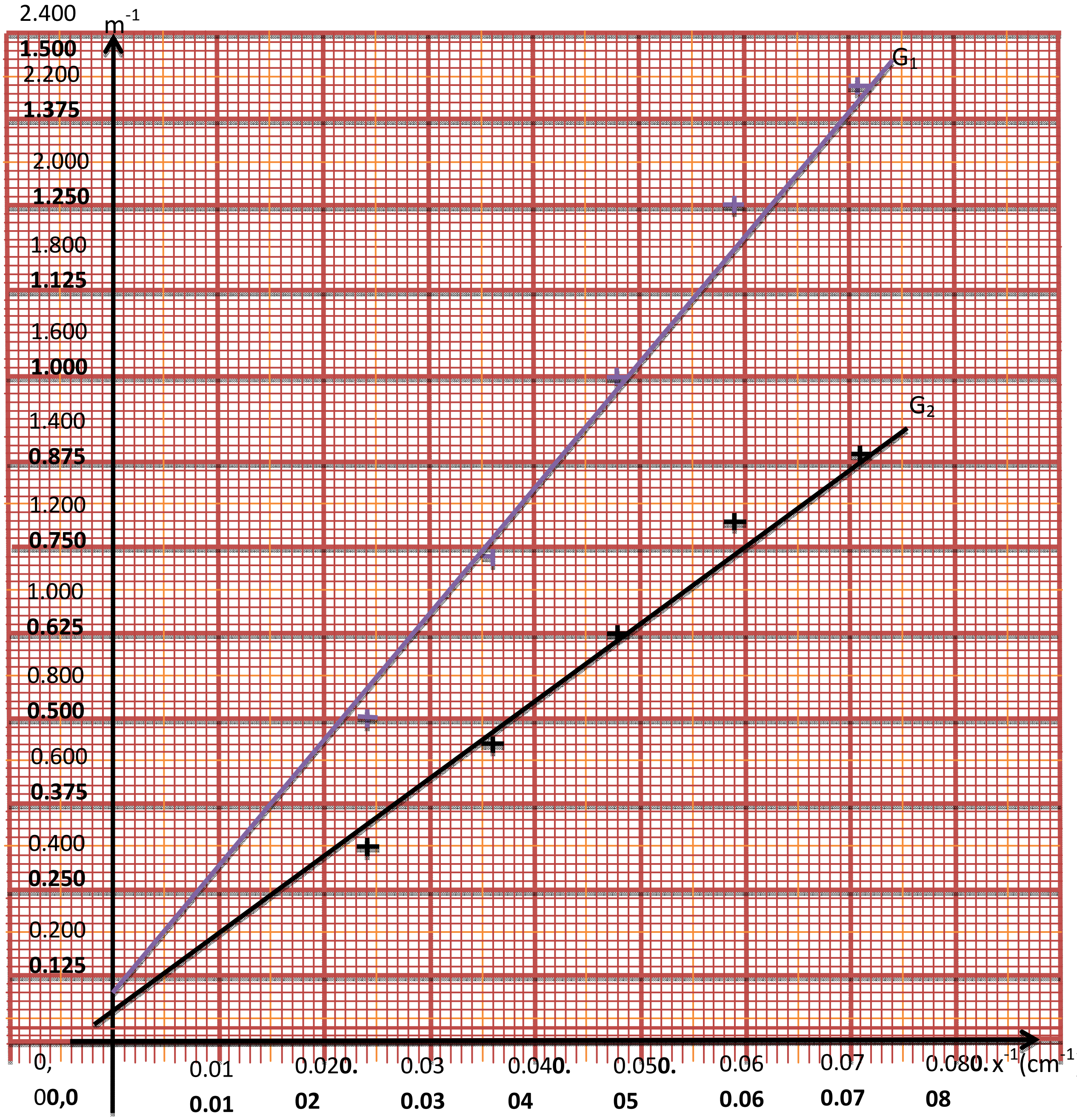

Figure 1: The graph of m^{-1} against $x^{-1}(cm^{-1})$

4.0 Discussion

1 The calculated scale has an advantage in that it determines exactly the decimal places; that have good contribution on the data under consideration, which will otherwise be ignored, e.g in column m^{-1} in the table above, the calculated scale is 0.125 showing that up to the 3rd decimal place has good contribution on the data but the chosen scale is 0.2 ignoring the 2nd and 3rd decimal places.

2 The graph G_1 is the graph plotted on the calculated scales, it is clear that it covers most of the space provided for graph work which is the most recommended by examination bodies[4][5]but graph G_2 which is plotted on chosen scale by the author of the data[15] has fallen short of this recommended standard.

3 The variation between graph G_1 and G_2 is clear hence the physical quantities of interest such as SLOPE and INTERCEPT will vary significantly. Therefore, any stochastic process of the abovedata base on graph G_2 can be misleading.

4 The calculated scale is logical hence easier to teach[13]; and different candidates can have the same scale factors for the same data and paper size,than the chosen method which is judgmental and different candidates can have different scale factors for the same data and paper size.

5.0 Summary

Scale factor in graphical and computational analysis is very important for obtaining results which are reasonable and accurate from a data or observation collected during research activities.

In this paper we defined and discussed some of the vital parameters and operational mathematic operators to give good scale factors when drawing graphs to represent the data or observational values from an experiment.

A mathematical expression for determination of scale factor is derived as equation (6) and(7)

Test of the above equations with data obtain from an experiment on light gives a better result compared to the existing process of choosing scale factors at random.

Other advantages of the scale factor formula include reduction in abstractness associated with the concept of choosing scale factors to logic which makes it easy to teach and comprehend. It also helps and easily preserves the decimal places that are vital in plots which would be difficult with randomly choosing method.

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