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On the Structure of Rhotrix

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Abstract

Rhotrices were first defined by Ajibade [1], over the set of real numbers. In Tudunkaya and Makanjuola [2], they were also defined over Z_p (the set of integers modulo p), under the same operations defined by Ajibade [1], but addition and multiplication were done modulo p. The purpose of this note is to present rhotrices defined over the set R of real rhotrices and to explore their properties. The hope is that these rhotrices may be useful in the development of Mathematics and its application as those defined in Ajibade [1] and Tudunkaya and Makanjuola [2].

1.0 Introduction

A rhotrix was defined by Ajibade [1], as the set

$$R = \left\{ \left\langle \begin{matrix} & a & \\ b & c & d \\ & e & \end{matrix} \right\rangle : a, b, c, d, e \in \Re \right\} \tag{1}$$

The entry at the centre of a rhotrix is called heart, which is 'c' in the above definition. The addition of two rhotrices A and $Q = \left\langle \begin{matrix} & f & \\ g & h(Q) & j \\ & k & \end{matrix} \right\rangle$ was given by

$$A + Q = \left\langle \begin{matrix} & a & \\ b & c & d \\ & e & \end{matrix} \right\rangle + \left\langle \begin{matrix} & f & \\ g & h & j \\ & k & \end{matrix} \right\rangle = \left\langle \begin{matrix} & a+f & \\ b+g & c+h & d+j \\ & e+k & \end{matrix} \right\rangle \tag{2}$$

The additive inverse of *A* is

$$-A = -\left\langle \begin{matrix} & a & \\ b & c & d \\ & e & \end{matrix} \right\rangle = \left\langle \begin{matrix} & -a & \\ -b & -c & -d \\ & -e & \end{matrix} \right\rangle \tag{3}$$

Such that the additive identity is

$$0 = \left\langle \begin{matrix} & 0 & \\ 0 & 0 & 0 \\ & 0 & \end{matrix} \right\rangle \tag{4}$$

with the multiplication of *A* by *Q* as

$$A \bullet Q = \left\langle \begin{matrix} & a & \\ b & c & d \\ & e & \end{matrix} \right\rangle \bullet \left\langle \begin{matrix} & f & \\ g & h & j \\ & k & \end{matrix} \right\rangle = \left\langle \begin{matrix} & ah+fc & \\ bh+gc & ch & dh+jc \\ & eh+kc & \end{matrix} \right\rangle \tag{5}$$

The multiplicative identity of *R* was given as:

$$.I = \left\langle \begin{matrix} & 0 & \\ 0 & 1 & 0 \\ & 0 & \end{matrix} \right\rangle \tag{6}$$

Also, if

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$$A \bullet Q = \left\langle \begin{matrix} & a & \\ b & c & d \\ & e & \end{matrix} \right\rangle \bullet \left\langle \begin{matrix} & f & \\ g & h & j \\ & k & \end{matrix} \right\rangle = \left\langle \begin{matrix} & 0 & \\ 0 & 1 & 0 \\ & 0 & \end{matrix} \right\rangle, \text{ then}$$

$$Q = A^{-1} = -\frac{1}{c^2} \left\langle \begin{matrix} & a & \\ b & -c & d \\ & e & \end{matrix} \right\rangle, \quad c \neq 0 \qquad (7)$$

Another definition that is more general was given by Mohammed [3] as

$$R = \left\{ \left\langle \begin{matrix} & & & a_1 & & & \\ & & a_2 & a_3 & a_4 & & \\ & \dots & \dots & \dots & \dots & \dots & \\ a_{\left\{\frac{(t+1)}{2}\right\}-\frac{n}{2}} & \dots & \dots & a_{\left\{\frac{(t+1)}{2}\right\}} & \dots & \dots & a_{\left\{\frac{(t+1)}{2}\right\}+\frac{n}{2}} \\ & \dots & \dots & \dots & \dots & \dots & \\ & & a_{t-3} & a_{t-2} & a_{t-1} & & \\ & & & a_t & & & \end{matrix} \right\rangle a_i \in \Re \right\} \qquad (8)$$

$t = \frac{(n^2+1)}{2}$, $n \epsilon 2Z^+ + 1$ and $\frac{n}{2}$ is the integer value upon division of n by 2, $= \left\{\frac{(t+1)}{2}\right\} - \frac{n}{2}$, $\beta = \left\{\frac{(t+1)}{2}\right\}$, $\pi = \left\{\frac{(t+1)}{2}\right\} + \frac{n}{2}$.

The discussions of concepts in this work, will be presented according to Jaisingh [4].

1.1 Definition

$$\text{Let } \boldsymbol{\xi} = \left\{ \left\langle \begin{matrix} & & & A_1 & & & \\ & & A_2 & A_3 & A_4 & & \\ & \dots & \dots & \dots & \dots & \dots & \\ A & \dots & \dots & A_\beta & \dots & \dots & A_\pi \\ & \dots & \dots & \dots & \dots & \dots & \\ & & A_{t-3} & A_{t-2} & A_{t-1} & & \\ & & & A_t & & & \end{matrix} \right\rangle : A_1, A_2, \dots, A_t \in R \right\} \qquad (9)$$

then $\boldsymbol{\xi}$ will be called a rhotrix rhotrix. In other words, $\boldsymbol{\xi}$ is a rhotrix with rhotrices as entries.

The additive identity is

$$0_\xi = \left\langle \begin{matrix} & & & 0_1 & & & \\ & & 0_2 & 0_3 & 0_4 & & \\ & \dots & \dots & \dots & \dots & \dots & \\ 0 & \dots & \dots & 0_\beta & \dots & \dots & 0_\pi \\ & \dots & \dots & \dots & \dots & \dots & \\ & & 0_{t-3} & 0_{t-2} & 0_{t-1} & & \\ & & & 0_t & & & \end{matrix} \right\rangle \qquad (10)$$

Where each

$$0_i = \left\langle \begin{matrix} & & & 0_1 & & & \\ & & 0_2 & 0_3 & 0_4 & & \\ & \dots & \dots & \dots & \dots & \dots & \\ 0 & \dots & \dots & 0_\beta & \dots & \dots & 0_\pi \\ & \dots & \dots & \dots & \dots & \dots & \\ & & 0_{t-3} & 0_{t-2} & 0_{t-1} & & \\ & & & 0_t & & & \end{matrix} \right\rangle, i = 1,2,3, \dots, t-1, t \qquad (11)$$

and hence the additive inverse of

$$\xi = \left\langle \begin{matrix} & & & A_1 & & & \\ & & A_2 & A_3 & A_4 & & \\ & \dots & \dots & \dots & \dots & \dots & \\ A & \dots & \dots & A_\beta & \dots & \dots & A_\pi \\ & \dots & \dots & \dots & \dots & \dots & \\ & & A_{t-3} & A_{t-2} & A_{t-1} & & \\ & & & A_t & & & \end{matrix} \right\rangle$$

will be

$$-\xi = \left\langle \begin{array}{ccccccc} & & & -A_1 & & & \\ & & -A_2 & -A_3 & -A_4 & & \\ & \cdots & \dots & \dots & \dots & \cdots & \\ -A & \cdots & \dots & -A_\beta & \dots & \cdots & -A_\pi \\ & \cdots & \dots & \dots & \dots & \cdots & \\ & & -A_{t-3} & -A_{t-2} & -A_{t-1} & & \\ & & & -A_t & & & \end{array} \right\rangle \tag{12}$$

1.2 Proposition

(ξ, +) is a group.

Proof: The proof follows from equations (2), (11) and (12).

1.3 Proposition

(ξ, +, •) is a ring.

Proof: The proof follows from equations (5) and (7). The multiplicative identity of ξ will be given by

$$I_i = \left\langle \begin{array}{ccccccc} & & & 0_1 & & & \\ & & 0_2 & 0_3 & 0_4 & & \\ & \cdots & \dots & \dots & \dots & \cdots & \\ 0 & \cdots & \dots & I_\beta & \dots & \cdots & 0_\pi \\ & \cdots & \dots & \dots & \dots & \cdots & \\ & & 0_{t-3} & 0_{t-2} & 0_{t-1} & & \\ & & & 0_t & & & \end{array} \right\rangle \tag{13}$$

Where

$$I = \left\langle \begin{array}{ccccccc} & & & 0_1 & & & \\ & & 0_2 & 0_3 & 0_4 & & \\ & \cdots & \dots & \dots & \dots & \cdots & \\ 0 & \cdots & \dots & 1_\beta & \dots & \cdots & 0_\pi \\ & \cdots & \dots & \dots & \dots & \cdots & \\ & & 0_{t-3} & 0_{t-2} & 0_{t-1} & & \\ & & & 0_t & & & \end{array} \right\rangle \tag{14}$$

Now looking at the multiplicative inverse of a rhotrix

$$B = \left\langle \begin{array}{ccccccc} & & & b_1 & & & \\ & & b_2 & b_3 & b_4 & & \\ & \cdots & \dots & \dots & \dots & \cdots & \\ b & \cdots & \dots & b_\beta & \dots & \cdots & b_\pi \\ & \cdots & \dots & \dots & \dots & \cdots & \\ & & b_{t-3} & b_{t-2} & b_{t-1} & & \\ & & & b_t & & & \end{array} \right\rangle$$

which was defined to be

$$B^{-1} = -\frac{1}{{b_\beta}^2} \left\langle \begin{array}{ccccccc} & & & b_1 & & & \\ & & b_2 & b_3 & b_4 & & \\ & \cdots & \dots & \dots & \dots & \cdots & \\ b & \cdots & \dots & -b_\beta & \dots & \cdots & b_\pi \\ & \cdots & \dots & \dots & \dots & \cdots & \\ & & b_{t-3} & b_{t-2} & b_{t-1} & & \\ & & & b_t & & & \end{array} \right\rangle \tag{15}$$

such that $b_\beta \neq 0$ by Ajibade [1], we can observe a difference in the way the heart is treated. This difference may bring some obstacles when dealing with rhotrices under this multiplication method. To take care of these obstacles, the following were proposed:

1.4 Definition

Let

$$B = \left\langle \begin{array}{ccccccc} & & & b_1 & & & \\ & & b_2 & b_3 & b_4 & & \\ & \cdots & \dots & \dots & \dots & \cdots & \\ b & \cdots & \dots & b_\beta & \dots & \cdots & b_\pi \\ & \cdots & \dots & \dots & \dots & \cdots & \\ & & b_{t-3} & b_{t-2} & b_{t-1} & & \\ & & & b_t & & & \end{array} \right\rangle,$$

Then

$$Neg(B) = \left\langle \begin{array}{ccccccc} & & & b_1 & & & \\ & & b_2 & b_3 & b_4 & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ b & \cdots & \cdots & -b_\beta & \cdots & \cdots & b_\pi \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & b_{t-3} & b_{t-2} & b_{t-1} & & \\ & & & b_t & & & \end{array} \right\rangle$$

In other words, $Neg(B)$ is the rhotrix got by multiplying the heart of the rhotrix B by a negative sign.

1.5 Definition

Let

$$C = \left\langle \begin{array}{ccccccc} & & & c_1 & & & \\ & & c_2 & c_3 & c_4 & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ c & \cdots & \cdots & c_\beta & \cdots & \cdots & c_\pi \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & c_{t-3} & c_{t-2} & c_{t-1} & & \\ & & & c_t & & & \end{array} \right\rangle,$$

the rhotrix

$$Min(C) = \left\langle \begin{array}{ccccccc} & & & -c_1 & & & \\ & & -c_2 & -c_3 & -c_4 & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ -c & \cdots & \cdots & c_\beta & \cdots & \cdots & -c_\pi \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & -c_{t-3} & -c_{t-2} & -c_{t-1} & & \\ & & & -c_t & & & \end{array} \right\rangle$$

In other words, $Min(C)$ is the rhotrix got by multiplying the entire entries (except the heart) of a rhotrix C by a negative sign.

1.6 Lemma

If X and Y are any two rhotrices of the same size, then

$$XY^{-1} + Neg\left(\frac{1}{h(Y)^2} Min(X)Y\right) = \left\langle \begin{array}{ccccccc} & & & 0_1 & & & \\ & & 0_2 & 0_3 & 0_4 & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ 0 & \cdots & \cdots & 0_\beta & \cdots & \cdots & 0_\pi \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & 0_{t-3} & 0_{t-2} & 0_{t-1} & & \\ & & & 0_t & & & \end{array} \right\rangle,$$

$$h(Y) \neq \left\langle \begin{array}{ccccccc} & & & 0_1 & & & \\ & & 0_2 & 0_3 & 0_4 & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ 0 & \cdots & \cdots & 0_\beta & \cdots & \cdots & 0_\pi \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & 0_{t-3} & 0_{t-2} & 0_{t-1} & & \\ & & & 0_t & & & \end{array} \right\rangle.$$

Proof:

Let

$$X = \left\langle \begin{array}{ccccccc} & & & x_1 & & & \\ & & x_2 & x_3 & x_4 & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ x & \cdots & \cdots & x_\beta & \cdots & \cdots & x_\pi \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & x_{t-3} & x_{t-2} & x_{t-1} & & \\ & & & x_t & & & \end{array} \right\rangle$$

And

$$Y = \left\langle \begin{array}{ccccccc} & & & y_1 & & & \\ & & y_2 & y_3 & y_4 & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ y & \cdots & \cdots & y_\beta & \cdots & \cdots & y_\pi \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & y_{t-3} & y_{t-2} & y_{t-1} & & \\ & & & y_t & & & \end{array} \right\rangle$$

this means

$$Y^{-1} = -\frac{1}{{y_\beta}^2} \left\langle \begin{array}{ccccccc} & & & y_1 & & & \\ & & y_2 & y_3 & y_4 & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ y & \cdots & \cdots & -y_\beta & \cdots & \cdots & y_\pi \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & y_{t-3} & y_{t-2} & y_{t-1} & & \\ & & & y_t & & & \end{array} \right\rangle$$

$$= \left\langle \begin{array}{ccccccc} & & & -\frac{y_1}{{y_\beta}^2} & & & \\ & & -\frac{y_2}{{y_\beta}^2} & -\frac{y_3}{{y_\beta}^2} & -\frac{y_4}{{y_\beta}^2} & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ -\frac{y}{{y_\beta}^2} & \cdots & \cdots & \frac{y_\beta}{{y_\beta}^2} & \cdots & \cdots & -\frac{y_\pi}{{y_\beta}^2} \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & -\frac{y_{t-3}}{{y_\beta}^2} & -\frac{y_{t-2}}{{y_\beta}^2} & -\frac{y_{t-1}}{{y_\beta}^2} & & \\ & & & -\frac{y_t}{{y_\beta}^2} & & & \end{array} \right\rangle$$

therefore,

$$XY^{-1} = \left\langle \begin{array}{ccccccc} & & & \tau_1 & & & \\ & & \tau_2 & \tau_3 & \tau_4 & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ \tau & \cdots & \cdots & \tau_\beta & \cdots & \cdots & \tau_\pi \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & \tau_{t-3} & \tau_{t-2} & \tau_{t-1} & & \\ & & & \tau_t & & & \end{array} \right\rangle$$

Where

$$\tau_1 = \frac{x_1 y_\beta - y_1 x_\beta}{{y_\beta}^2}$$

$$\tau_2 = \frac{x_2 y_\beta - y_2 x_\beta}{{y_\beta}^2}$$

$$\tau_3 = \frac{x_3 y_\beta - y_3 x_\beta}{{y_\beta}^2}$$

$$\cdots$$
$$\cdots$$
$$\cdots$$

$$\tau_3 = \frac{x_\beta y_\beta}{{y_\beta}^2}$$

$$\cdots$$
$$\cdots$$
$$\cdots$$

$$\tau_{t-2} = \frac{x_{t-2} y_\beta - y_{t-2} x_\beta}{{y_\beta}^2}$$

$$\tau_{t-1} = \frac{x_{t-2}y_{\beta} - y_{t-2}x_{\beta}}{y_{\beta}^2}$$

$$\tau_t = \frac{x_t y_{\beta} - y_t x_{\beta}}{y_{\beta}^2}$$

Now,

$$Min(X) = \left\langle \begin{array}{ccccccccc} & & & & -x_1 & & & & \\ & & & -x_2 & -x_3 & -x_4 & & & \\ & \cdots & \cdots & & \cdots & & \cdots & \cdots & \\ -x & \cdots & \cdots & & x_{\beta} & & \cdots & \cdots & -x_{\pi} \\ & \cdots & \cdots & & \cdots & & \cdots & \cdots & \\ & & & -x_{t-3} & -x_{t-2} & -x_{t-1} & & & \\ & & & & -x_t & & & & \end{array} \right\rangle$$

$$Neg\left(\frac{1}{h(Y)^2} Min(X)Y\right) = \left\langle \begin{array}{ccccccccc} & & & & \sigma_1 & & & & \\ & & & \sigma_2 & \sigma_3 & \sigma_4 & & & \\ & \cdots & \cdots & & \cdots & & \cdots & \cdots & \\ \sigma & \cdots & \cdots & & \sigma_{\beta} & & \cdots & \cdots & \sigma_{\pi} \\ & \cdots & \cdots & & \cdots & & \cdots & \cdots & \\ & & & \sigma_{t-3} & \sigma_{t-2} & \sigma_{t-1} & & & \\ & & & & \sigma_t & & & & \end{array} \right\rangle$$

where

$$\sigma_1 = \frac{y_1 x_{\beta} - x_1 y_{\beta}}{y_{\beta}^2}$$

$$\sigma_2 = \frac{y_2 x_{\beta} - x_2 y_{\beta}}{y_{\beta}^2}$$

$$\sigma_3 = \frac{y_3 x_{\beta} - x_3 y_{\beta}}{y_{\beta}^2}$$

…
…
…

$$\sigma_{\beta} = -\frac{y_{\beta} x_{\beta}}{y_{\beta}^2}$$

…
…
…

$$\sigma_{t-1} = \frac{y_{t-1} x_{\beta} - x_{t-1} y_{\beta}}{y_{\beta}^2}$$

$$\sigma_t = \frac{y_t x_{\beta} - x_t y_{\beta}}{y_{\beta}^2}$$

Hence,

$$XY^{-1} + Neg\left(\frac{1}{h(Y)^2} Min(X)Y\right) = \left\langle \begin{array}{ccccccccc} & & & & 0_1 & & & & \\ & & & 0_2 & 0_3 & 0_4 & & & \\ & \cdots & \cdots & & \cdots & & \cdots & \cdots & \\ 0 & \cdots & \cdots & & 0_{\beta} & & \cdots & \cdots & 0_{\pi} \\ & \cdots & \cdots & & \cdots & & \cdots & \cdots & \\ & & & 0_{t-3} & 0_{t-2} & 0_{t-1} & & & \\ & & & & 0_t & & & & \end{array} \right\rangle$$

With the above definitions and results, we can now define inverse for a rhotrix rhotrix as follows:
The multiplicative inverse of

$$\Gamma = \left\langle \begin{array}{ccccccc} & & & A_1 & & & \\ & & A_2 & A_3 & A_4 & & \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ A & \cdots & \ldots & I_\beta & \ldots & \cdots & A_\pi \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ & & A_{t-3} & A_{t-2} & A_{t-1} & & \\ & & & A_t & & & \end{array} \right\rangle$$

Where

$$I_\beta = \left\langle \begin{array}{ccccccc} & & & 0_1 & & & \\ & & 0_2 & 0_3 & 0_4 & & \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ 0 & \cdots & \ldots & 1_\beta & \ldots & \cdots & 0_\pi \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ & & 0_{t-3} & 0_{t-2} & 0_{t-1} & & \\ & & & 0_t & & & \end{array} \right\rangle$$

will simply be given by

$$\Gamma^{-1} = \left\langle \begin{array}{ccccccc} & & & -A_1 & & & \\ & & -A_2 & -A_3 & -A_4 & & \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ -A & \cdots & \ldots & I_\beta & \ldots & \cdots & -A_\pi \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ & & -A_{t-3} & -A_{t-2} & -A_{t-1} & & \\ & & & -A_t & & & \end{array} \right\rangle$$

But generally, if

$$\gamma = \left\langle \begin{array}{ccccccc} & & & A_1 & & & \\ & & A_2 & A_3 & A_4 & & \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ A & \cdots & \ldots & A_\beta & \ldots & \cdots & A_\pi \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ & & A_{t-3} & A_{t-2} & A_{t-1} & & \\ & & & A_t & & & \end{array} \right\rangle$$ then its inverse will be

$$\gamma^{-1} = \left\langle \begin{array}{ccccccc} & & & F_1 & & & \\ & & F_2 & F_3 & F_4 & & \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ F & \cdots & \ldots & F_\beta & \ldots & \cdots & F_\pi \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ & & F_{t-3} & F_{t-2} & F_{t-1} & & \\ & & & F_t & & & \end{array} \right\rangle$$

where

$$F_1 = (Neg)\left(\frac{1}{{A_\beta}^2} Min(F_1)\right)$$

$$F_2 = (Neg)\left(\frac{1}{{A_\beta}^2} Min(F_2)\right)$$

$$\ldots$$

$$\ldots$$

$$\ldots$$

$$F_\beta = {A_\beta}^{-1}$$

$$\ldots$$

$$\ldots$$

$$F_{t-1} = (Neg)\left(\frac{1}{{A_\beta}^2} Min(F_{t-1})\right)$$

$$F_t = (Neg)\left(\frac{1}{{A_\beta}^2} Min(F_t)\right)$$

1.7 Definition

$$\text{Let ZD}=\left\{\left\langle \begin{array}{ccccccc} & & & A_1 & & & \\ & & A_2 & A_3 & A_4 & & \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ A & \cdots & \ldots & 0_\beta & \ldots & \cdots & A_\pi \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ & & A_{t-3} & A_{t-2} & A_{t-1} & & \\ & & & A_t & & & \end{array} \right\rangle : \text{ at least one of } A_1, A_2, \ldots, A_t \neq 0 \in R \right\}$$

1.8 Proposition

(ξ/ZD, •) is a commutative group.

Proof:

The proof follows from equations (5), (13), (15) and Lemma (1.6).

The left and the right distributivities also hold. That is, if

$\nu, \omega, \iota \in \xi$

Then $\nu(\omega + \iota) = \nu\omega + \nu\iota$ and $(\omega + \iota)\nu = \omega\nu + \iota\nu$ *(16)*

This can be shown as follows: suppose

$$\nu = \left\langle \begin{array}{ccccccc} & & & H_1 & & & \\ & & H_2 & H_3 & H_4 & & \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ H & \cdots & \ldots & H_\beta & \ldots & \cdots & H_\pi \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ & & H_{t-3} & H_{t-2} & H_{t-1} & & \\ & & & H_t & & & \end{array} \right\rangle$$

$$\omega = \left\langle \begin{array}{ccccccc} & & & J_1 & & & \\ & & J_2 & J_3 & J_4 & & \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ J & \cdots & \ldots & J_\beta & \ldots & \cdots & J_\pi \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ & & J_{t-3} & J_{t-2} & J_{t-1} & & \\ & & & J_t & & & \end{array} \right\rangle$$

and

$$\iota = \left\langle \begin{array}{ccccccc} & & & K_1 & & & \\ & & K_2 & K_3 & K_4 & & \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ K & \cdots & \ldots & K_\beta & \ldots & \cdots & K_\pi \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ & & K_{t-3} & K_{t-2} & K_{t-1} & & \\ & & & K_t & & & \end{array} \right\rangle$$

then

$$\nu(\omega + \iota) = \left\langle \begin{array}{ccccccc} & & & L_1 & & & \\ & & L_2 & L_3 & L_4 & & \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ L & \cdots & \ldots & L_\beta & \ldots & \cdots & L_\pi \\ & \cdots & \ldots & \ldots & \ldots & \cdots & \\ & & L_{t-3} & L_{t-2} & L_{t-1} & & \\ & & & L_t & & & \end{array} \right\rangle$$

$$= \nu\omega + \nu\iota$$

where

$$L_1 = H_1(J_\beta + K_\beta) + H_\beta(J_1 + K_1) = H_1 J_\beta + H_1 K_\beta + H_\beta J_1 + H_\beta K_1$$

$$= H_1 J_\beta + H_\beta J_1 + H_1 K_\beta + H_\beta K_1$$

$$L_2 = H_2(J_\beta + K_\beta) + H_\beta(J_2 + K_2) = H_2 J_\beta + H_2 K_\beta + H_\beta J_2 + H_\beta K_2$$

$= H_2 J_\beta + H_\beta J_2 + H_2 K_\beta + H_\beta K_2$

$L_3 = H_3\,(J_\beta + K_\beta) + H_\beta\,(J_3 + K_3) = H_3 J_\beta + H_3 K_\beta + H_\beta J_3 + H_\beta K_3$
$= H_3 J_\beta + H_\beta J_3 + H_3 K_\beta + H_\beta K_3$
…
…
…
$L_\beta = H_\beta J_\beta + H_\beta K_\beta$

…
…
…
$L_{t-1} = H_{t-1}\,(J_\beta + K_\beta) + H_\beta\,(J_{t-1} + K_{t-1}) = H_{t-1} J_\beta + H_{t-1} K_\beta + H_\beta J_{t-1} + H_\beta K_{t-1}$
$= H_{t-1} J_\beta + H_\beta J_{t-1} + H_{t-1} K_\beta + H_\beta K_{t-1}$
$L_t = H_t\,(J_\beta + K_\beta) + H_\beta\,(J_t + K_t) = H_t J_\beta + H_t K_\beta + H_\beta J_t + H_\beta K_t$
$= H_t J_\beta + H_\beta J_t + H_t K_\beta + H_\beta K_t$

Similarly, it can be shown for $(\omega + \iota)v = \omega v + \iota v$.

1.9 Definition

A nonid field is defined in [5] as the set F_N with two binary operations (+) and (•) defined on it such that;

(1) $(F_N, +)$ is a commutative group.
(2) $\exists\, a \neq 0,\, b \neq 0 \in F_N \ni ab = 0$.
(3) $(F_N - ZD, \bullet)$ is commutative group.
(4) $\forall, b, c \in F_N$, $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$

1.10 Definition

A nonid field is said to be finite if it has finite number of elements.
ZD here is the set of all zero divisors of F_N.

1.11 Proposition

$(\xi, +, \bullet)$ is a nonid field.
Proof: The proof follows from propositions (1.2), (1.8) and the discussion on equation (16) above.
Moreover, it can be shown that the set ξ of all rhotrix rhotrices forms a vector space over the set *R* of all rhotrices. The vector space ξ is spanned by the following vectors:

$$v_1 = \left\langle \begin{array}{ccccccc} & & & I_1 & & & \\ & & 0_2 & 0_3 & 0_4 & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ 0 & \cdots & \cdots & 0_\beta & \cdots & \cdots & 0_\pi \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & 0_{t-3} & 0_{t-2} & 0_{t-1} & & \\ & & & 0_t & & & \end{array} \right\rangle$$

$$v_2 = \left\langle \begin{array}{ccccccc} & & & 0_1 & & & \\ & & I_2 & 0_3 & 0_4 & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ 0 & \cdots & \cdots & 0_\beta & \cdots & \cdots & 0_\pi \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & 0_{t-3} & 0_{t-2} & 0_{t-1} & & \\ & & & 0_t & & & \end{array} \right\rangle$$

$$v_3 = \left\langle \begin{array}{ccccccc} & & & 0_1 & & & \\ & & 0_2 & I_3 & 0_4 & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ 0 & \cdots & \cdots & 0_\beta & \cdots & \cdots & 0_\pi \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & 0_{t-3} & 0_{t-2} & 0_{t-1} & & \\ & & & 0_t & & & \end{array} \right\rangle$$

…
…

$$v_{t-2}=\left\langle\begin{array}{c} 0_1 \\ 0_2 \quad 0_3 \quad 0_4 \\ \cdots \quad \ldots \quad \ldots \quad \ldots \quad \cdots \\ 0 \quad \cdots \quad \ldots \quad 0_\beta \quad \ldots \quad \cdots \quad 0_\pi \\ \cdots \quad \ldots \quad \ldots \quad \ldots \quad \cdots \\ 0_{t-3} \quad I_{t-2} \quad 0_{t-1} \\ 0_t \end{array}\right\rangle$$

$$v_{t-1}=\left\langle\begin{array}{c} 0_1 \\ 0_2 \quad 0_3 \quad 0_4 \\ \cdots \quad \ldots \quad \ldots \quad \ldots \quad \cdots \\ 0 \quad \cdots \quad \ldots \quad 0_\beta \quad \ldots \quad \cdots \quad 0_\pi \\ \cdots \quad \ldots \quad \ldots \quad \ldots \quad \cdots \\ 0_{t-3} \quad 0_{t-2} \quad I_{t-1} \\ 0_t \end{array}\right\rangle$$

$$v_{t}=\left\langle\begin{array}{c} 0_1 \\ 0_2 \quad 0_3 \quad 0_4 \\ \cdots \quad \ldots \quad \ldots \quad \ldots \quad \cdots \\ 0 \quad \cdots \quad \ldots \quad 0_\beta \quad \ldots \quad \cdots \quad 0_\pi \\ \cdots \quad \ldots \quad \ldots \quad \ldots \quad \cdots \\ 0_{t-3} \quad 0_{t-2} \quad 0_{t-1} \\ I_t \end{array}\right\rangle$$

such that

$$A_1V_1 + A_2V_2 \ldots + A_{t-2}V_{t-2} + A_{t-1}V_{t-1} + A_tV_t = 0$$

only when

$$A_1 = A_2 = \ldots = A_{t-2} = A_{t-1} = A_t = 0$$

This means they are linearly independent, hence the set

$$B = \{V_1, V_2, \ldots + V_{t-2}, V_{t-1}, V_t\}$$

forms a basis for ξ.

So

$$\xi = \{A_1V_1 + A_2V_2 \ldots + A_{t-2}V_{t-2} + A_{t-1}V_{t-1} + A_tV_t \;\; A1V1\!: \; A_1, A_2, \ldots, A_t \in R\}$$

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