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Dynamic Response Under A Linearly Varying Distributed Moving Loads Of An Elastically Supported Euler Bernoulli Beam On Variable Elastic Foundation

[1]Akinpelu F.O, [2]R.A Mustapha, and [3]I. A. Idowu

[1]Department of Pure and Applied Mathematics,
Ladoke Akintola University of Technology, Ogbomoso, Nigeria.
[2]Department of Mathematics,
Lagos state University Ojo, Lagos, Nigeria.
[3]Department of Mathematics,
Lagos state Polytechnic Ikorodu, Lagos, Nigeria.

Abstract

A theory describing the dynamic response under a linearly varying distributed moving loads of an elastically supported Bernoulli -Euler beam on variable elastic foundation is considered. The governing equation of fourth order partial differential equation was solved by an assumed solution in series form to reduce the coupled equation to ordinary second order differential equations. Two cases considered are moving masses and moving forces which are solved with Mathematical software (Maple). The numerical results are presented in a plotted curve which shows that the response amplitude of the elastically supported Bernoulli-Euler Beam decrease as the foundation modulli K increases. The response amplitude of the beam increase as the value of the mass increases. The result again shows that the response amplitudes for moving mass of the elastically supported Bernoulli Euler beam is reached earlier than that of the moving force.

Keywords: dynamic responses, Bernoulli Euler beam, elastic foundation, moving force, moving mass, foundation modulli

1.0 Introduction

Moving loads causes solid bodies to vibrate intensively at high velocities. Thus, the study of the behaviors of bodies subjected to moving loads has been the concern of several researchers. Among the earliest work in this area of study was the work of Timoshenko et al [1] who considered the problem of simply supported finite beams resting on an elastic foundation and traversed by moving loads. In his analysis, he assumed that the loads were moving with constant velocities along the beam. Oni and Awodola [2] considered a problem of the vibrations under a concentrated moving mass of a non-uniform Rayleigh beam resting on a variable elastic foundation. The technique used is based on the Generalized Galerking Method and struble's asymptotic technique. Numerical results in plotted curves are presented. The results show that the response amplitudes of the non-uniform Rayleigh beam decrease as the rotatory inertia correction factor r increases. Similarly, for fixed value of the displacements of a non-uniform Rayleigh beam resting on variable elastic foundation decease as the foundation modulli F increases. Furthermore, the critical speed for the moving mass problem is reached prior that of the moving force problem for both illustrative. Dada [3] Studied uniform distributed moving masses vibration for Euler Bernoulli beams on elastic foundation. The partial differential equation governing the beam's motion is reduced to ordinary differential equation and then expressed as a system of linear equations by finite difference scheme. The analysis is valid for Euler Bernoulli beams with various boundary conditions. However, simply supported boundary conditions were used as an illustrated example. The numerical results are presented in graphical forms and the limiting cases compared well with known existing results. The numerical analysis shows that the foundation stiffness and loads'

distribution have significant effects on the dynamic deflection of the beam. Oni and Omolofe [4] investigates the dynamics behavior of non-uniform Bernoulli-Euler beams subjected to concentrated loads raveling at variable velocities. The solution technique is based on the generalized Galerkins Method and the use of the generating function of the Bessel function type. The results show that, for all the illustrative examples considered, for the same natural frequency, the critical speed for the system consisting of a non-uniform beam traversed by a force moving at a non-uniform velocity is greater than that of the corresponding moving mass problem. It was also found that, for fixed axial force, an increase in foundation moduli reduces the response amplitudes of the dynamical system. Furthermore, it was shown that the transverse-displacement amplitude of a

Corresponding author ***Akinpelu F. O.,*** E-mail: rilwandemus@yahoo.com-, Tel. +234 8033370470

clamped-clamped non-uniform Bernoulli-Euler beam traversed by a load moving at variable velocities is lower than that of the cantilever. The response amplitude of the same dynamical systems which is simply supported is higher than those which consist of clamped-clamped or clamped-free (Cantilever) end conditions. Finally, an increase in the values of foundation moduli and axial force reduces the critical speed for all variants of the boundary conditions recently, Oni [6] used the Galerkin method to obtain the response to several moving masses of a non-uniform beam resting on an elastic foundation. The effects of the elastic foundation in the transverse displacement of the non-uniform beam were analyzed for both the moving mass and associated moving force problems. Akinpelu et al [7] investigates the response under moving load of an elastically supported Euler-Bernoulli Beam on pre-stressed and variable Elastic Foundation .The technique used is based on the analytical and numerical method in terms of series solution. Numerical results in tables and plotted curves are presented. The results shows that the response amplitude of the moving mass increases as the mass of the load M increases. It was also found that the response amplitude due to the moving mass for pre-stressed is greater than that due to moving force. Furthermore, Milormir et al [5] developed a theory describing the response of Bernoulli-Euler beam under an arbitrary number of concentrated moving masses. The theory is based on the fourier technique and shows that, for a simply supported beam, the resonance frequency is lower with no corresponding decrease in maximum amplitude when the inertia is considered. More recent Oni and Awodola [7] considered the dynamic response under a moving load of an elastically supported non- prismatic Bernoulli-Euler beam resting on variable elastic foundation were investigated. The technique based on the Generalized Galerkin's method and the struble's asymptotic technique. The results show that response amplitudes of the elastically supported non-prismatic Bernoulli Euler beam decrease as the foundation Modulli increases. Therefore, a theory describing the dynamic response under a partially distributed moving load of an elastically supported pre stressed Bernoulli- Euler beam on variable elastic foundation is investigated in this research.

2.0 Formulation of Problem

The problem of the dynamic response under a partially distributed moving load of an elastically supported Bernoulli-Euler beam on variable elastic foundation is governed by the fourth order partial differential equation given by

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 W(x,t)}{\partial x^2}\right]+2\omega_b\mu(x)\frac{\partial W(x,t)}{\partial t}+\mu(x)\frac{\partial^2 W(x,t)}{\partial t^2}+K(x)W(x,t)$$
$$=W_1\langle x-\alpha_1\rangle^0+\frac{W_2-W_1}{d}\langle x-\alpha_1\rangle^1+W_2\langle x-\alpha_2\rangle^0-\frac{W_2-W_1}{d}\langle x-\alpha_2\rangle \tag{1}$$

Where E is the Young's Modulus (x,t) is the transverse displacement, $K(x)$ is the variable elastic foundation, $\mu(x)$ is the Variable mass per unit length of the beam, $I(x)$ is the variable moment of inertia, ω_b is the damping coefficient, x and t are respectively spatial and time coordinates. $q(x,t)$ is defined by Macaulay notation as the load.

V is the velocity of load, g is the acceleration due to gravity.

Macaulay notation is defined as

$$\langle x-\alpha_i\rangle^n=\begin{cases}0 & x\langle\alpha_i\\ (x-\alpha_i) & x\rangle\alpha_i\end{cases} \tag{2}$$

With $q(x,t)$ defined as

$$q(x,t)=W_1\langle x-\alpha_1\rangle^0+\frac{W_2-W_1}{d}\langle x-\alpha_1\rangle^1+W_2\langle x-\alpha_2\rangle^0-\frac{W_2-W_1}{d}\langle x-\alpha_2\rangle \tag{3}$$

Where $W_1=M_1g$ and $W_2=M_2g$ are the forces produced by masses M_1 and M_2 respectively at the end points of the load as shown in the figure below

$$d=a_2-a_1 \tag{4}$$

The variable elastic foundation and variable moment of inertia by [8] is adopted, where K is the foundation modulus.

$$I(x)=I_0\left(1+Sin\frac{\pi x}{L}\right)^3 \tag{5}$$

$$\mu(x)=\mu_0\left(1+Sin\frac{\pi x}{L}\right) \tag{6}$$

Where I_0 and μ_0 are constant. Substituting equation (4), (5) and (6) into (1), then becomes
The equation (7) becomes

$$\frac{\partial^2}{\partial x^2}\left[EI_0(x)\frac{\partial^2 W(x,t)}{\partial x^2}\right]+\mu_0(x)\frac{\partial^2 W(x,t)}{\partial t^2}+2\omega_b\mu_0(x)\frac{\partial W(x,t)}{\partial t}+K(x)=q(x,t)----(7)$$

$$\frac{EI_0}{4}\left[\begin{array}{l}\left(10+15Sin\frac{\pi x}{L}-6Cos\frac{2\pi x}{L}-Sin\frac{3\pi x}{L}\right)\frac{\partial^4 W(x,t)}{\partial x^4}+\left(\frac{30\pi}{L}Cos\frac{\pi x}{L}+\frac{24\pi}{L}Sin\frac{2\pi x}{L}-\frac{6\pi}{L}Cos\frac{3\pi x}{L}\right)\frac{\partial^3 W(x,t)}{\partial x^3}+\\ \left(\frac{24\pi^2}{L^2}Cos\frac{2\pi x}{L}-\frac{15\pi^2}{L^2}Sin\frac{3\pi x}{L}+\frac{9\pi^2}{L^2}Sin\frac{\pi x}{L}\right)\frac{\partial W^2(x,t)}{\partial x^2}\end{array}\right]+ \quad (8)$$

$$+2\omega_b\mu_0\left(1+Sin\frac{\pi x}{L}\right)\frac{\partial W(x,t)}{\partial t}+\mu_0\left(1+Sin\frac{\pi x}{L}\right)\frac{\partial W^2(x,t)}{\partial t^2}+K\left(4x-3x^2+x^3\right)\frac{w(x,t)}{\partial t}=q(x,t)$$

With boundary conditions:

$$\left.\begin{array}{l}W(0,t)=W''(0,t)=0\\ W(L,t)=W''(L,t)=0\end{array}\right\} \quad (9)$$

And the initial conditions

$$W(0,t)=\frac{\partial W(x,t)}{\partial t}=0 \quad (10)$$

The variable elastic foundation and variable moment of inertia by [7] is adopted, where K is the foundation modulus.

$$K(x)=K\left(4x-3x^2+x^3\right) \quad (11)$$

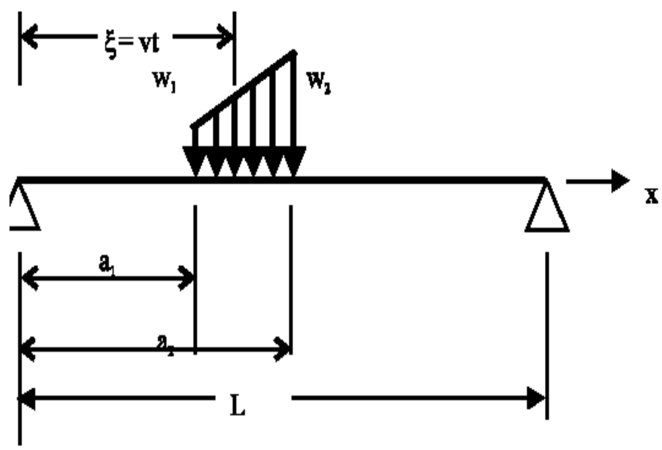

Fig 1, Schematic diagram showing linearly varying distributed moving loads on

3.0 Method of Solution

Evidently, a closed form solution of the partial differential equation (11) does not exist. The assumed solution of the form

$$W(x,t)=\sum_{n=1}^{\infty}X_n(x)T_n(t)--------------(12)$$

Were considered. Where $X_n(x)$ is chosen such that desired boundary condition are satisfied.
Substituting equation (12) into equation (11) this yield

$$\frac{EI_0}{4}\left[\begin{array}{l}\sum_{n=1}^{\infty}X_n^{iiii}(x)T_n(t)\left(10+15Sin\frac{\pi x}{L}-6Cos\frac{2\pi x}{L}-Sin\frac{3\pi x}{L}\right)+\sum_{n=1}^{\infty}X_n^{iii}(x)T_n(t)\left(\frac{30\pi}{L}Cos\frac{\pi x}{L}+\frac{24\pi}{L}Sin\frac{2\pi x}{L}-\frac{6\pi}{L}Cos\frac{3\pi x}{L}\right)+\\ \sum_{n=1}^{\infty}X_n^{ii}(x)T_n(t)\left(\frac{24\pi^2}{L^2}Cos\frac{2\pi x}{L}-\frac{15\pi^2}{L^2}Sin\frac{3\pi x}{L}+\frac{9\pi^2}{L^2}Sin\frac{\pi x}{L}\right)+2\omega_b\mu_0\left(1+Sin\frac{\pi x}{L}\right)\sum_{n=1}^{\infty}X_n(x)\dot{T}_n(t)+\\ \mu_0\left(1+Sin\frac{\pi x}{L}\right)\sum_{n=1}^{\infty}X_n(x)\ddot{T}_n(t)+K\left(4x-3x^2+x^3\right)\sum_{n=1}^{\infty}X_n(x)T_n(t)=W_1\langle x-\alpha_1\rangle^0+\frac{W_2-W_1}{d}\langle x-\alpha_1\rangle^1+W_2\langle x-\alpha_2\rangle^0-\frac{W_2-W_1}{d}\langle x-\alpha_2\rangle----(13)\end{array}\right]$$

Multiplying both sides of the equation (13) by $X_k(x)$ and then integrating it along the entire length of the beam.

$$\frac{EI_0}{4}\int\left(\left[\begin{array}{l}\sum_{n=1}^{\infty}X_n^{iiii}(x)T_n(t)\left(10+15Sin\frac{\pi x}{L}-6Cos\frac{2\pi x}{L}-Sin\frac{3\pi x}{L}\right)+\sum_{n=1}^{\infty}X_n^{iii}(x)T_n(t)\left(\frac{30\pi}{L}Cos\frac{\pi x}{L}+\frac{24\pi}{L}Sin\frac{2\pi x}{L}-\frac{6\pi}{L}Cos\frac{3\pi x}{L}\right)+\\ \sum_{n=1}^{\infty}X_n^{ii}(x)T_n(t)\left(\frac{24\pi^2}{L^2}Cos\frac{2\pi x}{L}-\frac{15\pi^2}{L^2}Sin\frac{3\pi x}{L}+\frac{9\pi^2}{L^2}Sin\frac{\pi x}{L}\right)\end{array}\right]\right)X_k(x)dx+ \qquad (14)$$

$$\left[2\omega_b\mu_0\left(1+Sin\frac{\pi x}{L}\right)\sum_{n=1}^{\infty}X_n(x)\dot{T}_n(t)+\mu_0\left(1+Sin\frac{\pi x}{L}\right)\sum_{n=1}^{\infty}X_n(x)\ddot{T}_n(t)+K\left(4x-3x^2+x^3\right)\sum_{n=1}^{\infty}X_n(x)T_n(t)\right]X_k(x)dx=$$

$$\left(W_1\langle x-\alpha_1\rangle^0+\frac{W_2-W_1}{d}\langle x-\alpha_1\rangle^1+W_2\langle x-\alpha_2\rangle^0-\frac{W_2-W_1}{d}\langle x-\alpha_2\rangle\right)X_k(x)dx$$

and applying orthogonality condition where necessary.

$$\int_0^L X_K(x)X_n(x)dx=\alpha, n=k \qquad (15)$$
$$0, n\neq K$$

and α is a constant, equation (14) becomes,

$$P_1=\frac{10EI_0}{4}\int_0^L X_n^{iiii}(x)X_k(x)dx$$

$$P_2=\frac{15EI_0}{4}\int_0^L X_n^{iiii}(x)X_k(x)Sin\left(\frac{\pi x}{L}\right)dx$$

$$P_3=\frac{6EI_0}{4}\int_0^L X_n^{iiii}(x)X_k(x)Cos\left(\frac{2\pi x}{L}\right)dx$$

$$P_4=\frac{EI_0}{4}\int_0^L X_n^{iiii}(x)X_k(x)Sin\left(\frac{2\pi x}{L}\right)dx$$

$$P_5=\frac{30EI_0}{4}\int_0^L X_n^{iii}(x)X_k(x)\frac{\pi}{L}Cos\left(\frac{\pi x}{L}\right)dx$$

$$P_6=\frac{24EI_0}{4}\int_0^L X_n^{iii}(x)X_k(x)\frac{\pi}{L}Sin\left(\frac{2\pi x}{L}\right)dx$$

$$P_7=\frac{6EI_0}{4}\int_0^L X_n^{iii}(x)X_k(x)\frac{\pi}{L}Cos\left(\frac{3\pi x}{L}\right)dx$$

$$P_8=\frac{24EI_0}{4}\int_0^L X_n^{ii}(x)X_k(x)\frac{\pi^2}{L^2}Cos\left(\frac{2\pi x}{L}\right)dx$$

$$P_9=\frac{15EI_0}{4}\int_0^L X_n^{ii}(x)X_k(x)\frac{\pi^2}{L^2}Sin\left(\frac{\pi x}{L}\right)dx$$

$$P_{10}=\frac{9EI_0}{4}\int_0^L X_n^{ii}(x)X_k(x)\frac{\pi^2}{L^2}Sin\left(\frac{3\pi x}{L}\right)dx$$

$$P_{11}=2\omega_b\mu_0\int_0^L X_n(x)X_k(x)dx$$

$$P_{12} = 2\omega_b\mu_0\int_0^L X_n(x)X_k(x)Sin\left(\frac{\pi x}{L}\right)dx$$

$$P_{13} = 2\omega_b\mu_0\int_0^L X_n(x)X_k(x)dx$$

$$P_{14} = \mu_0\int_0^L X_n(x)X_k(x)Sin\left(\frac{\pi x}{L}\right)dx$$

$$P_{15} = K\int_0^L X''_n(x)X_k(x)4xdx$$

$$P_{16} = W_1\int_0^L \langle x-\alpha_1\rangle^0 X_k(x)dx$$

$$P_{17} = \frac{W_2 - W_1}{d}\int_0^L \langle x-\alpha_1\rangle X_k(x)d$$

$$P_{18} = W_2\int_0^L \langle x-\alpha_1\rangle^0 X_k(x)dx$$

$$P_{19} = \frac{W_2 - W_1}{d}\int_0^L \langle x-\alpha_1\rangle X_k(x)dx$$

By collecting the differential equation of the order and degree together, equation (14) becomes,

$$(P_1+P_2-P_3-P_4+P_5+P_6-P_7+P_8-P_9+P_{10})T_n(t)+(P_{11}+P_{12})\dot{T}_n(t)+(P_{13}+P_{14})\ddot{T}_n(t)=(P_{18}+P_{19}+P_{20}-P_{21})-(P_{15}-P_{16}+P_{17})------(16)$$

For the general solution of the equation (16) and for the purpose of the solution, we consider mass (M) traveling with constant velocity (V).Solutions for the greater number of mass can be obtained in the same manner. Thereafter, the following special cases for the equation (16) follows;

(a) Moving force

If we neglect the inertia term, we have the classical case of the moving force, under the Assumption of equation (14), we have

$$(P_1+P_2-P_3-P_4+P_5+P_6-P_7+P_8-P_9+P_{10}-P_{15})T_n(t)+(P_{13}+P_{14})\ddot{T}_n(t)=(P_{16}+P_{17}+P_{18}-P_{19})-------(17)$$

(b) Moving mass

If we consider all the inertia terms we have classical case of moving mass under the assumption of equation (14), we have

$$(P_1+P_2-P_3-P_4+P_5+P_6-P_7+P_8-P_9+P_{10}-P_{15})T_n(t)+(P_{11}+P_{12})\dot{T}_n(t)+(P_{13}+P_{14})\ddot{T}_n(t)=(P_{16}+P_{17}+P_{18}-P_{19})---(18)$$

4.0 Numerical Method and Discussion of Results

To illustrate the foregoing analysis, the uniform Bernoulli - Euler beam of length 15m were considered , $M = 3kg, 6kg, 9kg$, $\mu = 75$, $EI = 2785\ NM^{-1}$, $K = 1,2,3,...$, $V = 3.3ms^{-1}$, $g = 10\ ms^{-2}$, $L = 15\ m$, $\pi = 3.142$, $n = 1,2,3$. The results are shown graphically below for the various values of foundation moduli and masses.

Figure 2, displays the Deflection against time response of simply supported Bernoulli beam on variable elastic foundation under the action of moving mass for the various values of foundation Moduli. The graph shows that the response deflection of the beam decreases as the value of the foundation modulus increases. And the maximum deflection was attained at foundation modulus K = 0. Also

Figure 3, shows the deflection against time of simply supported Bernoulli beam on variable elastic foundation under the action of moving mass 10 for various masses M. From the graph, the response deflection of the beam increases as the value of the masses increases.

Figure 4, shows the deflection against time response of simply supported Bernoulli beam on variable elastic foundation under the action of moving force for the various values of foundation Moduli. The graph shows that the response deflection of the beam decreases as the value of the foundation modulus increases. And the maximum deflection was attained at foundation modulus K = 0. Also

Figure 5, shows the deflection against time of simply supported Bernoulli beam on variable elastic foundation under the action of moving force for various masses M. From the graph, the response deflection of the beam increases as the value of the masses increases.

Figure 6 shows the comparison of moving mass and moving force of the elastically supported Bernoulli Euler beam. Which shows that the response amplitudes for moving mass of the elastically supported Bernoulli Euler beam is reached earlier than that of the moving force.

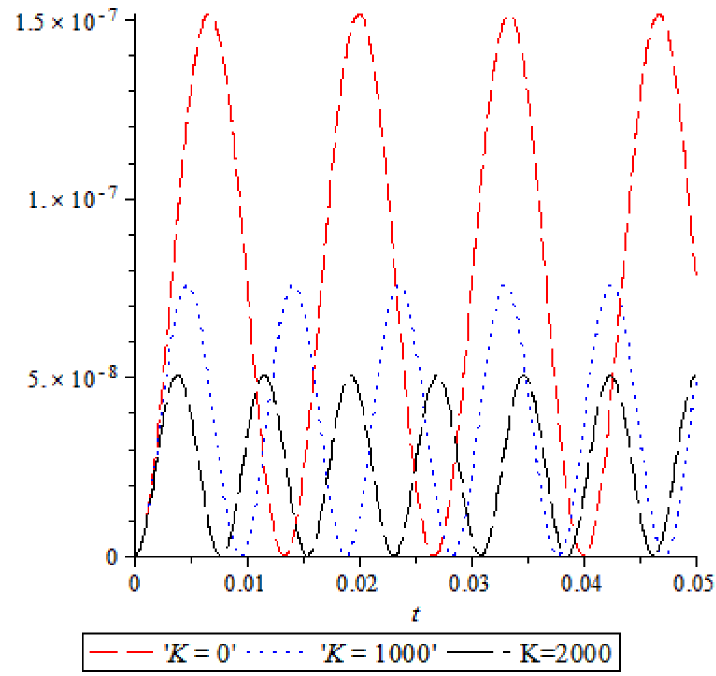

Figure 2: Displacement response of moving mass for simply uniform Bernoulli beam on variable elastic foundation for different values of K

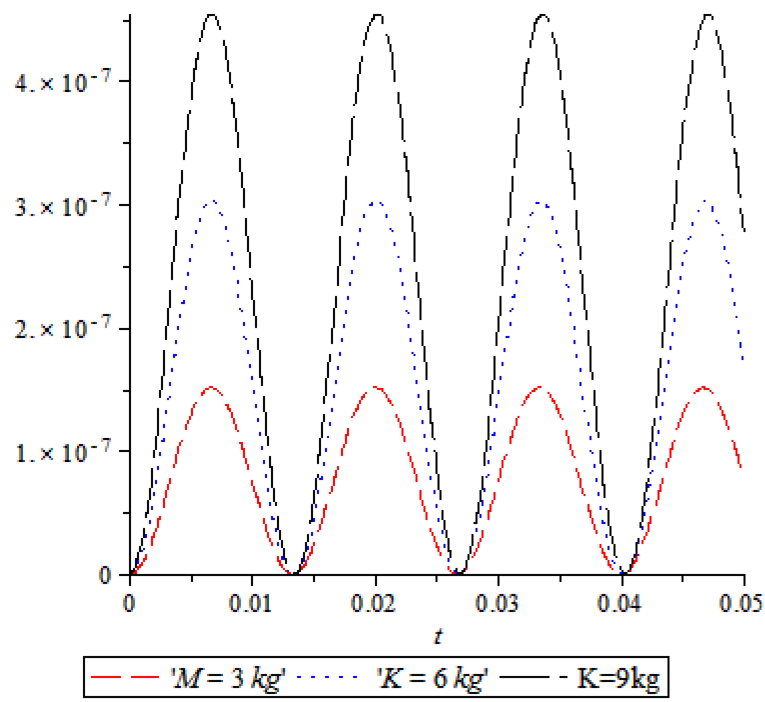

Figure 3: Displacement response of moving mass for simple uniform Bernoulli beam on variable elastic foundation for different values of mass M

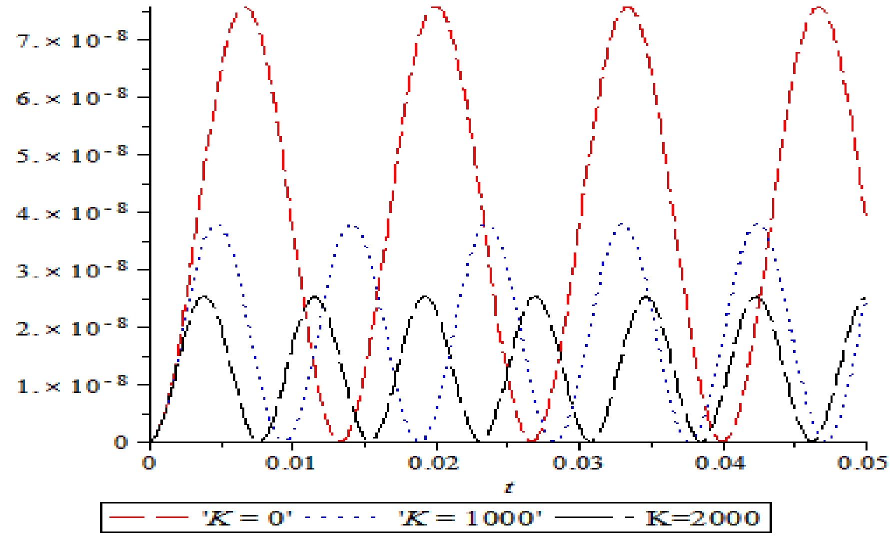

Figure 4: Displacement response of moving force for simply uniform Bernoulli beam on variable elastic foundation for different values of K

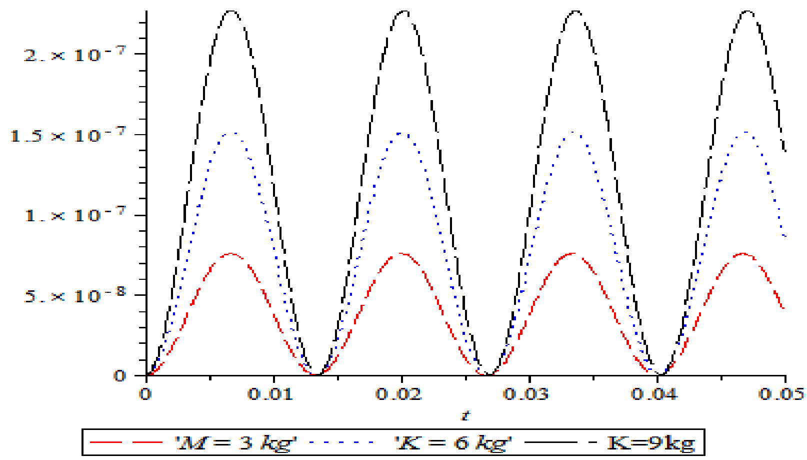

Figure 5: Displacement response of moving froce for simply uniform Bernoulli beam on variable elastic foundation for different values of mass M

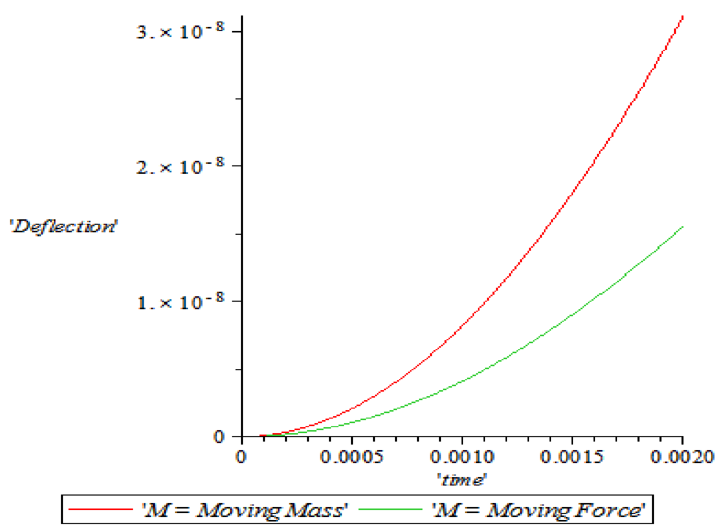

Figure 6: Comparison of moving mass and moving force for simple uniform Bernoulli beam on variable elastic foundation for masses

5.0 Conclusion

The problem of assessing the dynamic response under a linearly varying distributed moving loads of an elastically supported variable Euler-Bernoulli Beam. The governing equation for the mathematical model is analytically simplified into a set of ordinary differential equations that are solved by a mathematical software (Maple).Clearly, the elastic foundation, have considerable effects on the dynamic behavior of the beam.

The results have been able to show that the response amplitude of the elastically supported Bernoulli-Euler Beam decrease as the Foundation modulli K increases. Also that the response amplitude of the beam increase as the value Mass M increases.

The result again shows that the response amplitude for moving mass of elastically supported Bernoulli-Euler Beam is reached earlier than that of the moving force.

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