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A Note on the Mathematical Model on a Cigarette-Like Combustion

Waheed A.A and Ayandokun O.O.

Department of mathematics,
Emmanuel College of Education Oyo,Oyo State –Nigeria.

Abstract

This paper presents a mathematical model of a cigarette-like combustion using high activation energy asymptotic of particular interest are question of existence and uniqueness. It is shown that the steady temperature increase as the Frank-Kamenetskii $\boldsymbol{\lambda_1}$ ***increases.***

1.0 Introduction

Many scientists have contributed to the theory of combustion. Some have done a lot particularly on the criteria for existence and uniqueness of solution of some combustion problems [1-9].

Ayeni and Agusto [1] investigated the existence and uniqueness of a solution of self-similar diffusion equations. They examined the models of fast flow of a gas in a porous medium and microwaves heating of various materials. They gave the criteria for the existence and uniqueness of solution of the problem. Koriko and Ayeni [6] investigated the existence and unique of a system of partial differential equations arising from cigarette-like combustion.

Their results are useful in making deductions on porous medium combustion. Olanrewaju and Ayeni [8] investigated the effect of Frank-Kamenetskii parameter on strong detonations in converging vessels in which they established the fact that there is an appreciable difference in the temperature along the vessel when Frank-Kamenetskii parameters differ only by 1/30. In this paper, we extend the model of Koriko and Ayeni [6] to a situation when the Frank-Kamenetskii λ_1 and λ_2 are not equal.

2.0 Mathematical Formulation

A cigarette-like combustion is a two stage reaction [6] the two stages are first the diffusion of gas (air) between the gas mainstream and the reaction sites in the solid and secondly the conventional Arrhenius reaction.

$$\frac{Kd^2T}{dx^2} + \frac{(T-T_0)^n QAe^{-E/RT}}{a+be^{-E/RT}} = 0 \qquad (1)$$

As boundary conditions we take $T(-L) = T(L) = T_o$ (2)

Where:

T is the temperature of the medium,
T_o is the initial temperature,
K is the thermal conductivity,
Q is the combustion energy,
A is the pre-exponential factor,
E_1 is the activation energy of the first stage ,
E is the activation energy of the second reaction,
a and b are permeability constants(dimensionless),
n is the other of reaction.

We non-dimensionalize the temperature by writing:

$T = \frac{E}{RT_o}(T - T_o),\ y = \frac{x}{L}$ and we assumed $E = E_1$

$$\frac{d^2\theta}{dy^2} + \frac{\lambda_1\theta^n e^{\frac{\theta}{1+\varepsilon\theta}}}{1+\lambda_2 e^{\frac{\theta}{1+\varepsilon\theta}}} \qquad (3)$$

$$\theta(-1) = \theta(1) = 0 \qquad (4)$$

Where:

Corresponding author: ***Waheed A. A.***, E-mail: Tel. +2347031199441

$\lambda_1 = \frac{L^2 \varepsilon^n Q\, A\, e^{-\frac{E}{RT_o}}}{a}$ $-$ Frank-Kamenetskill

$\lambda_2 = \frac{b}{a}\, e^{-\frac{E}{RT_o}}$ $-$ Permeability Parameter

$\varepsilon = \frac{RT_o}{E}$ $-$ Dimensionless Parameter

$L =$ half length of the medium

In the this paper, $n = 0, E = E_1$. The case $n \neq 0, E \neq E_1$ is the subject of another paper.

Method of Solution

We let:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} y \\ \theta \\ \theta' \end{pmatrix} \quad (5)$$

Then we obtain

$$\begin{pmatrix} {x_1}' \\ {x_2}' \\ {x_3}' \end{pmatrix} = \begin{pmatrix} 1 \\ \theta' \\ \theta'' \end{pmatrix} \quad (6)$$

Where

$$\begin{pmatrix} {x_1}' \\ {x_2}' \\ {x_3}' \end{pmatrix} = \begin{pmatrix} 1 \\ x_3 \\ \frac{-\lambda_1\, e^{\frac{x_2}{1+\varepsilon x_2}}}{1+\lambda_2\, e^{\frac{x_2}{1+\varepsilon x_2}}} \end{pmatrix} \quad (7)$$

The initial conditions are:

$$\begin{pmatrix} x_1(-1) \\ x_2(-1) \\ x_3(-1) \end{pmatrix} = \begin{pmatrix} x_1(-1) \\ \theta(-1) \\ \theta(-1) \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ \beta \end{pmatrix} \quad (8)$$

Let

$$Z = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad (9)$$

and

$$E(z) = \begin{pmatrix} 1 \\ x_3 \\ \frac{-\lambda_1\, e^{\frac{x_2}{1+\varepsilon x_2}}}{1+\lambda_2\, e^{\frac{x_2}{1+\varepsilon x_2}}} \end{pmatrix}$$

Then $z^1 = E(z)$ and

$$z(-1) = \begin{pmatrix} -1 \\ 0 \\ \beta \end{pmatrix}$$

We guess β. We continue to change β. The required β is the one for which $\theta(1) = 0$ (shooting method). We present in the figure 1 the graph of θ against y for various values of λ_1.

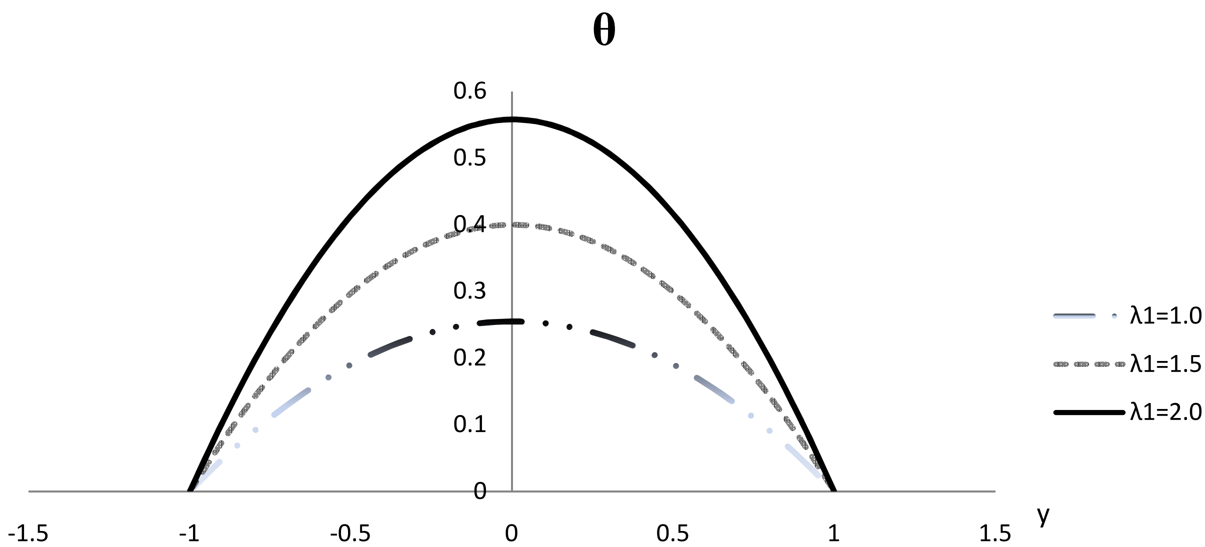

Figure 1: Graph of temperature θ(y) against position y for various values of λ_1 when λ_2=1.25, h=0.1 and ε =0.01.

3.0 Discussion of the Result

We observe that the maximum temperature increases with the increase in the frank-Kamenetskiis parameter λ_1. Koriko [5] showed there are multiple solutions when only the initial and end points are fixed. The figure 1 shows that there exists only one solution when the derivative $\theta^1(-1)$ is fixed. That is, the problem for which $\theta(-1)=0,\ \theta^1(-1)=\beta$ and $\theta(-1)=0$ is unique.

References

[1]Ayeni, R. O. and Agusto, F. B., On the existence and uniqueness of self-similar diffusion equations, *J. Nig. Ass. Maths Phys.* 4, (2000), 183 – 186.

[2]Ayeni, R. O., Thermal Runaway, Ph.D Thesis, Cornell University, U.S.A. (1978).

[3]Ayeni, R. O., Criteria for branched chain explosion, *Nig J. of Natural Sciences*, (1981), 45 – 50.

[4]Buckmaster and Ludford, G.S.S., Theory of laminar flames, Cambridge University Press, London, (1981).

[5]Koriko, O. K., *A mathematical model for a radial porous medium combustion*, Ph.D Thesis, Federal University of Technology, Akure, (1995).

[6] Koriko, O. K. and Ayeni, R. O., Existence and uniqueness of solution for a mathematical model for cigarette-like combustion, *African Journal of Science*, Vol 1.1, No. 1, (2001), 84 – 94.

[7]Okoya, S. S., Boundedness for a system of reaction diffusion equation, mathematika, 41,(1994), 293 – 300.

[8]Olanrewaju, P. O. and Ayeni, R. O., Effect of Frank-Kamenetskii parameter on strong detonation in a converging vessel, *Journal of Nigeria Association of Mathematical Physics, 4, (2006),67.*

[9] Waheed A.A., A Mathematical Model of a cigarette-like Combustion, M. Tech.Thesis,LAUTECH,Ogbomoso,(2005.