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Determining the Importance of the Carrying Capacities of a Prey-Predator Interaction with Harvesting

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Abstract

The dynamics of interaction between two prey species and a predator can be understood using the process of a complex mathematical modelling of several parameters [1]. Having studied the relative importance of the migration rates of this ecological system in one of the published papers in the present volume of this journal, it is imperative in this context to computationally examine the importance of the two carrying capacities which sustain the growth of these prey and predator populations because of the inevitable role which the notion of a carrying capacity plays in the functioning and stability of this ecological system.

In this study, we consider the following secondary data such as $N_1(0) = 50$, $N_2(0) = 50$, $P(0) = 45$, carrying capacities K_1=110 and K_2 = 100, the migration rate of prey species in the free zone and the migration rate of prey species in the reserve zone have precise values of 0.5 and 0.4 respectively [1]. We considered the instance when the duration of interaction is 180 days. On the basis of our present sensitivity analysis, we have found that the carrying capacity K_1 of the prey in the free zone is a dominant sensitive parameter than the carrying capacity K_2 of the prey in the reserve zone irrespective of the three popular mathematical norms which we have implemented in a Matlab program to calculate the sensitivity measures of these carrying capacities. These two carrying capacities can be considered as relatively equally sensitive or relatively equally important parameters which define the dynamics of the prey-predator interaction with harvesting. In order to minimise prediction uncertainty, a further model validation is suggested to guide further research and strengthen knowledge-base in this ecosystem modelling. We have also found in this study that the coefficient of variation for the carrying capacity K_1 is a better estimate than the coefficient of variation for the carrying capacity K_2 irrespective of the type of the 1-norm, 2-norm and infinity-norm calculated values. Detailed numerical results of sensitivity measures are presented and discussed.

Keywords: Carrying Capacity, Prey-Predator Interaction, Harvesting, Mathematical Norms, Sensitivity Analysis

1.0 Introduction

It is an established scientific fact that the carrying capacity of an ecological system defines the maximum population size which can support the growth of interacting populations. In its own long history in the study of ecological populations, its sensitivity was not considered in the work of Khamis et al. [1]. This model formulation defines two carrying capacities namely the carrying capacity of the prey in the free zone and the carrying capacity of the prey in the reserve zone. While the carrying capacity of the prey in the free zone defines the maximum mass density of prey biomass in the free fishing zone at any time t, the carrying capacity of the prey in the reserve zone defines the maximum mass density of prey biomass in the reserve zone at any time t. Having successfully studied the sensitivities of the migration rate of prey species in the free zone and that of the migration rate of prey species in the reserve zone, we will attempt to conduct the sensitivity analysis of the two carrying capacities of the prey-predator interaction with harvesting which was unfortunately not considered in the work of Khamis et al. [1].

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While the migration rates tend to be associated with some sort of spread between the prey and predator populations, the notion of a carrying capacity measures the maximum population size which sustains the growth of these populations. From the ecological perspective, a carrying capacity is a more significant model parameter than the migration rate. The carrying capacity plays a key role in the logistic model formulation which inhibits the popular Malthus exponential growth phenomenon that has been unanimously agreed to provide a meaningless ecological insight because the resources that species depend on are not inexhaustible. It is on the basis of this idea that the theory of competition takes on a dominant contribution in the formulation of mathematical models of most complex ecological systems. In terms of applications, the carrying capacity of a population system can play a key role in the stabilization of an unstable steady-state solution of a population system. Another interesting application in a population system concerns the study of the bifurcation of a steady-state solution involving a variation of a carrying capacity when other model parameters are fixed. The fundamental qualitative changes in the behaviour of the steady-state solution which is the core output of this sophisticated numerical mathematics have potential and attractive applications in the functioning and planning of the ecological system. It is against this background that we are motivated to separately study the sensitivity of the carrying capacities as a distinct research investigation from the sensitivity of the migration rates of the interaction between two preys and a predator.

2.0 Materials and Methods

The primary source of data for this complex computational analysis is based on the data provided by Khamis et al. [1]. Their model is a system of continuous first order nonlinear ordinary differential equations with the following mathematical structure:

$$dN_1(t)/dt = r_1 N_1(t)(1-N_1(t)/K_1) + \beta_2 N_2(t) - \beta_1 N_1(t) - m_1 N_1(t) P(t) - qEN_1(t)$$

$$dN_2(t)/dt = r_2 N_2(t)(1-N_2(t)/K_2) - \beta_2 N_2(t) + \beta_1 N_1(t) - m_2 N_2(t)P(t)$$

$$dP(t)/dt = P(t)(-d - \sigma P(t) + \alpha_1 N_1(t) + \alpha_2 N_2(t)),$$

Here, the initial conditions are $N_1(0) > 0$, $N_2(0) > 0$ and $P(0) > 0$. The other model parameters are considered as positive constants. For the purpose of this simulation sensitivity analysis, the precise values of the carrying capacity of prey species in the free zone denoted by K_1 and the carrying capacity of prey species in the reserve zone denoted by K_2 are 110 and 100 respectively. The above model formulation describes a prey-predator interaction with harvesting in the context of aquatic ecosystem. Following Khamis et al. [1], we consider the prey in patch 1 denoted by $N_1(t)$ to be free for fishing and preys in patch 2 denoted by N_2 (t) as prey refuge which constitutes a reserve area and no fishing is permitted in that area. The predator population (density P(t)) has no barrier between the two patches in terms of fishing.

Our computational method of calculating the sensitivity of the carrying capacity of prey species in the free zone and the carrying capacity of_prey_species in the reserve zone is based on the proposed method of Ekaka-a and Nafo [2] and Ekaka-a [3]. This 01114tactical numerical method is based on the hypothesis of varying a model parameter a little one-at-a-time and observing its cumulative effect on the solution trajectories or model outputs. These sensitivity values can be calculated by using the three popular mathematical norms of 1-norm, 2-norm and infinity-norm which are based on the ODE 45 Matlab programming language. The detailed definition of the step by step algorithm of our sensitivity analysis method can be seen in one of our published in the present volume of this journal. The other versions of the sensitivity analysis of model parameters which is based on the popular one-at-a-time formulation can be read in the works of [4, 5, 6, 7, 8, 9, 3]. It is worth mentioning that our present method of sensitivity analysis is clearly different from the approaches of these researchers.

3.0 Discussion of Results

In this section, we will present and discuss the results which we have obtained by using the numerical technique of sensitivity analysis.

RSV represents range of sensitivity values; W_m represents the weighted mean of sensitivity values; Var represents the variance of sensitivity values; Std represents the standard deviation of sensitivity values; MN represents mathematical norms.

What do we learn from Table 1? Based on our choice of sensitivity hypothesis, our calculations clearly show that the carrying capacity of prey species in the free zone produce higher values of the cumulative effects on the solution trajectories or model outputs when this parameter is varied a little one-at-a-time while the other model parameters are fixed. Similarly, our calculations show that the carrying capacity of prey species in the reserve zone produce relatively higher values of the cumulative effects on the solution trajectories or model outputs when this parameter is varied a little

Table 1: Calculating the sensitivity values of the carrying capacity of prey species in the free zone and the carrying capacity of prey species in the reserve zone

MN	Sensitivity values of carrying capacities with percentage variations					RSV	W_m	Var	Std
	$K_1 = 1.1$ $K_2 = 1$	$K_1 = 2.2$ $K_2 = 2$	$K_1 = 3.3$ $K_2 = 3$	$K_1 = 4.4$ $K_2 = 4$	$K_1 = 5.5$ $K_2 = 5$				
1-norm	245.05 131.70	229.65 120.50	217.40 112.23	206.87 105.47	197.47 99.75	47.58 31.95	211.43 108.67	351.2 158.6	18.7 12.6
2-norm	144.45 88.13	135.10 80.60	127.70 75.05	121.36 70.53	115.72 66.70	28.73 21.43	124.12 72.67	128.1 71.35	11.3 8.45
Infinity-norm	166.80 96.77	160.99 88.55	155.60 82.46	153.13 77.18	150.11 73.41	16.69 23.36	154.78 79.80	43.96 86.07	6.63 9.28

one-at-a-time while the other model parameters are fixed. Therefore, the carrying capacities can be classified as relatively sensitive or important parameters in this context. These present observations are consistently the same irrespective of the values of the 1-norm, 2-norm and infinity-norm ODE 45 sensitivity values. Our present contribution to knowledge complements and extends the current mathematical analysis of Khamis et al. [2011].

In this study, the 1-norm calculation of the statistical coefficient of variation (CV) which is defined as the value of the standard deviation divided by the value of the weighted mean shows that the CV of the carrying capacity K_1 is 0.0884 while the CV of the carrying capacity K_2 is 0.1159. On the basis of this calculation, the 1-norm sensitivity for the carrying capacity K_1 is a better estimate than the 1-norm sensitivity for the carrying capacity K_2. Following the same procedure, the 2-norm calculated values of the CV for the carrying capacities K_1 and K_2 are 0.0910 and 0.1163 respectively. In this scenario, the 2-norm sensitivity for the carrying capacity K_1 is a better estimate than the 2-norm sensitivity for the carrying capacity K_2. Similarly, the infinity-norm calculated values of the CV for the carrying capacities K_1 and K_2 are 0.0428 and 0.1163 respectively. Therefore, the infinity-norm sensitivity for the carrying capacity K_1 is a better estimate than the infinity-norm sensitivity for the carrying capacity K_2.

Conclusion

On the basis of this contribution which is quite different from our earlier contribution, the carrying capacity of prey species in the free zone is a relatively more sensitive parameter than the carrying capacity of prey species in the reserve zone. These two parameters will need to be estimated efficiently in order to minimise model prediction uncertainty. We will expect these observations to guide further research and strengthen knowledge-base in model validation and parameter estimation theory. The statistical measures of range, weighted mean, variance and standard deviation of each sequence of sensitivity values are quantitatively specified.

We have found in this study that the coefficient of variation (CV) for the carrying capacity K_1 is a better estimate than the CV for the carrying capacity K_2 irrespective of the 1-norm, 2-norm and infinity-norm calculated values of the coefficient of variation. The pattern of changes in the values of the CV for the carrying capacities are different from the values of the CV for the migration rates which we have studied in one of our published papers in the present volume of this journal. From our experience, the study of the sensitivity of the carrying capacities is a distinct and sophisticated level of numerical mathematics because the variation of the carrying capacities can play key roles in the computational analysis of the doubling time for the biogas solids production and in the determination of the limiting population sizes for the three interacting populations. This challenging aspect of computational mathematics will be attempted in our future extension of this paper.

Reference

[1] S. A. Khamis, J. M. Tchuenche, M. Lukka and M. Heilio (2011), Dynamics of fisheries with prey reserve and harvesting, International Journal of Computer Mathematics, Vol. 88, No. 8, pp. 1776-1802.

[2] E.N. Ekaka-a and N.F. Nafo (2012), Parameter Ranking of Stock Market Dynamics: A Comparative Study of the Mathematical Models of Competition and Mutualistic Interactions, Scientia Africana, Vol. 11 (No. 1), pp. 36-43.

[3] E.N. Ekaka-a (2009), Computational and Mathematical Modelling of Plant Species Interactions in a Harsh Climate, PhD Thesis, University of Liverpool and University of Chester, 2009.

[4] R.H. Gardner, D.D. Huff, R.V. O'Neill, J.B. Mankin, J. Carney and J. Jones (1980), 'Application of Error Analysis to a Marsh Hydrology Model', Water Resources Res. 16, pp. 659-664.

[5] R.V. O'Neill, R.H. Gardner and J.B. Mankin (1980), 'Analysis of Parameter Error in a Nonlinear Model', Ecol. Modelling. 8, pp. 297-311.
[6] D.J. Downing, R.H. Gardner and E.O. Hoffman (1985), 'An Examination of Response-Surface Methodologies for Uncertainty Analysis in Assessment Models', Technometrics. 27, pp. 151-163.
[7] D.D. Breshears (1987), Uncertainty and sensitivity analyses of simulated concentrations of radionuclides in milk. Fort Collins, CO: Colorado State University, MS Thesis, pp. 1-69.
[8] M.J. Crick, M.D. Hill and D. Charles (1987), 'The Role of Sensitivity Analysis in Assessing Uncertainty. In: Proceedings of an NEA Workshop on Uncertainty Analysis for Performance Assessments of Radioactive Waste Disposal Systems, Paris, OECD, pp. 1-258.
[9] C. Yu, J-J Cheng and A-J Zielen (1991), 'Sensitivity Analysis of the RESRAD, a Dose Assessment Code' , Trans. Am. Nuc. Soc. 64; pp. 73-74.