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Nuclear cross section calculations in the Glauber model

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Abstract

We present here cross section calculations for various nucleus-nucleus reactions for elastic and inelastic scatterings in the framework of the Glauber theory. The CSC_GM code used in the computations was obtained from the CPC Program Library, Queen's University of Belfast, N. Ireland. The CSC_GM code is a Fortran 90 program that was originally run on UNIX operating system. The code was modified and run on Windows xp. The projectile nucleus is assumed to be structure of a core plus valence nucleon. Total cross sections for $^{13}C+^{12}C$ ***and one nucleon-removal cross sections for*** $^{19}C+^{12}C$ ***are computed. The input data needed for the calculations are the core and target densities and the nucleon-nucleon profile function. Results of the*** $^{13}C+^{12}C$ ***reactions system are found to agree with the experimental data, especially at high incident energies.***

Keywords: Glauber model, Cross section, One-nucleon halo, Neutron skin, Nucleon-removal cross section, Differential cross section

1.0 Introduction

Experimental study of unstable radioactive nuclei has advanced considerably through the technique of using secondary radioactive beams. The quantities measured in this type of study are the various reaction cross sections that include the total reaction cross section, nucleon-removal cross sections, momentum distribution of a fragment, etc. These quantities play important role in revealing the nuclear structure of unstable nuclei, particularly the halo structure, proton and neutron skins. Halo and skin are nuclear properties or structures that are peculiar to only unstable radioactive nuclei. Sizes and density distributions (of both nuclear matter and charge) of unstable nuclei are therefore quite different from those of stable nuclei.

In this paper total and one nucleon-removal cross sections for the ^{13}C + ^{12}C and ^{19}C + ^{12}C reactions are calculated in the framework of the Glauber theory. The projectile nucleus is assumed to have structure of a core nucleus plus a valence nucleon. Measurement of these cross sections is now a standard work for the study of unstable nuclei

The Glauber model is a microscopic reaction theory of high-energy collision based on the eikonal approximation and on the bare nucleon-nucleon interaction. It is now a standard tool to calculate the cross sections because it can account for a significant part of breakup effects which play an important role in the reaction of a weakly bound nucleus [1,2].

2.0 Theoretical Background

The reaction of a projectile nucleus P with a target nucleus T is considered. At the initial stage of the reaction, the projectile in the ground state, described with an intrinsic wave function Ψ_0, impinges with momentum $hK = (0, 0, hK)$ on the target in its ground state, described with an intrinsic wave function Θ_0 . The center-of-mass wave function is removed from $\Psi 0$ (Θ_0). At the final stage of the reaction, the projectile goes to the state *a* specified by a wave function Ψa and the target goes to the state *c* specified by a wave function Θc . The state *a* is not necessarily a bound state but may be a continuum state that includes some fragments. The momentum transferred from the target to the projectile is *hq*. The scattering amplitude for this reaction is written in the Glauber theory as an integral over the impact parameter ***b*** between the projectile and the target [3] as

$$F_{ac}(q) = \frac{iK}{2\pi}\int dbe^{-iq\cdot b}\left\langle \psi_a \Theta_c \middle| 1-\prod_{i\in P}\prod_{j\in T}\left(1-\Gamma_{ij}\right) \middle| \psi_0 \Theta_0 \right\rangle \qquad (1)$$

The integrated cross section for this reaction is given by

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$$\sigma_{ac} = \int \frac{dq}{K^2} \left| F_{ac}(q) \right|^2 \tag{2}$$

The profile function *Γ in* Eq. (1) is given by:

$$\Gamma(b) = \frac{1 - i\alpha}{4\pi\beta} \sigma_{NN} e^{-b^2/2\beta} \tag{3}$$

The parameters σ_{NN}, *α*, and *β* usually depend on either the proton–proton (neutron–neutron) or proton–neutron case. The argument of Γij in Eq. (2) is $\boldsymbol{b} + \boldsymbol{s}\mathrm{P} - \boldsymbol{s}\mathrm{T}$, which stands for the impact parameter between *i*th and *j*th nucleons. Here $\boldsymbol{s}\mathrm{P}$ ($\boldsymbol{s}\mathrm{T}$) is the two-dimensional coordinates comprising the *x*- and *y*-components of the *i*th nucleon coordinate in the projectile (target) relative to its center-of-mass coordinate.

2.1 Total reaction cross section

The total reaction cross section is obtained by summing *σac* over all possible final states *ac* except for the elastic channel:

$$\sigma_{reac}(P+T) = \sum_{ac \neq 00} \sigma_{ac} \tag{4}$$

With the use of the closure relation, $\sum_{ac} \left| \psi_a \Theta_c \right\rangle \left\langle \psi_a \Theta_c \right| = 1$, and the unitarity condition, $\left| 1 - \Gamma \right|^2 = 1$, the reaction cross section reduces to [4]

$$\sigma_{reac}(P+T) = \int db \left(1 - \left| e^{i\mathrm{X}_{PT}(b)} \right|^2 \right) \tag{5}$$

where the phase-shift function χ_{PT} is given by:

$$e^{i\mathrm{X}_{PT}(b)} = \left\langle \psi_0 \Theta_0 \left| \prod_{i \in P} \prod_{j \in T} \left[1 - \Gamma(b + s_i^P - s_j^T \right] \right| \psi_0 \Theta_0 \right\rangle \tag{6}$$

Similarly the core-target total reaction cross section is given by [4]

$$\sigma_{reac}(C+T) = \int db \left(1 - \left| e^{i\mathrm{X}_{CT}(b)} \right|^2 \right) \tag{8}$$

The core-target phase-shift, $\chi_{\mathrm{CT},}$ and the nucleon-target phase-shift, $\chi_{\mathrm{NT},}$ are defined through the relevant densities by the integrals:

$$i\chi_{CT}(b) = -\int dr \int dr' \rho_C(r) \rho_T(r') \Gamma(b + s - s'), \tag{9}$$

$$i\chi_{NT}(b) = -\int dr \rho_T(r) \Gamma(b - s) \tag{10}$$

where the density, ρ, is given by:

$$\rho(r) = \sum_i c_i e^{-a_i r^2} \tag{11}$$

The projectile is assumed to be a system of a core nucleus coupled with a valence nucleon. Its ground-state wave function is given by

$$\Psi_0 = □_0 \Phi_0, \tag{12}$$

where Φ_0 is the ground-state wave function of the core, and $□_0$ is the valence-nucleon wave function [4]

2.2 One-nucleon removal cross section

The one-nucleon removal cross section, $\sigma_{-\mathrm{N}}$, is obtained by [3]

$$\sigma_{-N} = \sum_{C} dk \sigma_{a=(k,g=0),c} \tag{14}$$

where the projectile nucleus breaks up into the core with an internal wave function *Φg and* one-nucleon in the continuum state with asymptotic momentum, ***hk***, elative to the core. It is assumed that the core remains in its ground state, *g* = 0, during the collision, or in other words, the projectile has no particle-bound state

The one-nucleon removal cross section is separated into the elastic part (c = 0) and the inelastic part $(c \neq 0)$ as $\sigma_{-N} = \sigma_{-N}^{el} + \sigma_{-N}^{inel}$, where

$$\sigma_{-N}^{el} = \int db \left\{ \left\langle \varphi_0 \left| e^{-2Im\chi_{CT}(b_C) - 2Im\chi_{NT}(b_C + s)} \right| \varphi_0 \right\rangle - \left| \left\langle \varphi_0 \left| e^{i\chi_{CT}(b_C) + i\chi_{NT}(b_C + s)} \right| \varphi_0 \right\rangle \right|^2 \right\} \tag{15}$$

and

$$\sigma_{-N}^{inel} = \int db \left\langle \varphi_0 \left| e^{-2Im\chi_{CT}(b_C)} - e^{-2Im\chi_{CT}(b_C)-2Im\chi_{NT}(b_C - s)} \right| \varphi_0 \right\rangle \qquad (16)$$

The one-nucleon removal cross section for halo nuclei is approximately equal to the difference between the two reaction cross sections, σ_{reac}(P + T) − σ_{reac}(C + T).

3.0 Methodology

The *Cross Section Calculations in the Glauber Model* (CSC_GM) code is a Fortran 90 program that was originally run on UNIX operating system. The program is modified and run on Windows xp. The code calculates the cross sections of various reactions for a core plus one valence-nucleon system in the framework of the Glauber model. The cross sections which are available in this code are the total reaction cross section, the one-nucleon removal cross section. The input data specify the reaction system, the type of cross section, and the target and core densities. The input data, read from a file **csc.inp.** in Table 1, represents the input data for the $^{13}C+^{12}C$ reaction system in the format of the csc.inp file. The first line, according to this format, gives the mass numbers of the target, projectile and core (A_T, A_P, and A_C), the second line gives the charge numbers of those nuclei (Z_T, Z_P, and Z_C). The code assumes $A_P - A_T = 1$. For a proton target, A_T = 1 and Z_T = 1. The third line defines the incident energy of the projectile per nucleon (in MeV). The fourth line defines the parameters of the nucleon–nucleon profile function, Eq. (3): σ_{NN} (in fm2), α, and β (in fm2). The fifth line gives the orbital angular momentum of the valence nucleon. The sixth line gives the number of Gaussians used to fit the core and target densities, and the following lines give the coefficients *ci* and the ranges *ai (*in fm−2) as defined by Eq. (5). Results of the computations are written on the file **reac.out**

4.0 Results

4.1 Reaction cross sections for $^{13}C+^{12}C$

The ^{13}C nucleus is described with a ^{12}C + neutron system. The parameters of the profile function Eq. (4) are taken from Ref. [6]. Results of calculation at 800 MeV/nucleon are given in Table 2.

The cross sections calculated at different incident energies are compared with the experiment in Fig. 1. The experimental data are obtained from Ref. [7].

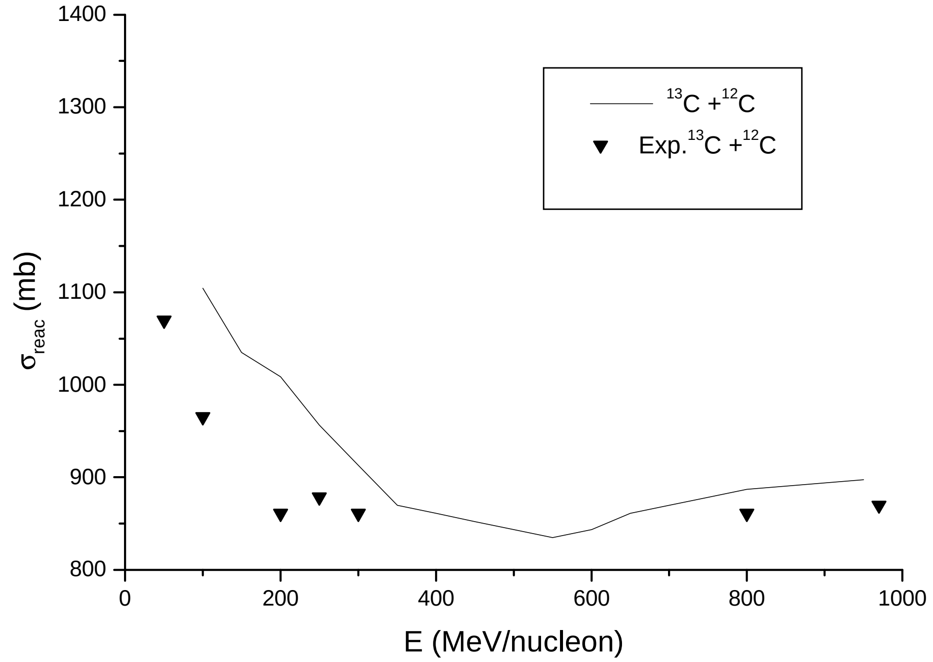

Fig. 1. Total reaction cross sections for $^{13}C+^{12}C$. Experimental data are taken from Refs. [7].

4.2 Cross sections for $^{19}C+^{12}C$

The energy dependence of the one-nucleon removal cross section for the $^{19}C+^{12}C$ system is also studied. The ground state of ^{19}C is assumed to be a ^{18}C + neutron system: The valence neutron is in the $1s_{1/2}$ orbit with the separation energy of ε = 0.24 MeV [8, 9]. Thus ^{19}C is considered to be a good example for a one-neutron halo nucleus. In fact, the rms radius of the neutron orbit becomes very large (8.5 fm). The density of ^{18}C is constructed from the harmonic-oscillator shell model. Fig. 2 shows the energy dependence of the elastic and inelastic one-nucleon removal cross sections. Note that the one-nucleon removal cross section is given by the sum of the two parts as discussed in section 3.2.

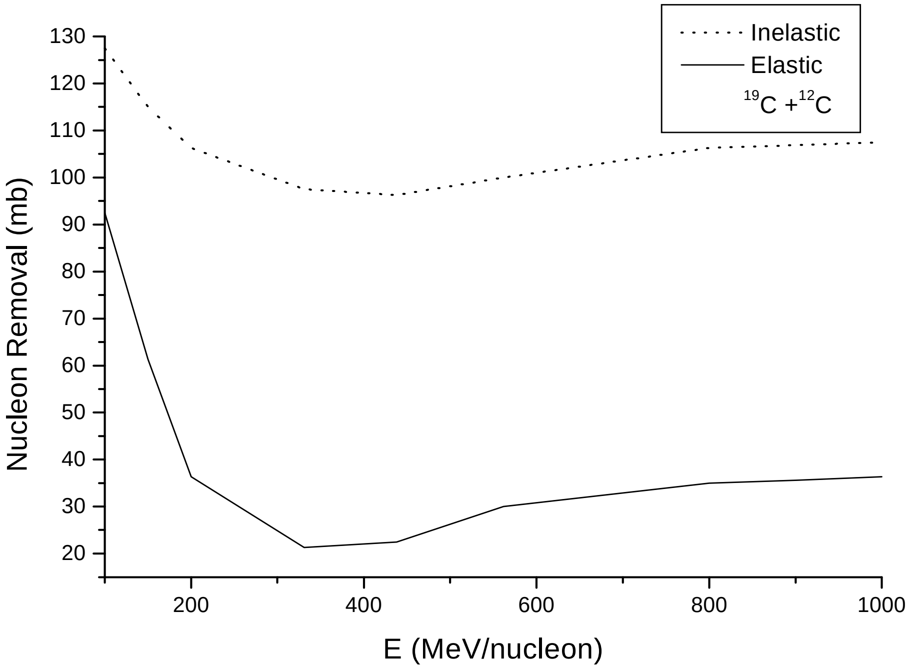

Fig.2. Energy dependence of the neutron-removal cross section for 19C+12C system. Both contributions of the elastic and inelastic processes are shown.

Table 1

csc.inp input file for the $^{13}C+^{12}C$ system

12.0	13.0	1.	A_T, A_P, A_C
6.0	6.0	6.0	Z_T, Z_P, Z_C
800			Incident energy per nucleon (in MeV)
4.26	-0.07	0.21008	Profile function ($\sigma_{NN}, \alpha, \beta,$ in fm^2)
1			*l* (orbital angular momentum)
2			Number of Gaussians used to fit the core and target densities
-1.23342874 1.38536085	0.462770142 0.373622826		Coefficient, range (in fm^{-2})

Table 2

Results **(reac.out)** for the $^{13}C+^{12}C$ system

projectile	mass = 13.0, charge = 6.0
target	mass = 12.0, charge = 6.0
Energy (MeV/nucleon	800
Proj. reaction cross section (mb)	924.6
Core reaction cross section (mb)	894.7
N-removal cross section (mb)	29.8
N-removal (elastic) (mb)	3.5
N-removal (inelastic) (mb)	26.2

5.0 Conclusion

A Fortran program was used to calculate various cross sections for reactions induced by the projectile which has a core plus one valence-nucleon structure in the framework of the Glauber theory. Results were compared with available data for the total reaction cross section, and the one-nucleon removal cross section.

The Glauber model is obviously suitable for high energy reactions. For reactions at lower energy, (less than a hundred MeV/nucleon), predictions of the Glauber model are rather poor. Contributing factors might be inappropriate choice of the effective interactions between the nucleon and the target for the low energy reactions. With appropriate choice of the interaction potential the code can be modified to replace χ_{CT} and χ_{NT} with the corresponding phase-shift functions constructed from the new potential.

Clearly the results for the $^{13}C+^{12}C$ reaction compared well with the experimental data especially at high energies. No much difference was observed between the experimental data for the $^{13}C+^{12}C$ and $^{13}C+^{12}C$ reactions.[10, 11] The ^{13}C nucleus thus shows no sign of halo or skin structures. This experimental observation is confirmed by the present calculations

The one-nucleon removal cross section in $^{19}C+^{12}C$ system is large reflecting the halo structure. The contribution from the inelastic breakup process is much larger than that from the elastic process in the energy region beyond a few hundred MeV per nucleon. The elastic breakup cross section is less than 30% of the total one-neutron removal cross section. For example, at incident energy of 800 MeV, the elastic and inelastic parts of the one-neutron removal cross section are 35.0 mb and 106.3 mb respectively, and the total one-neutron removal cross section is 141.3 mb. The high value of this cross section corresponds to a low binding energy for the last neutron in ^{19}C and a halo structure of the nucleus. No experimental data was found in available literatures for the $^{19}C+^{12}C$ system.

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