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Testing for a Sensitivity Analysis in the Biogas Production Policies

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Abstract

Scholars have identified sensitivity analysis as an integral component of model development which has several interesting petroleum and chemical engineering applications particularly in the biogas production sector. This numerical tool has a long history of neglect in Nigeria. For the first time, we propose this powerful scientific technique which can be utilized to tackle the expected problems of parameter estimation, data collection, model validation and policy formulations in biogas production. The model parameter when varied a little one-at-a-time which will produce a big cumulative effect on the solution trajectories can be considered as a most sensitive parameter. In this context, we have observed that the model parameters based on our range of variations are to be considered as all relatively equally sensitive or relatively equally least sensitive. The implications of our novel contributions in the biogas production have the capability to propel stakeholders to identify the role of effective implementation of data collection in order to improve the quality of biogas production as one of the means of sustaining national development in Nigeria.

Keywords: Sensitivity Analysis, Biogas Production, Policies

1.0 Introduction

The notion of a sensitivity analysis in parameter estimation, model validation and related policies can have diverse interesting applications in several aspects of natural sciences, physical sciences and engineering to mention a few [1- 11]. Sensitivity analysis measures the relationship between a model parameter and its corresponding impact on the solution trajectory due to a variation of a parameter one-at-a-time. It is an integral component of model development. In this talk, we propose to apply the notion of a sensitivity analysis in order to quantify the relative importance of one parameter over the other parameter in the context of biogas production [11]. Other related cited references and data on biogas production in sub-Saharan Africa, Nigeria and Asian countries can be obtained and read from the classic work of Iyagba et al. [11].

2.0 Materials and Methods

The method of sensitivity analysis has been clearly defined, discussed and applied in our most recent work [2]. A detailed description of the algorithm which clearly defines the method of sensitivity analysis can be read in one of our papers in the present volume of this journal.

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In this paper, this numerical technique is applied to analyse an interaction model between two components of biogas solids [11] which we have successfully constructed. This system of continuous first order ordinary differential equations does not admit a closed-form solution but involves a sophisticated MATLAB programming with which this interesting environmental physics related problem was solved within our multi-disciplinary skills. Considering the fact that biogas production is an alternative source of energy makes it imperative to involve the joint application of chemical engineering, statistics and mathematics, computer and environmental physics in order to provide a sustainable effort of tackling this scientific problem.

The complex interaction between two biogas solids can be described using a Lotka-Volterra type of model equations which has the following mathematical structure

$$dB_1(t)/dt = B_1(t)(a-bB_1(t)-cB_2(t)) \qquad (1)$$

$$dB_2(t)/dt = B_2(t)(d-eB_1(t)-fB_2(t)) \qquad (2)$$

Here, $B_1(t)$ and $B_2(t)$ define the populations of two biogas solids at time t in days. The model parameters a, d, b, f, c, and e are assumed to be positive constants which we will specify their precise values later in this paper. The initial conditions or the starting biomasses for these biogas solids are specified by $B_1(0)$ and $B_2(0)$ which are also assumed to be positive.

In the work of Iyagba et al. [11], the data of bio gas production were obtained but the model which describes the interaction between two solids of biogas was not constructed. The proposed model in equations 1 and 2 is called a system of continuous nonlinear first order ordinary differential equations of the Lotka-Volterra type in the mathematical ecology literatures. Without the self-interaction or intraspecific interaction specified by the terms b $B_1(t)$ $B_1(t)$ and f $B_2(t)$ $B_2(t)$ as well as the interspecific interaction terms c $B_1(t)$ $B_2(t)$ and $eB_1(t)$ $B_2(t)$, the rate of change equations using the theory of calculus will grow exponentially obeying the Malthus growth law which does not provided a meaningful scientific insight because the two biogas solids in question will tend to compete for limited resources in the environment. The inclusion of both the intraspecific and interspecific terms acts to inhibit the growth of these two biogas solids. It is worth mentioning that the above system of equations does not generally have closed-form solutions but do have four steady-state solutions [2].

The model parameters a and d define the growth rates for the $B_1(t)$ and $B_2(t)$ populations. The parameters b and f define the logistic parameters otherwise called the intraspecific or self-interaction coefficients which inhibit the popular Maltusian growth whereas the parameters c and e are called the interspecific coefficients. The parameter c is the effort of the second biogas population to inhibit the growth of the first biogas population while the parameter e is the effort of the first biogas population to inhibit the growth of the second biogas population. The interspecific coefficients clearly obey the popular mass action law which has its root in molecular physics.

The growth rate parameters and the intraspecific coefficients were determined from the experimental data collected by Iyagba et al [11]. On the basis of these time series biogas production data, the values of a and d were calculated to be 0.1067 ml/g TS and 0.1064 ml/gVS; the values of b and f were calculated to be 0.0099 and 0.0079 with similar units. Under some simplifying assumptions that the precise values of the interspecific coefficients must be relatively smaller than the values of the intraspecific coefficients, we have chosen the values of c and e to be 0.01 and 0.0002. The initial conditions or initial data for this system of model equations were chosen to be 1.2 ml/g TS and 1.5 ml/gVS over a biogas production period of 100 days.

Our sensitivity analysis of these model parameters which is based on these parameter estimations are displayed in the series of Tables below:

Discussion of ODE45 Sensitivity Analysis for Biogas Production in Nigeria

For the first time in Nigeria and in the context of simulation sensitivity analysis, we hereby present the following results which we have not seen elsewhere with the expectation of providing sufficient insights which are capable of guiding stakeholders who provide funding and make policies about biogas production in Nigeria. Our results in terms of 1-norm, 2-norm and infinity-norm will be presented in the form of rows. For example, our ODE45 1-norm sensitivity values when a model

parameter is varied one-at-a-time by 1 percent, 2 percent, 3 percent, 4 percent and 5 percent will be shown in the form of five co-ordinates (mp_{1p}, mp_{2p}, mp_{3p}, mp_{4p}, mp_{5p}) where mp_{1p} represents a sensitivity value when a model parameter is varied by 1 percent, mp_{2p} represents a sensitivity value when a model parameter is varied by 2 percent, mp_{3p} represents a sensitivity value when a model parameter is varied by 3 percent, mp_{4p} represents a sensitivity value when a model parameter is varied by 4 percent and mp_{5p} represents a sensitivity value when a model parameter is varied by 5 percent.

TABLE 1: ODE45 Sensitivity analysis of parameter a = 0.1067 ml/g TS

Mathematical Norms	a=0.0011	a=0.0021	a=0.0032	a=0.0043	a=0.0053
1-norm	93.34	93.06	92.76	92.46	92.14
2-norm	93.97	93.70	93.42	93.12	92.81
Infinity-norm	99.12	98.88	98.62	98.34	98.04

We observe from Table 1 that a variation of parameter a will produce 1-norm sensitivity values of 93.34, 93.06, 92.76, 92.46 and 92.14, 2-norm sensitivity values of 93.97, 93.70, 93.42, 93.12 and 92.81 and the infinity-norm sensitivity values of 99.12, 98.88, 98.62, 98.34 and 98.04. We observe from these ODE 45 calculated sensitivity values that the intrinsic growth a of the first biogas solid can be considered as a more sensitivity parameter. On the basis of our sensitivity values for the parameters a, $B_1(0)$ and $B_2(0)$, we observe that the sensitivity values for parameter a outweigh the sensitivity values for the two initial condition parameters. This result is consistent with other related scientific results which have reported that the intrinsic growth rate a is associated with the carrying capacity which defines the maximum population size for a growing population. This observation can also be applied in a growing population of biogas solids. Therefore, the parameter a is expected to be relatively more sensitive than the initial condition parameters.

TABLE 2: ODE45 Sensitivity analysis of parameter $B_1(0)$ = 1.2 ml/g TS

Mathematical Norms	$B_1(0)$= 0.0120	$B_1(0)$ = 0.0240	$B_1(0)$ =0.0360	$B_1(0)$= 0.0480	$B_1(0)$ = 0.060
1-norm	61.18	52.22	46.90	43.09	40.13
2-norm	65.30	57.61	52.81	49.25	46.40
Infinity-norm	86.99	80.96	76.50	72.84	69.72

In this scenario, Table 2 shows that a variation of parameter IC_1 will produce 1-norm sensitivity values of 61.18, 52.22, 46.90, 43.09 and 40.13, 2-norm sensitivity values of 65.30, 57.61, 52.81, 49.25 and 46.40 and the infinity-norm sensitivity values of 86.99, 80.96, 76.50, 72.84 and 69.72. In summary, the model parameter $B_1(0)$ can be considered as a relatively sensitive parameter when compared with the two intrinsic growth rates parameters.

TABLE 3: ODE45 Sensitivity analysis of parameter $B_2(0)$ = 1.5 ml/gVS

Mathematical Norms	$B_2(0)$=0.015	$B_2(0)$ = 0.030	$B_2(0)$ = 0.045	$B_2(0)$ =0.060	$B_2(0)$ = 0.075
1-norm	62.41	53.18	47.73	43.85	40.83
2-norm	61.95	54.52	49.89	46.47	43.73
Infinity-norm	94.66	87.46	82.26	78.06	74.51

In this scenario, Table 3 shows that a variation of parameter IC_2 will produce 1-norm sensitivity values of 62.41, 53.18, 47.73, 43.85 and 40.83, 2-norm sensitivity values of 61.95, 54.52, 49.89, 46.47 and 43.73 and the infinity-norm sensitivity values of 94.66, 87.46, 82.26, 78.06 and 74.51. In summary, the sensitivity values of the first initial condition parameter slightly differ from the sensitivity values of the second initial condition parameter. Therefore, the parameters $B_1(0)$ and $B_2(0)$ can be considered as relatively and equally sensitive parameters. It is interesting to mention that the sensitivity of these parameters presupposes their relative contribution in the growth of these two biogas solids as starting weights of the biogas solids which we have chosen to make sense with the original data of Iyagba et al. [11].

TABLE 4: ODE45 Sensitivity analysis of parameter d = 0.1064 ml/gVS

Mathematical Norms	d = 0.0011	d = 0.0021	d = 0.0032	d = 0.0043	d = 0.0053
1-norm	101.63	101.24	100.84	100.41	99.97
2-norm	93.32	92.97	92.60	92.22	91.81
Infinity-norm	108.50	108.03	107.56	107.05	106.51

In this scenario, Table 4 shows that a variation of parameter d will produce 1-norm sensitivity values of 101.63, 101.24, 100.84, 100.41 and 99.97, 2-norm sensitivity values of 93.32, 92.97, 92.60, 92.22 and 91.81 and the infinity-norm sensitivity values of 108.50, 108.03, 107.56, 107.05 and 106.51. These sensitivity values clearly show that the parameter d can be considered as another relatively more sensitive parameter than the two initial condition parameters $B_1(0)$ and $B_2(0)$. In terms of the sensitivity of the intrinsic growth rate parameters and the sensitivity of the initial condition parameters, we have observed on the basis of this analysis that the sensitivity of the intraspecific coefficients b and f can be categorised as more sensitive [see Table 5 and Table 6].

TABLE 5: ODE45 Sensitivity analysis of parameter b = 0.0099 ml/gTS

Mathematical Norms	b = 0.000099	b = 0.000198	b = 0.000297	b = 0.000396	b = 0.000495
1-norm	4326	2519.9	1804.3	1413.0	1163.5
2-norm	6071	3308.2	2286.6	1749.9	1417.1
Infinity-norm	10804	5315.3	3461.6	2544.9	2001.1

In this scenario, Table 5 shows that a variation of parameter b will produce 1-norm sensitivity values of 4326, 2519.9, 1804.3, 1413.0 and 1163.5, 2-norm sensitivity values of 6071, 3308.2, 2286.6, 1749.9 and 1417.1 and the infinity-norm sensitivity values of 10804, 5315.3, 3461.6, 2544.9 and 2001.1.

TABLE 6: ODE45 Sensitivity analysis of parameter f = 0.0079 ml/gVS

Mathematical Norms	f = 0.000079	f = 0.000158	f = 0.000237	f = 0.000316	f = 0.000395
1-norm	4591.7	2686.5	1938.8	1529.4	1267.8
2-norm	6183.8	3391.9	2356.5	1808.4	1466.8
Infinity-norm	9958.2	5034.4	3368.7	2531.1	2027.0

Table 6 shows that a variation of parameter f will produce 1-norm sensitivity values of 4591.7, 2686.5, 1938.8, 1529.4 and 1267.8, 2-norm sensitivity values of 6183.8, 3391.9, 2356.5, 1808.4 and 1466.8 and the infinity-norm sensitivity values of 9958.2, 5034.4, 3368.7, 2531.1 and 2027.0. The sensitivity values of the intraspecific coefficients b and f are clearly bigger than the sensitivity values of the interspecific coefficients c and e [see Table 7 and Table 8].

TABLE 7: ODE45 Sensitivity analysis of parameter c = 0.001

Mathematical Norms	c = 0.00001	c = 0.00002	c = 0.00003	c = 0.00004	c = 0.00005
1-norm	12.24	12.12	11.99	11.87	11.75
2-norm	12.92	12.80	12.66	12.52	12.40
Infinity-norm	14.26	14.11	13.97	13.82	13.68

Table 7 shows that a variation of parameter c will produce 1-norm sensitivity values of 12.24, 12.12, 11.99, 11.87 and 11.75, 2-norm sensitivity values of 12.92, 12.80, 12.66, 12.52 and 12.40 and the infinity-norm sensitivity values of 14.26, 14.11, 13.97, 13.82 and 13.68. Comparing the sensitivity values of the interspecific coefficients c and e, we observe that the parameter c can be considered more as a slightly least sensitive than the parameter e which we can say that it is the most least sensitive [Table 7 and Table 8].

TABLE 8: ODE45 Sensitivity analysis of parameter e = 0.0002

Mathematical Norms	e = 0.000002	e = 0.000004	e = 0.000006	e = 0.000008	e = 0.000010
1-norm	1.7952	1.7771	1.7589	1.7408	1.7227
2-norm	1.7141	1.6968	1.6795	1.6622	1.6449
Infinity-norm	2.0418	2.0212	2.0006	1.9800	1.9595

Table 8 shows that a variation of parameter e will produce 1-norm sensitivity values of 1.7952, 1.7771, 1.7589, 1.7408 and 1.7227, 2-norm sensitivity values of 1.7141, 1.6968, 1.6795, 1.6622 and 1.6449 and the infinity-norm sensitivity values of 2.0418, 2.0212, 2.0006, 1.9800 and 1.9595. On the basis of these calculations, we observe that the interspecific coefficient parameter e has the most lower sensitivity values.

TABLE 9: ODE45 Sensitivity analysis of parameter T [the duration of biogas production] = 100 days

Mathematical Norms	T = 1 day	T = 2 days	T = 3 days	T = 4 days	T = 5 days
1-norm	168.48	166.95	165.34	163.65	161.86
2-norm	121.60	120.45	119.24	117.98	116.65
Infinity-norm	173.86	171.93	170.05	168.17	166.30

Table 9 shows that a variation of parameter T will produce 1-norm sensitivity values of 168.48, 166.95, 165.34, 163.65 and 161.86, 2-norm sensitivity values of 121.60, 120.45, 119.24, 117.98 and 116.65 and the infinity-norm sensitivity values of 173.86, 171.93, 170.05, 168.17 and 166.30. Therefore, the duration of biogas production parameter T can be considered as another relatively sensitive parameter than the two intrinsic growth rate parameters a and d.

What do we learn from this detailed and systematic sensitivity analysis? Our contribution unanimously point to the key fact that when a model parameter is varied a little one-at-a-time, two parameters b and f are dominantly more sensitive while the other parameters such as a, d, and the duration of biogas production T can be classified as relatively equally sensitivity.

Conclusion

In this pioneering study, our sensitivity analysis of the model parameters of biogas interaction reinforces a key hypothesis which clearly states that a model parameter which when varied a little and produces a biggest cumulative effect on the solution trajectories is classified as a sensitive parameter whereas a model parameter which is similarly varied a little and produces a smallest cumulative effect on the solution trajectories is classified as a least sensitive parameter. On the basis of this proven numerical technique, the model parameters b and f are clearly dominant sensitive parameters. In contrast, the model parameters T, d, a, B_2, B_1, and c can be classified as relatively equally sensitive. In the context of biogas production, the model parameter e is the least sensitive.

Therefore, the sensitive parameters which we have identified would require to be estimated efficiently in order to provide better predictions and reduce uncertainty which can characterise most biogas estimated data. It is worth mentioning that the intraspecific coefficient e which defines the effort of the first biogas population to inhibit the growth of the second biogas population is hereby called the least sensitive parameter and in the context of parameter estimation, this least sensitive parameter needs to be taken as a rough estimate.

For our present simulation analysis to be fully translated into a sustainable biogas production and national development, we will recommend a further data collection from which our proposed model can be validated. The outcome of our contribution reinforces the urgent need for stakeholders to fund and sustain a long-term biogas production in Nigeria as it is presently being implemented in other fast and slow developing countries where support for biogas production as an alternative source of energy and agricultural production are empowered to function side by side.

References

[1] E.N. Ekaka-a (2009), Computational and Mathematical Modelling of Plant Species Interactions in a Harsh Climate, Ph.D. Thesis, Department of Mathematics, The University of Liverpool and The University of Chester, United Kingdom.

[2] E.N. Ekaka-a and N.F. Nafo (2012), Parameter Ranking of Stock Market Dynamics: A Comparative Study of the Mathematical Models of Competition and Mutualistic Interactions, Scientia Africana, Vol. 11 (No. 1), pp. 36-43.

[3] L.A. Alcazar and J. Ancheyta (2007), Sensitivity analysis based methodology to estimate the best set of parameters for heterogeneous kinetic models, Chemical Engineering Journal 128 (2-3), pp. 85-93.

[4] S. Amod, M. Battaglio, and N. Bllomo (1996), Nonlinear models in soils consolidation theory parameter sensitivity analysis, Mathematical and Computer Modelling 24(3), pp. 11-20.

[5] C.T.H. Baker and F.A. Rihan (1999), Sensitivity Analysis of Parameter in Modelling With Delay-Differential Equations, MCCM Numerical Analysis Report, No. 349, Manchester University.

[6] J. Cariboni, D. Gatelli, R. Liska, and A. Saltelli (2007), The role of sensitivity analysis in ecological modelling, Ecological Modelling 203, pp. 167-182.

[7] M. Kleiber, H. Antuney, T.D. Hlen, and P. Kowalczck (1998), Parameter sensitivity in nonlinear mechanics , European Journal of Mechanics-A/Solids 17 (4), pp. 702-703.

[8] W.S. Overton (1977), A Strategy of Model Construction, In C.A.S. Hall and J.W. Day (Editors), Ecosystem Modelling in Theory and Practice, Wiley, (New York).

[9] A. Sieber, S. Uhlenbrook (2005), Sensitivity analysis of a distributed catchment model to verify the model structure, Journal of Hydrology 310 (1-4), pp. 216-235.

[10] G. Xenakis, D. Ray, and M. Mencuccini (2008), Sensitivity and uncertainty analysis from a coupled 3-PG and soil organic matter decomposition model, Ecological Modelling 219, pp. 1-16.

[11] E. T. Iyagba I. A. Mangibo and Y. S. Mohammad (2009)**,** The study of cow dung as co-substrate with rice husk in biogas production, Scientific Research and Essay Vol.4 (9), pp. 861-866.