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Mathematical modeling and exact solution for unsteady MHD two phase flow: An open circuit case

Basant K. Jha And Haruna M. Jibril

Department of Mathematics, Faculty of Science,
Ahmadu Bello University, Zaria, NIGERIA

Abstract

In this work the analytical solutions for the time-dependent MHD two phase flows in parallel plate channel are obtained using the Green's function approach. The time dependent flow formation inside the channel is due to either by a sudden change in the imposed pressure gradient or (and) by a sudden change in the velocity of the channel boundaries. The two conducting regions are coupled by equating the velocity and stress at the interface. The mathematical model relevant to the problem is solved using the Green's function approach. Expressions for velocity in both regions are derived for general class of time dependent movement of boundaries or time dependent pressure gradient. As a special case, expressions for velocities in both the conducting phases are obtained due to sudden change in the constant pressure gradient.

Keywords*:* Magnetohydrodynamics (MHD), two-phase flow, Green's function, time dependent, parallel plates, open circuit, pressure gradient

2010 Mathematics Subject Classification 76W05

1. 0 Introduction

The study of two-phase MHD flow is of great importance because of its applications in several industrial and physical fields. These applications abide in the petroleum industries especially in petroleum extraction from crude oil. The interest in MHD (magneto-hydrodynamic) flow began in 1937, when Hartman [1] studied the influence of a transverse magnetic field on the flow of conducting fluid between two infinite parallel, stationary, and, insulated plates. Since then a lot of research work concerning the Hartmann flow was conducted under different physical situations [2-9].In recent past years there has been some theoretical and experimental work on the stratified laminar flow of two immiscible liquids in horizontal pipe. The interest in these types of problems stems from the possibility of reducing the power required to pump oil in a pipe line by suitable addition of water. Hartmann flow of conducting fluid in a channel between two horizontal insulating plates of infinite extent with a layer of non-conducting fluid between upper channel wall and conducting fluid has been studied by Shail [10]. He found that an increase of order 30% can be obtained in the flow rate for suitable ratios of depths and viscosities of the two fluids and realistic values of the Hartmann number. Lohrasbi and Sahai [11] studied the heat transfer aspect of problem [10] by taking in to account the effects of viscous and Joule dissipations. Malashetty and Leela [12-13], further extended the problem [11] by considering both phases as electrically conducting having different viscosities and electrical conductivities. They found that the effect of increasing the Hartmann number is to accelerate the velocity and to increase the temperature in case of open circuit case while the result is just reverse in case of short circuit case. Chamkha[14] discussed the flow of two-immiscible and electrically conducting fluids in porous and non-porous channels. He observed that increase in Hartmann number reduced flow velocities. Furthermore Umavathi *et al [15]* investigated the unsteady two-fluid flow and heat transfer in a horizontal channel. In another article Umavathi et al [16] investigated the unsteady Hartmann flow of two immiscible fluids through a horizontal channel with time dependent oscillatory wall transpiration velocity. Recently Zivojin et al [17] studied the MHD flow and heat transfer of two immiscible electrically conducting fluids between moving plates in the presence of an applied electric field.

Corresponding author: ***Haruna M. Jibril,*** E-mail: alharun2004@yahoo.comTel.: +2348037268879

In all the above discussed works the momentum and energy equations were solved under steady state operating condition. The lack of analytical solutions for time dependent fully developed MHD two phase flows in horizontal parallel plate channel, with different operating conditions, motivated the present work.

The purpose of this work is to present, in closed forms, transient fully developed MHD two phase flow solutions, corresponding to open circuit case, in a horizontal channel filled with different conducting fluids in the presence of uniform transverse magnetic field. The study of such flow gives the limiting conditions for MHD two phase developing flows and provides an analytical check on numerical solutions

2. GOVERNING EQUATIONS AND SOLUTIONS

A schematic diagram of problem under consideration is exhibited in Fig.1.The problem considers unsteady two phase MHD flow between two infinite horizontal parallel plates filled with two conducting fluids. The direction of the flow is taken along the x'-axis coinciding with the lower horizontal plate while y'-axis is taken normal to the flow direction. The regions $0 \le y \le d$ and $d \le y \le h$ are occupied by two different conducting fluids having different densities , ρ_1 and ρ_2, viscosities, μ_1 and μ_2, and electrical conductivities, σ_1 and σ_2 respectively. An external magnetic field of uniform strength B_0, is applied parallel to y'-axis. The magnetic field is assumed to be small so that the induced magnetic field can be neglected compared to the applied magnetic field. The unsteadiness in the fluid motion is caused either by a sudden change in the imposed pressure gradient along x'-axis or (and) by a sudden change in the velocity of the channel boundaries.

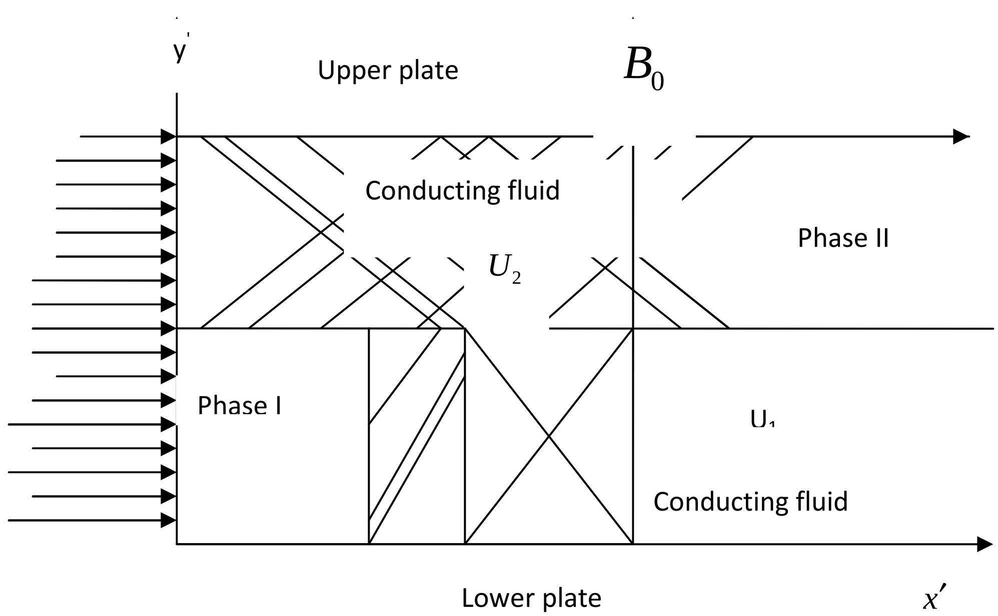

Fig.1 Physical configuration of the flow.

Under the assumptions discussed above the one dimensional time dependent flow for both phases in open circuit case are governed by the following equations:

$$\rho_1 \frac{\partial u_1}{\partial t} = -\frac{\partial p}{\partial x} + \mu_1 \frac{\partial^2 u_1}{\partial y^2} - B_0{}^2 \sigma \left[u_1 + \frac{E_0}{B_0} \right] \tag{1}$$

$$\rho_2 \frac{\partial u_2}{\partial t} = -\frac{\partial p}{\partial x} + \mu_2 \frac{\partial^2 u_2}{\partial y^2} - \delta B_0{}^2 \sigma \left[u_2 + \frac{E_0}{B_0} \right] \tag{2}$$

The Initial, boundary and interface conditions are:

$$u_1(0,y)=\overline{F_1}(y); \qquad u_2(0,y)=\overline{F_2}(y),$$
$$u_1(t,0)=f_1^*(\tau), \qquad u_2(t,H)=f_2^*(\tau), \tag{3}$$
$$u_1(t,1)=u_2(t,1); \ \frac{\partial u_1(t,1)}{\partial y}=\gamma\frac{\partial u_2(t,1)}{\partial y}, \tag{4}$$

Equations (1) - (4) can be written in a dimensionless form by using the non-dimensionless quantities

$$U_1=\frac{u_1 d}{\nu_1},\ U_2=\frac{u_2 d}{\nu_1},\ Y=\frac{y}{d},\ \mathrm{H}=\frac{h}{d},\ P=\frac{\partial p}{\partial x}\left(\frac{d^3}{\rho_1\nu_1^2}\right),\ M=B_0 d\sqrt{\frac{\sigma}{\mu_1}},\ \mathrm{Re}=\frac{E_0\rho_1 d}{B_0\mu_1},$$
$$f_1(t)=\frac{f_1^*(\tau)d}{\nu_1},\ f_2(t)=\frac{f_2^*(\tau)d}{\nu_1},\ F_1(Y)=\frac{\overline{F_1}(y)d}{\nu_1},\ F_2(Y)=\frac{\overline{F_2}(y)d}{\nu_1},\ t=\frac{\tau\nu_1}{d^2}$$

Equations (1) and (2) in dimensionless form are

$$\frac{\partial U_1}{\partial t}=-P+\frac{\partial^2 U_1}{\partial Y^2}-M^2[U_1+\mathrm{Re}] \tag{5}$$
$$\frac{\partial U_2}{\partial t}=-\alpha P+\gamma\alpha\frac{\partial^2 U_2}{\partial Y^2}-\alpha\delta M^2[U_2+\mathrm{Re}] \tag{6}$$

And the Initial, boundary (3) and interface conditions (4) in dimensionless form are:

$$U_1(0,Y)=F_1(Y), \quad U_2(0,Y)=F_2(Y),$$
$$U_1(t,0)=f_1(t), \qquad U_2(t,H)=f_2(t), \tag{7}$$
$$U_1(t,1)=U_2(t,1); \quad \frac{\partial U_1(t,1)}{\partial Y}=\gamma\frac{\partial U_2(t,1)}{\partial Y}, \tag{8}$$

Equations (7) and (8) represent the general nature of initial and boundary conditions. A specific situation can be handled by proper selection of initial and boundary conditions.

It is more convenient to solve the problem with homogeneous boundary conditions. To attain this let

$$U_i(Y,t)=V_i(Y,t)+\phi_i(Y)f_1(t)+\varepsilon_i(Y)f_2(t),\ i=1,2 \tag{9}$$

This relation transforms the governing equations (5) and (6) with the initial and boundary conditions (7) and (8) in to the following set of equations

$$\frac{d^2\phi_1}{dY^2}-M^2\phi_1=0 \tag{10}$$
$$\gamma\frac{d^2\phi_1}{dY^2}-\delta M^2\phi_1=0 \tag{11}$$

With initial and boundary conditions

$$\left.\begin{aligned}&\phi_1(0)=1; \qquad \phi_2(H)=0,\\ &\phi_1(1)=\phi_2(1), \quad \frac{d\phi_1(1)}{dY}=\gamma\frac{d\phi_2(1)}{dY}\end{aligned}\right\}. \tag{12}$$

And the differential equations

$$\frac{d^2\varepsilon_1}{dY^2}-M^2\varepsilon_1=0 \tag{13}$$

$$\gamma \frac{d^2\varepsilon_2}{dY^2} - \delta M^2 \varepsilon_2 = 0 \tag{14}$$

With the initial and Boundary conditions

$$\left.\begin{aligned} &\varepsilon_1(0) = 0; \qquad \varepsilon_2(H) = 1, \\ &\varepsilon_1(1) = \varepsilon_2(1), \quad \frac{d\varepsilon_1(1)}{dY} = \gamma \frac{d\varepsilon_2(1)}{dY} \end{aligned}\right\} . \tag{15}$$

$$\frac{\partial V_1}{\partial t} = -P_1 + \frac{\partial^2 V_1}{\partial Y^2} - M^2 V_1 \tag{16}$$

$$\frac{\partial V_2}{\partial t} = -P_2 + \gamma\alpha \frac{\partial^2 V_2}{\partial Y^2} - \alpha\delta M^2 V_2 \tag{17}$$

$$V_1(t,0) = 0,\ V_2(t,H) = 0, \quad V_1(t,1) = V_2(t,1)$$

$$\frac{dV_1(t,1)}{dY} = \mu_r \frac{dV_2(t,1)}{dY} \tag{18}$$

$$V_i(0,Y) = F_i(Y) - \phi_i(Y) f_1(0) - \varepsilon_i(Y) f_2(0) = F_i^*, i = 1,2 \tag{19}$$

$$P_1 = P + M^2 \,\mathrm{Re} + \phi_1 \frac{df_1}{dt} + \varepsilon_1 \frac{df_2}{dt} \tag{20}$$

$$P_2 = \left[P\alpha + \alpha\delta M^2 \,\mathrm{Re}\right] + \phi_2 \frac{df_1}{dt} + \varepsilon_2 \frac{df_2}{dt} \tag{21}$$

The solutions of equation (10 -11) are

$$\phi_1(Y) = A_1 Exp(MY) + A_2 Exp(-MY) \tag{22}$$

$$\phi_2(Y) = A_3 Exp\left(MY\sqrt{\frac{\delta}{\gamma}}\right) + A_4 Exp\left(-MY\sqrt{\frac{\delta}{\gamma}}\right) \tag{23}$$

where A_1, A_2, A_3, and A_4 are found from the solution of the following set of equations:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ Exp(M) & Exp(-M) & -Exp\left(M\sqrt{\frac{\delta}{\gamma}}\right) & -Exp\left(M\sqrt{\frac{\delta}{\gamma}}\right) \\ Exp(M) & -Exp(-M) & -\sqrt{\gamma\delta}Exp\left(M\sqrt{\frac{\delta}{\gamma}}\right) & \sqrt{\gamma\delta}Exp\left(-M\sqrt{\frac{\delta}{\gamma}}\right) \\ 0 & 0 & Exp\left(MH\sqrt{\frac{\delta}{\gamma}}\right) & Exp\left(-MH\sqrt{\frac{\delta}{\gamma}}\right) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{24}$$

Also, the solutions of equation (13-14) are

$$\varepsilon_1(Y) = A_5 Exp(MY) + A_6 Exp(-MY) \tag{25}$$

$$\varepsilon_2(Y) = A_7 Exp\left(MY\sqrt{\frac{\delta}{\gamma}}\right) + A_8 Exp\left(-MY\sqrt{\frac{\delta}{\gamma}}\right) \tag{26}$$

where $A_5, A_6, A_7,$ and A_8 are found from the solution of the following set of equations:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ Exp(M) & Exp(-M) & -Exp\left(M\sqrt{\frac{\delta}{\gamma}}\right) & -Exp\left(M\sqrt{\frac{\delta}{\gamma}}\right) \\ Exp(M) & -Exp(-M) & -\sqrt{\gamma\delta}Exp\left(M\sqrt{\frac{\delta}{\gamma}}\right) & \sqrt{\gamma\delta}Exp\left(-M\sqrt{\frac{\delta}{\gamma}}\right) \\ 0 & 0 & Exp\left(MH\sqrt{\frac{\delta}{\gamma}}\right) & Exp\left(-MH\sqrt{\frac{\delta}{\gamma}}\right) \end{bmatrix} \begin{bmatrix} A_5 \\ A_6 \\ A_7 \\ A_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{27}$$

Using Green's function approach, Equation (16) and (17) assumes the following form:

$$\begin{aligned} V_1(t,Y) = & \int_0^1 G_{11}(Y,t/Y',t')_{t'=0} F_1^*(Y')dY' \\ & - \int_{t'=0}^{t} dt' \int_0^1 G_{11}(Y,t/Y',t')(P_1)dY' \\ & + \int_1^H G_{12}(Y,t/Y',t')_{t'=0} F_2^*(Y')dY' \\ & - \int_{t'=0}^{t} dt' \int_1^H G_{12}(Y,t/Y',t')(P_2)dY' \end{aligned} \tag{28}$$

$$\begin{aligned} V_2(t,Y) = & \int_0^1 G_{21}(Y,t/Y',t')_{t'=0} F_1^*(Y')dY' \\ & - \int_{t'=0'}^{t} dt' \int_0^1 G_{21}(Y,t/Y',t')(P_1)dY' \\ & + \int_1^H G_{22}(Y,t/Y',t')_{t'=0} F_2^*(Y')dY' \\ & - \int_{t'=0}^{t} dt' \int_1^H G_{22}(Y,t/Y',t')(P_2)dY' \end{aligned} \tag{29}$$

Where G_{ij} is the appropriate Green's function found from the homogeneous version of the governing equations (12-15) are:

$$G_{ij}\left(Y,t/Y^{'},t^{'}\right)=\sum_{n=1}^{\infty}\frac{Exp\left(-\beta_n^2\left(t-t^{'}\right)\right)}{N_n}\eta_i\psi_{in}(Y)\psi_{jn}\left(Y^{'}\right), \ .$$

where

$$\text{i}=1,2 \ \& \ \text{j}=1,2, \eta_1=1, \eta_2=\frac{1}{\alpha}. \tag{30}$$

$$N_n=\int_0^1\psi_{1n}^2\left(Y^{'}\right)dY^{'}+\left(\frac{1.}{\alpha}\right)\int_1^H\psi_{2n}^2\left(Y^{'}\right)dY^{'} \tag{31}$$

$$\psi_{1n}(Y)=Sin(C_nY), \text{ where } C_n^2=\beta_n^2-M^2 \tag{32}$$

$$\psi_{2n}=A_{2n}Cos(\lambda_nY)+B_{2n}Sin(\lambda_nY), \text{ where } \lambda_n^2=\left[\frac{\beta_n^2}{\alpha\gamma}-\frac{\delta M^2}{\gamma}\right] \tag{33}$$

The values of constant A_{2n} and B_{2n} are found from the solution of following set of equations

$$\begin{bmatrix} Cos(\lambda_n) & Sin(\lambda_n) \\ -\gamma\lambda_n Sin(\lambda_n) & \gamma\lambda_n Sin(\lambda_n)\end{bmatrix}\begin{bmatrix} A_{2n} \\ B_{2n}\end{bmatrix}=\begin{bmatrix} Sin(C_n) \\ C_nCos(C_n)\end{bmatrix} \tag{34}$$

The eigen-values λ_n and β_n are found as the roots of

$$A_{2n}Cos(H\lambda_n)+B_{2n}Sin(H\lambda_n)=0. \tag{35}$$

3. Particular Case

The physical situation, in which flow inside the channel is solely caused by uniform pressure gradient, i.e., $F_1(Y)=F_2(Y)=f_1(t)=f_2(t)=0$, which yields:

$$P_1=\left[P+M^2\,\text{Re}\right], P_2=\left[\alpha P+\alpha\delta M^2\,\text{Re}\right] \text{and } U_i=V_i, i=1,2.$$

Using these values in equations (22) and (23) the dimensional velocity in both phases are:

$$U_1(Y,t)=-\int_{t^{'}}^{t}dt^{'}\int_0^1 G_{11}\left(Y,t/Y^{'},t^{'}\right)\psi_{1n}(Y)\psi_{1n}\left(Y^{'}\right)\left(P+M^2\,\text{Re}\right)dY^{'}$$
$$-\int_{t^{'}}^{t}dt^{'}\int_1^H G_{12}\left(Y,t/Y^{'},t^{'}\right)\psi_{1n}(Y)\psi_{2n}\left(Y^{'}\right)\left(P+\delta M^2\,\text{Re}\right)dY^{'} \tag{36}$$

$$U_2(Y,t) = -\int_{t'}^{t} dt' \int_0^1 G_{21}(Y,t/Y',t')\psi_{2n}(Y)\psi_{1n}(Y')(P+M^2\,\mathrm{Re})dY'$$

$$-\int_{t'}^{t} dt' \int_1^H G_{22}(Y,t/Y',t')\psi_{2n}(Y)\psi_{2n}(Y')(P+\delta M^2\,\mathrm{Re})dY' \tag{37}$$

$$U_1(Y,t) = -(P+M^2\,\mathrm{Re})\sum_{n=1}^{\infty}\frac{\left[1.-Exp\left(-\beta_n^2 t\right)\right]}{\beta_n^2 N_n C_n}Sin(C_n Y)[1.-Cos(C_n)]$$

$$-(P+\delta M^2\,\mathrm{Re})\sum_{n=1}^{\infty}\frac{\left[1.-Exp\left(-\beta_n^2 t\right)\right]}{\beta_n^2 N_n \lambda_n}Sin(C_n Y)$$

$$[A_{2n}\{Sin(\lambda_n H)-Sin(\lambda_n)\}-B_{2n}\{Cos(\lambda_n H)-Cos(\lambda_n)\}] \tag{38}$$

$$U_2(Y,t) = -(P+M^2\,\mathrm{Re})\sum_{n=1}^{\infty}\frac{\left[1.-Exp\left(-\beta_n^2 t\right)\right][A_{2n}Cos(C_n Y)+B_{2n}Sin(C_n Y)][1.-Cos(C_n)]}{\beta_n^2 N_n C_n}$$

$$-(P+\delta M^2\,\mathrm{Re})\sum_{n=1}^{\infty}\frac{\left[1.-Exp\left(-\beta_n^2 t\right)\right][A_{2n}Cos(C_n Y)+B_{2n}Sin(C_n Y)]}{\beta_n^2 N_n \lambda_n}$$

$$[A_{2n}\{Sin(\lambda_n H)-Sin(\lambda_n)\}-B_{2n}\{Cos(\lambda_n H)-Cos(\lambda_n)\}] \tag{39}$$

$$N_n = \int_0^1 \psi_{1n}^2(Y')dY' + \left(\frac{1.}{\alpha}\right)\int_1^H \psi_{2n}^2(Y')dY'$$

$$= \left[\frac{Y'}{2} - \frac{Sin(2C_n Y')}{4}\right]_0^1 + \left(\frac{A_{2n}^2}{\alpha}\right)\left[\frac{Y'}{2} - \frac{Sin(2\lambda_n Y')}{4\lambda_n}\right]_1^H$$

$$+\left(\frac{B_{2n}^2}{\alpha}\right)\left[\frac{Y'}{2} - \frac{Sin(2\lambda_n Y')}{4\lambda_n}\right]_1^H - \left(\frac{A_{2n}B_{2n}}{\alpha\lambda_n}\right)\left[Cos(2\lambda_n Y')\right]_1^H \tag{40}$$

4. STEADY STATE SOLUTION

$$\frac{\partial^2 U_1}{\partial Y^2} - M^2 U_1 = P + M^2\,\mathrm{Re}\ , \tag{41}$$

$$\gamma \frac{\partial^2 U_2}{\partial Y^2} - M^2 U_2 = P + \mathrm{Re}\, \delta M^2 , \tag{42}$$

$$U_1 = C_1 Sinh\left(MY\right) - \frac{\left[P + M^2 \,\mathrm{Re}\right]}{M^2}\left[Cosh\left(MY\right) - 1.\right] , \tag{43}$$

$$U_2 = C_2 Cosh\left(MY\sqrt{\frac{\delta}{\gamma}}\right) + C_3 Sinh\left(MY\sqrt{\frac{\delta}{\gamma}}\right) - \frac{\left[P + \mathrm{Re}\,\delta M^2\right]}{M^2 \delta} , \tag{44}$$

Where

C_1, C_2 and C_3 are found from the solution of following set of equations:

$$\begin{bmatrix} 0 & Cosh\left(MH\sqrt{\frac{\delta}{\gamma}}\right) & Sinh\left(MH\sqrt{\frac{\delta}{\gamma}}\right) \\ -Sinh\left(M\right) & Cosh\left(M\sqrt{\frac{\delta}{\gamma}}\right) & Sinh\left(M\sqrt{\frac{\delta}{\gamma}}\right) \\ -MCosh\left(M\right) & M\sqrt{\gamma\delta}\, Sinh\left(M\sqrt{\frac{\delta}{\gamma}}\right) & M\sqrt{\gamma\delta}\, Cosh\left(M\sqrt{\frac{\delta}{\gamma}}\right) \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} \tag{45}$$

where

$$Z_1 = \frac{P + M^2 \delta \,\mathrm{Re}}{M^2 \delta}$$

$$Z_2 = \left[\frac{P + M^2 \,\mathrm{Re}}{M^2}\right]\left[Cosh(M) - 1.\right] + Z_1$$

$$Z_3 = \left[\frac{P + M^2 \,\mathrm{Re}}{M}\right] Sinh(M)$$

5. Concluding Remarks

The novel feature of the work is to present an analytical solution of unsteady MHD two phase flows in an open circuit case where both phases are electrically conducting using Green’s function approach.

Nomenclature

d = width of the non-conducting fluid in phase1
h = total width of channel
H = dimensionless total width of the channel $\left(\frac{h}{d}\right)$

B_0 = magnetic field strength

E_0 = Constant electric field along z -direction

u_1 = velocity field of conducting fluid in Phase I

u_2 = velocity field of conducting fluid in phase II

U_1 = dimensionless velocity field of conducing fluid in phase I $\left(\frac{u_1 d}{\nu_1}\right)$

U_2 =dimensionless velocity field of conducting fluid in phase II $\left(\frac{u_2 d}{\nu_1}\right)$

y =dimensional transverse co-ordinate

Y = dimensionless transverse co-ordinate $\left(\frac{y}{d}\right)$

$\frac{\partial p}{\partial x}$ =dimensional axial pressure gradient

P =dimensionless axial pressure gradient $\left(\frac{\partial p}{\partial x}\left(\frac{d^3}{\rho_1 \nu_1^2}\right)\right)$

M =Hartman number $\left(B_0 d\sqrt{\frac{\sigma}{\mu_1}}\right)$

$\mathrm{Re} =$ Electric load parameter $\left(\frac{E_0 \rho_1 d}{B_0 \mu_1}\right)$

$f_1^*(\tau)$ = velocity of lower bounding wall

$f_2^*(\tau)$ = velocity of upper bounding wall

$\overline{F_1}(y)$ = initial velocity in non-conducting fluid

$\overline{F_2}(y)$ = initial velocity in conducting fluid

$f_1(t)$ =dimensionless velocity of lower bounding wall $\left(\frac{f_1^*(\tau)d}{\nu_1}\right)$

$f_2(t)$ =dimensionless velocity of upper bounding wall $\left(\frac{f_2^*(\tau)d}{\nu_1}\right)$

$F_1(Y)$ =dimensionless initial velocity field of conducting fluid in phase I $\left(\frac{\overline{F_1}(y)d}{\nu_1}\right)$

$F_2(Y)$ =dimensionless initial velocity field of conducting fluid in phase II $\left(\frac{\overline{F_2}(y)d}{\nu_1}\right)$

t =dimensionless time $\left(\frac{\tau\nu_1}{d^2}\right)$

Greek Symbols

τ = time

μ =dynamic viscosity

ν = kinematic viscosity

ρ =density of fluid

σ =electrical-conductivity of conducting fluid

γ =ratio of dynamic viscosities $\left(\frac{\mu_2}{\mu_1}\right)$

α =ratio of densities of fluids $\left(\frac{\rho_1}{\rho_2}\right)$

δ =ratio of electrical conductivities of fluids $\left(\frac{\sigma_2}{\sigma_1}\right)$

Subscripts
1 = conducting fluid domain phase I
2 = conducting fluid domain phase II

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