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Controlling the HyperchaoticLorenz System using Integrator Backstepping

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Abstract

In this paper, an integrator backstepping technique that employs a single-control input is extended for the design of the control function that is capable of driving a hyperchaotic dynamical system to different desirable dynamical behaviors. The Lorenz hyperchaotic system is used as a typical model to illustrate the approach. The paper presents both theoretical and corresponding numerical simulations; and the result shows that, the hyperchaotic Lorenz system was effectively controlled. The control of this system to a specific fixed point as well as a bounded point was achieved. Trajectory tracking of a desired periodic function is also illustrated

1.0 Introduction

A deterministic nonlinear system is said to be chaotic whenever its evolution depends sensitively on the initial conditions [1,2]. This property implies that two trajectories emerging from two different closely initial conditions separate exponentially in the course of time. For a deterministic continuous system to be chaotic, the system must be nonlinear and must be at least three dimensional.

As at now, no definition of the term chaos is universally accepted yet [2], but almost all the researchers agree that chaos is a periodic long-term behaviour in a deterministic dynamical system that exhibits sensitive dependence on initial conditions. A measure of sensitive dependence on initial conditions, which is the hallmark of chaotic systems, is the Lyapunov exponent. In general, a positive Lyapunov exponent is an indicator of chaotic behavior; while a negative Lyapunov exponent denotes periodic behavior.

When a deterministic nonlinear system has more than three dimensions as could be found in lasers and Plasma models, it could exhibit hyperchaotic dynamics [3] in which case the system has at least two positive Lyapunov exponents corresponding to two of its dimensions and indicating complex instability. In recent times, interest in the generation, control and synchronization has witnessed tremendous increase. A large number of hyperchaotic systems has been identified, some of which were obtained from existing chaotic systems by artificial increase the systems' dimensions. One of such is the hyperchaotic Lorenz system [4].

Chaotic and hyperchaoticbehaviours occur naturally in many physical, biological, engineering, and social systems [5]. It could be beneficial in some applications, however, it is undesirable in many engineering and other physical applications and its control is therefore very important in order to improve the system performance.

The idea of chaos control was enunciated at the beginning of the last decade at the University of Maryland [6]. Control of chaos refers to a process wherein a perturbation is applied to a chaotic system, in order to realize a desirable (chaotic, periodic or steady-state) behaviour. This implies stabilization by means of small system perturbations of one of

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the unstable periodic orbits embedded in the chaotic attractor. The result of such control is to render an otherwise chaotic motion more stable and predictable which is often an advantage. Care must however be taken to ensure that such perturbation should be as small as possible, to avoid significant modification of the system's natural dynamics and ensure experimental realization [1,6].

Strategies for the control of chaos have grown in the last two decades since the pioneering work of Ott,Grebogi and Yorke (popularly known as OGY closed-loop method) [7]. Other methods includethePyragas feedback and non-feedback methods [8-10],differential geometric[11-13], active control [14, 15] inverse optimal control and adaptive control [16-22]. Feedback method could be classified as static, dynamic or open-loop feedback controls[23, 24]; and could also belong to either the linear or nonlinear feedback control forms. Backstepping techniques belong to nonlinear feedback control methods. They include back-stepping for strict feedback system [5, 25, 26], adaptive back-stepping [27, 28], robust back-stepping [29], recursive back-stepping and the integrator back-stepping[5, 23]which is used in this paper. The integrator backstepping is outstanding for its ability to achieve global stability, tracking, and transient performance for a broad class of strict-feedback nonlinear system. It has the advantages of wider applicability to a variety of chaotic systems with or without external excitation; and a controller could be sufficient to achieve stabilization of chaotic systems, thereby, reducing controller complexity. Furthermore, there are no derivatives in the controller [30]; the controller is singularity free from the nonlinear term of quadratic type, gives flexibility to construct a control law which can be extended to higher dimensional hyperchaotic systems, and the closed-loop system is globally stable; it requires less control effort in comparison with the differential geometric method.

In our very recent paper [27], we introduced a backstepping approach that uses a single-control function for the synchronization and control of a three-dimensional chaotic circuit. In this paper, we present preliminary results of an attempt to design a single-control input for the control of four-dimensional hyperchaotic systems. Here, we develop an integrator backsteppingmethod based on Lyapunov stability theory and obtain appropriatecontrol function for the control of hyperchaotic systems. A typical hyperchaotic Lorenz system was used to derive the control input. We illustrate numerically the effectiveness of the proposed method for the stabilization to the origin and bounded points. We also show that the method could be used to track a trajectory.

2. The Hyper-chaotic Lorenz System

The Lorenz chaotic system is given by

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= cx - xz - y \\ \dot{z} &= xy - bz \end{aligned} \tag{2.1}$$

Where *a* is the Pradtil number and *c* is the Rayleigh (or Reynolds) number[2, 4]. In the convention problem, *b* is related to the aspect ratio of the rolls. The variables *x*, *y* and *z* are dimensionless quantities, proportional to the circulatory fluid flow velocity, temperature difference between ascending and descending fluid elements, and the distortion of the vertical temperature profile from its equilibrium, respectively. For $a = 10$ and $b = 8/3$, the Lorenz system exhibits chaotic behavior when $c \approx 24.74$. Control of chaotic behavior in system (2.1) has been achieved by various means including backstepping. The Lorenz chaotic system (2.1) becomes hyper chaotic system if the following conditions are satisfied:

(i) The minimal dimension of the phase space that embeds the chaotic system becomes at least four, which will require the minimum number of coupled first order autonomous differential equations to be four[31-33] .

(ii) The number of terms in the coupled equations giving rise to instability would be at least two, of which at least one should have a non-linear function [32, 33].

Based on these conditions, the Hyperchaotic form of the Lorenz chaotic system (2.1) above was obtained by introducing a simple quadratic dynamic feedback term to system (2.1) above by Gao et al. [4] giving rise to the following fourth-order autonomous dynamical system:

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= cx + y - xz - w \\ \dot{z} &= xy - bz \\ \dot{w} &= kzy \end{aligned} \tag{2.2}$$

System (2.2) was proposed recently by Gao et al. [4] and it exhibits interesting dynamics including hyperchaosfor $a = 10$, $b = 8/3$, $c = 28$, and $k = 0.1$. A typical hyperchaotic orbit obtained using the above parameter values is shown in Figure 1

3. Theory of Integrator Back-stepping

The goal of this paper is to use the integrator backstepping technique to control a hyperchaoticLorenz system.The relevant technique is described below. Consider a second order system

$$\begin{aligned} \dot{x} &= x^2 - x^3 + u \\ \dot{\zeta} &= u \end{aligned} , \tag{3.1}$$

where$x$ is a variable and u is a parameter. The design objective is to stabilize the dynamics of system (3.1) such that $x(t) \to \varphi$ as $t \to \infty$, where φ is a fixed or bounded or a specified trajectory – the origin being a unique fixed or equilibrium point.

The control law can be synthesized in two steps. We denote ξ as a real control first, and by choosing the Lyapunov function

$$v_1 = \frac{1}{2} x^2 \tag{3.2}$$

and the control law $\xi_{des} = -x^2 - k_1 x \equiv \alpha(x)$,the control objective will be achieved. Nevertheless, ξ is a state and cannot be set to ξ_{des} .If we define the variable $z = \xi - \xi_{des}$ as the derivation of ξ from its desired value ξ_{des} with the definition of the error variable, we have

$$z = u - (2x + k_1)(k_1 x + x^3 - z) \tag{3.3}$$

The Lyapunov function candidate can be augmented as

$$v_2 = v_1 + \frac{1}{2} z^2 \tag{3.4}$$

Its time derivative is

$$\dot{v}_2 = x(-x^3 - k_1 x + z) + z[u - (2x + k_1)(k_1 x + x^3 - z)] \tag{3.5}$$

In principle, the control goal would be achieved if $\dot{v}_2$ isnegative definite.By choosing the control law as

$$u = -x + (2x + k_1(k_1 x + x^3 - z)) - k_2 z \tag{3.6}$$

then, we obtain

$$\dot{v}_2 = -x^4 - k_1 x^2 - k_2 z^2 \tag{3.7}$$

which is negative definite. This implies that $x(t) \to \varphi$ and $\xi \to \xi_{des}$ asymptotically. In this example ξ is called a virtual control, and its desired value $\alpha(x)$ is called a stabilizing function.

It should be noticed that system (1) above can also be stabilized by a linearizing control law

$$u = -(2x - 3x^2)x - k_1 x - k_2 x \tag{3.8}$$

However, the $-x^2$ term which helps in stabilizing equation (3.1) is cancelled by the linearizing control law (2.2). This shows the peculiarity of the integrator back stepping design as it can avoid cancellation of useful nonlinearities.The result of integrator back stepping is summarized in the following theorem.

Theorem : Consider the following system

$$\dot{x} = f(x) + g(x)\xi$$
$$\xi = u \qquad (3.9)$$

where $f(0) = 0$ is the fixed or equilibrium point. If there exist a stabilizing function $\dot{\xi} = \alpha(x)$ and a positive definite radially bounded Lyapunovfunction $v : R^n \to R$ such that

$$\frac{\partial v}{dx}[f(x) + g(x)]\alpha(x) < 0 \qquad (3.10)$$

then, the control

$$u = -c[\xi - \alpha(x)] + \frac{\partial v}{dx}[f(x) + g(x)]\alpha(x) < 0 \qquad (3.11)$$

will asymptotically stabilize the fixed or equilibrium point of system (2.2).

4.0 Control Formulation

The goal of this work is to design an effective control function based on integrator back stepping technique that will drive the hyper chaotic Lorenz system to a desired stable point. The idea is to add a single control input to any of the equations in (2.1) containing non – linear terms. In this way, control can be achieved to the origin and to bounded point. A trajectory can also be tracked in a systematic way

4.1 Stabilization to equilibrium point

In order to control Lorenz system to the origin point (0, 0, 0, 0) we add a control input u_1 to second equation of the system (2.1). Thus, the controlled system becomes:

$$\dot{x} = a(y - x)$$
$$\dot{y} = cx + y - xz - w + u_1$$
$$\dot{z} = xy - bz \qquad (4.1)$$
$$\dot{w} = kzy$$

Where u_1 is the required controller. For the virtual control y, we design a stabilizing function $\alpha_1(x)$ to make the derivative of

$$V_1(x) = \frac{x^2}{2} \qquad (4.2)$$

which is

$$\dot{V}_1(x) = -ax^2 + axy \qquad (4.3)$$

to be negative definite as $y = \alpha_1(x)$. Suppose $\alpha_1(x) = 0$ and define an error variable

$$\bar{y} = y - \alpha_1(x) \qquad (4.4)$$

Then, we obtain the $(x, \bar{y})$ subsystem as follows from *(3)*

$$\dot{x} = a(y - x)$$

But $\bar{y} = y - \alpha_1(x) = y$ and from (4.4) since $\alpha_1(x) = 0$, then $\dot{x} = a(\bar{y} - x)$

$$\dot{x} = -ax + a\bar{y}. \qquad (4.5)$$

Similarly,

$$\dot{y} = cx - xz + \bar{y} - w + u_1. \qquad (4.6)$$

Combining (4.5) and (4.6) we have the $(x, \bar{y})$ subsystem as follows

$$\dot{x} = -ax + a\bar{y}$$
$$\dot{y} = cx - xz + \bar{y} - w + u_1 \qquad (4.7)$$

Consider the following Lyapunov function;

$$v_2(x, \bar{y}) = v_1(x) + \frac{1}{2}\bar{y}^2$$

The time derivative of $v_2(x, \bar{y})$ is given by

$$\dot{v}_2(x,y) = \dot{v}_1(x) + \bar{y}\dot{\bar{y}} \qquad (4.8)$$

Substituting for *(4.3)* & *(4.7)* in *(4.8)* we have

$$\dot{v}_2(x,\bar{y}) = -ax^2 + ax\bar{y} + cx\bar{y} - xz\bar{y} + \bar{y}^2 - w\bar{y} + u_1\bar{y}) \qquad (4.9)$$

By introducing $-\bar{y}^2 + \bar{y}^2$ in the R.H.S. of the above equation, we have

$$\dot{v}_2(x,\bar{y}) = -ax^2 - \bar{y}^2 + \bar{y}(ax + cx - xz + 2\bar{y} - w + u_1) \qquad (4.10)$$

With this we have proved that in the $(x,\bar{y})$ subsystem, the equilibrium (0,0,0,0) of the subsystem is asymptotically stable.

According to *(4.1)*, $\alpha_1(x)=0$, $x \to 0$ and $y \to 0$ and the third equation of system *(4.1)*, we get that (x,y,z) the controlled system *(4.1)* tends to the fixed point when we choose the control input by making,

$$\dot{\bar{y}} = -\bar{y} \qquad (4.11)$$

so that

$$u_1 = w + x[z - (a+c)] - 3y \qquad (4.12)$$

4.2 Tracking desired trajectory

Here, we wish to find a control law u_2 so that a scalar output $\alpha(t)$ of the Hyperchaotic Lorenz system can track any desired trajectory $r(t)$. Replacing u_1 by u_2 in Eqn. (4.1) we now have

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= cx + y - xz - w + u_2 \\ \dot{z} &= xy - bz \\ \dot{w} &= kzy \end{aligned} \qquad (4.13)$$

Let $\bar{x}$ be the deviation between the output x and the desired trajectory $r(t)$. That is,

$$\bar{x} = x - r(t). \qquad (4.14)$$

Recalling the Lyapunov function given by *(4.3)*, it's time derivative along the controlled system

$$v_1(x) = \frac{x^2}{2} \qquad (4.15)$$

can be obtained by substituting x with $\bar{x}$ from *(4.15)* and $\dot{\bar{x}}$ given by

$$\dot{\bar{x}} = \dot{x} - \dot{r} \qquad (4.16)$$

to get

$$\dot{v}_1 = (x - r)[a(y-x) - r]. \qquad (4.17)$$

To make $v_1(x)$ negative definite, let $\dot{\bar{x}} = -\bar{x}$, then we obtain that

$$y = \frac{a-1}{a}x + \frac{r+\dot{r}}{a}. \qquad (4.18)$$

Equation *(4.16)* becomes negative definite by choosing the virtual control y as

$$y = \frac{\dot{r} + r - x}{a} + x. \qquad (4.19)$$

Define the second Lyapunov function as

$$v_2 = v_1 + \frac{\bar{y}^2}{2} \qquad (4.20)$$

where

$$\bar{y} = y - \left(x + \frac{\dot{r} + r - x}{a}\right) \qquad (4.21)$$

so that

$$\dot{\bar{y}} = \dot{y} - \left(\dot{x} + \frac{\dot{r} + \ddot{r} - \dot{x}}{a} \right) \qquad (4.22)$$

Then,

$$\dot{v}_2 = \dot{v}_1 + \dot{y}\dot{\bar{y}} \qquad (4.23)$$

is negative definite by choosing the control law u_2 as follows:

Let $\dot{\bar{y}} = -\bar{y}$ *(4.24)*

then,

$$\dot{y} - \left[\dot{x} + \frac{\dot{r} + \ddot{r} - \dot{x}}{a} \right] = -\left[y - \left(x + \frac{\dot{r} + r - x}{a} \right) \right] \qquad (4.25)$$

From (4.1), we have

$$cx + y - xz - w + u_2 - \left[a(y - x) + \frac{\dot{r} + \ddot{r} - \dot{x}}{a} \right] = -\left[y - \left(x + \frac{\dot{r} + \ddot{r} - x}{a} \right) \right]$$

From which we obtain the control function as

$$u_2 = xz + ay - (a + c + 1)x + w + \frac{\ddot{r} + 2\dot{r} - \dot{x} + r - x}{a} \qquad (4.26)$$

4.3 Stabilization to a Bounded Point

Our objective here is to find a control u_3 for stabilizing the state space of the controlled system (4.1) at a bounded point, say, px where *p* is an arbitrary constant. To achieve this goal, we add the control input u_3 to the fourth equation of Eqn. (4.1) and obtain

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= cx + y - xz - w \\ \dot{z} &= xy - bz \\ \dot{w} &= kzy + u_3 \end{aligned} \qquad (4.27)$$

Starting from the first equation, we define a stabilizing function $\alpha_1(x)$ for the virtual control variable

$$\alpha_1(x) = px \qquad (4.28)$$

and find a derivative of

$$v_1(x) = \frac{x^2}{2} \qquad (4.29)$$

as

$$\dot{v}(x) = x\dot{x} \qquad (4.30)$$

so that

$$\dot{v}_1(x) = axy - ax^2 \qquad (4.31)$$

Defining the error variable as

$$\bar{y} = y - \alpha_1(x) \qquad (4.32)$$

we obtain the $(x, \bar{y})$ subsystem as follows

$$\begin{aligned} \dot{x} &= a\bar{y} - a(1 - p)x \\ \dot{\bar{y}} &= cx - xz - (ap - 1)y + px - w + ap(1 - p)x \end{aligned} \qquad (4.33)$$

Consider the second Lyapunov function given by: $v_2(x, \bar{y}) = v_1(x) + \frac{1}{2}\bar{y}^2$.

The time derivative of

$v_2(x, \bar{y})$ is $\dot{v}_2(x, \bar{y}) = \dot{v}_1(x) + \frac{1}{2}\bar{y}\dot{\bar{y}}$. *(4.34)*

By substituting for $\dot{v}_1(x)$ given by *(4.31)* and $\dot{\bar{y}}$ given by *(4.33)* in *(4.34)* we have:

$$\dot{v}_2(x) = -a(1-p)x^2 - (ap-1)y^2 - yw + xy\{z - a - c - p - ap(1-p)\} \qquad (4.35)$$

We can choose a virtual control $\alpha_2(x,\bar{y})$ such that

$$z = \alpha_2(x,\bar{y}) = a + c + p + ap(1-p) \qquad (4.36)$$

Similarly, by defining the error state as

$$\bar{z} = z - \alpha_2(x,\bar{y}) \qquad (4.37)$$

We can obtain the sub-system in the $(x,\bar{y},\bar{z})$ coordinates. From *(4.37)* $\dot{\bar{z}} = \dot{z} - \alpha_2(x,\bar{y})$ and from the third equation of *(4.1)*

$\dot{\bar{z}} = xy - bz$. Then,

$$\bar{z} = xy - bz - \alpha_2(x,\bar{y}) \qquad (4.38)$$

From *(4.36)* $\alpha_2(x,\bar{y}) = a + c + p + ap(1-p)$ which are all fixed parameters of the system, so that

$$\dot{\alpha}_2(x,\bar{y}) = 0 . \qquad (4.39)$$

Hence, we have $\bar{z} = xy - bz$ and

$$\dot{\bar{z}} = x\bar{y} + p^x - b\bar{z} - b(\ a + c + p + ap(1-p)\ ) \qquad (4.40)$$

So, in the $(x,\bar{y},\bar{z})$ coordinate, we have the sub-system

$$\begin{aligned} \dot{x} &= a\bar{y} - a(1-p) \\ \dot{\bar{y}} &= cx - xz - \bar{y}(ap-1) + px - w + ap(1-p)x \\ \dot{\bar{z}} &= x\bar{y} + px^2 - b\bar{z} - b[a + c + p + ap(1-p)] \end{aligned} \qquad (4.41)$$

Consider a third Lyapunov function given by

$$v_3(x,\bar{y},\bar{z}) = v_2(x,\bar{y}) + \frac{1}{2}\bar{z}^2 \qquad (4.42)$$

By substituting for sub-system $\dot{\bar{z}}$ in *(4.41)* in the time derivative of *(4.42)*, we have

$$\dot{v}_3(x,\bar{y},\bar{z}) = -a(1-ap)x^2 - (ap-1)\bar{y}^2 - \bar{y}w - x\bar{y}[z - a - c - p - ap(1-p)] + \bar{z}[x\bar{y} + px^2 - bz - b\{\ a - c - ap(1-p)\ \}]$$

which can be written as

$$\dot{v}_3(x,\bar{y},\bar{z}) = a - (1-ap)x^2 - (ap-1)\bar{y}^2 - b\bar{z}^2 - \bar{z}\left[\bar{y}\frac{w}{z} + px^2 - b(a + c + p + ap(1-p)\right] \qquad (4.43)$$

To make $\dot{v}_3(x,\bar{y},\bar{z})$ negative definite, we let

$$\bar{y}\frac{w}{\bar{z}} + px^2 - b[a + c + p + ap(1-p)] = 0 \qquad (4.44)$$

So that

$$w = \frac{\bar{z}b[a + c + p + ap(1-p)] - \bar{z}px^2}{\bar{y}} \qquad (4.45)$$

We can also choose a third virtual control $\alpha_3(x,\bar{y},\bar{z})$ such that

$$w = \alpha_3(x,\bar{y},\bar{z}) \qquad (4.46)$$

$$= \frac{\bar{z}[b\{a + c + p + ap(1-p)\}] - \bar{z}px^2}{\bar{y}} \qquad (4.47)$$

Similarly let

$$\bar{w} = w - \alpha_3(x,\bar{y},\bar{z}) \qquad (4.48)$$

Then we obtain the $(x,\bar{y},\bar{z},\bar{w})$ subsystem so that

$$\dot{\bar{w}} = \dot{w} + \dot{\alpha}_3(x,\bar{y},\bar{z}) \qquad (4.49)$$

$$\dot{\bar{w}} = kyz + u_3 + \frac{px^2\dot{\bar{z}}}{\bar{y}} .$$

But $\dot{\bar{z}} = xy - bz$, then

$$\dot{\overline{w}} = kzy + u_3 + \frac{bz(xy - bz) - px^2(xy - bz)}{\overline{y}} \tag{4.50}$$

Finally we have the entire system in the complete $(x, \overline{y}, \overline{z}, \overline{w})$ as

$$\begin{aligned}
\dot{x} &= a\overline{y} - a(1 - p) \\
\dot{\overline{y}} &= cx - xz - \overline{y}(ap - 1) + px - w + ap(1 - p)x \\
\dot{\overline{z}} &= x\overline{y} + px^2 - b\overline{z} - b[a + c + p + ap(1 - p)] \\
\dot{\overline{w}} &= kzy + u_3 + \frac{(bz - px^2)(xy - bz)}{\overline{y}}
\end{aligned} \tag{4.51}$$

Consider the following Lyapunov function

$$\dot{v}_4(x, \overline{y}, \overline{z}, \dot{w}) = -a(1 - ap)x^2 - (ap - 1)\overline{y}^2 - b\overline{z}^2 + \overline{w}\left[kzy + \frac{(xy - bz)(bz - px^2)}{\overline{y}} + u_3\right]. \tag{4.52}$$

We choose the control u_3 as follows. Let, then

$$kzy + \frac{(xy - bz)(bz - px^2)}{\overline{y}} + u_3 = -[w - \alpha_3(x, \overline{y}, \overline{z})]$$ So that

$$u_3 = -kzy - w + \frac{(xy - bz)[a + c + p + ap(1 - p) - bz]}{\overline{y}} \tag{4.53}$$

$v_4(\overline{x}, \overline{y}, \overline{z}, \overline{w})$ becomes negative definite, that is

$$v_4(x, \overline{y}, \overline{z}, \overline{w}) = -a(1 - ap)x^2 - (ap - 1)\overline{y}^2 - b\overline{z}^2 \tag{4.54}$$

5.0 Numerical results

All the numerical results that follow were obtained using a standard 4th order Runge-Kutta algorithm with fixed integration time-step, $\Delta h = 0.005$.

5.1 Dynamics of the hyperchaotic Lorenz system

The Lorenz hyperchaotic described by system (2.2) exhibits varieties of dynamical behaviour, the most significant in this work being hyperchaos. For system (2.2), the parameters used are $a = 10, b = \frac{8}{3}, c = 28, \ k = 0.1$ as in Gao et al. [19]. When *k* is varied, chaotic periodic and hyperchaotic attractors are observed. For instance, in Fig. 1, phase space showing the hyperchaotic attractor for *k=0.1* is depicted in (a); while in (b) and (c) the corresponding time series for the x and z variables are displayed.

5.2 Stabilization to the equilibrium point

In Fig. 2, all the state variables (x, y, z, w) have been plotted simultaneously in a composite plot, the controller being activated at $t \geq 20$. Clearly, control to stable state was achieved. However, we find that the state variable *w* (in pink) is driven to a constant stable state at w≈ 50 as soon as the control is achieved. Theoretically, all the dimensions should approach zero equilibrium. The deviation from the expectation equilibrium state could be attributed to the position of the controller.

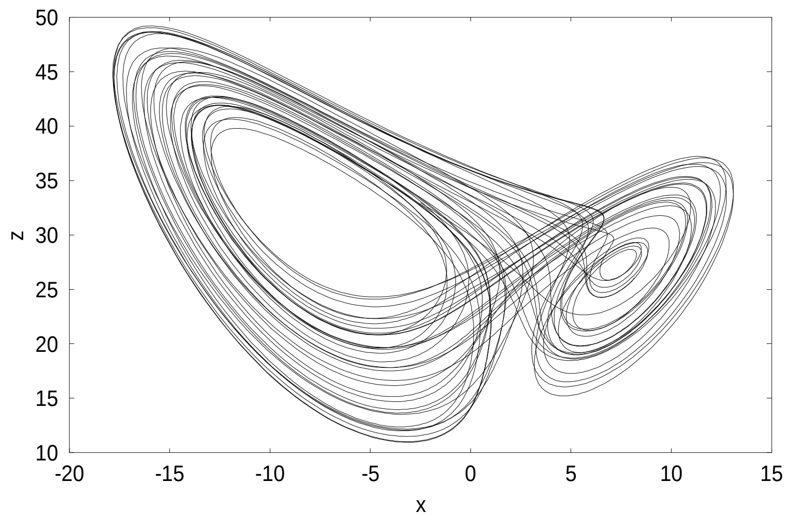
5.3 Tracking a desired trajectory

To track the trajectory $r = sint$, the controller u_2 was activated at $t \geq 20s$ .In Fig. 3 global tracking is achievedas soon as the control is activated.

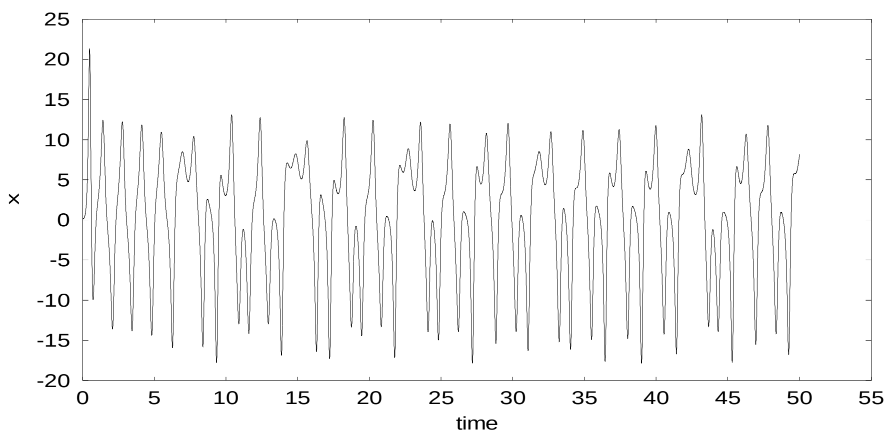
5.4 Stabilization to bounded points

To achieve stabilization to bounded points, which is not an equilibrium point, we set *p = 0.02* and the control u_3 $t \geq 20s$ .The effect of the control is seen in Fig. 4. All the variables are stabilized at various points as soon as the control action is on.

(a)



(b)



(c)

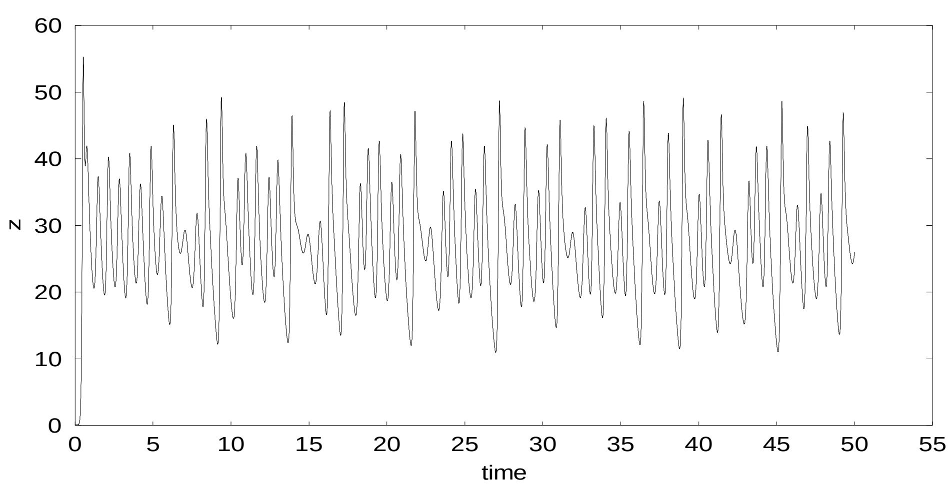


Figure 1: Chaotic dynamics of the Hyperchaotic Lorenz System (a) phase portrait (b) time series of the *x* – variable (c) time series of the *z* – variable.

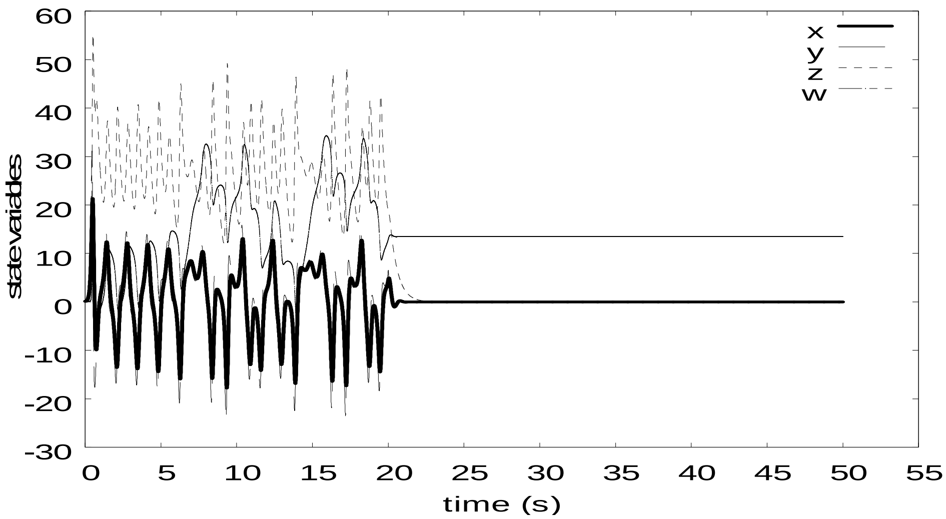


Figure 2: Control to the origin. The controller U_1 has been activated at $t = 20$.

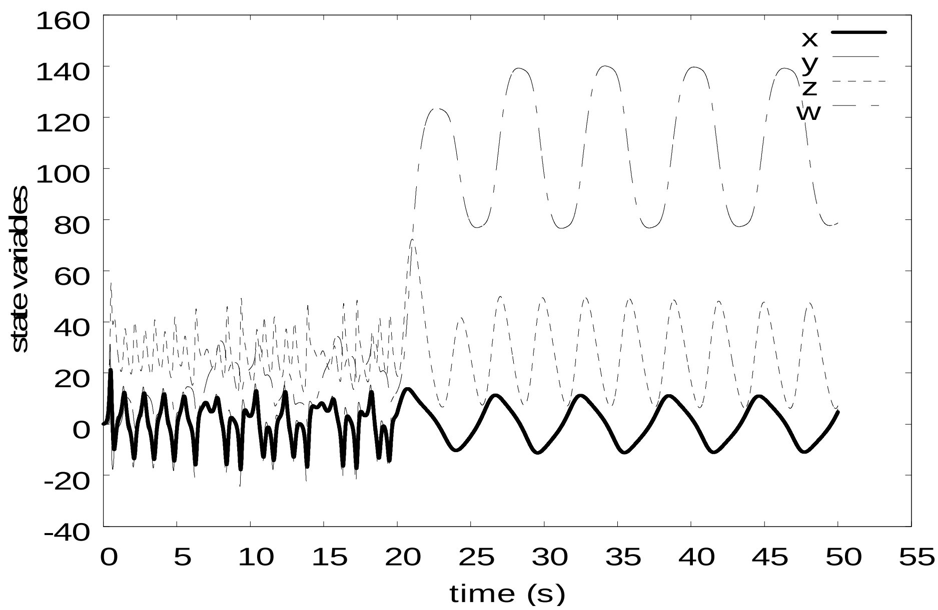


Figure 3: Tracked orbit $r = \sin t$. The controller u_2 has been activated at $t \geq 20s$.

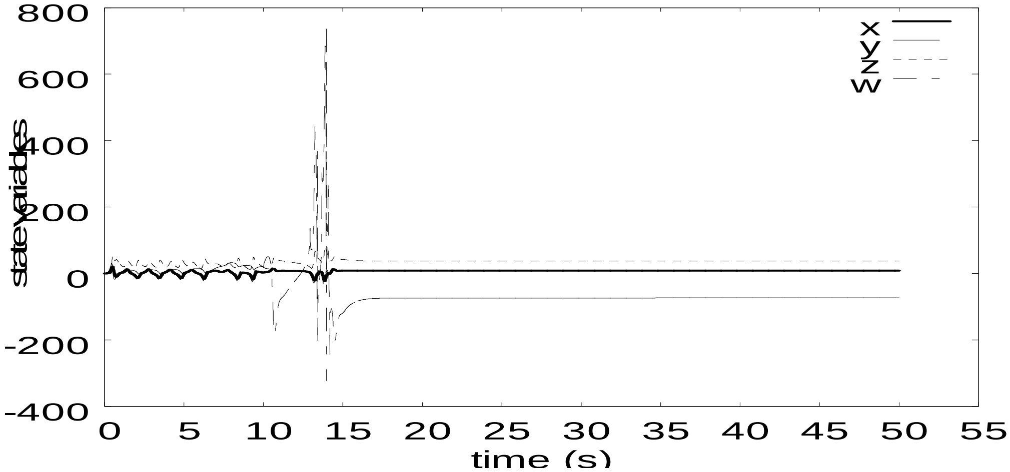


Figure 4: Stabilization to bounded points. The controller U_2 has been activated at $t \geq 20s$ and $p = -0.6$.

5.0 Concluding Remarks

We have presented in this paper the results of an attempt to address the problem of controller complexity by designing a single-control input for the control of hyperchaotic systems based on an integrator backsteppingmethod which uses the Lyapunov stability theory. A singledcontrol function was obtained for the control of a typical hyperchaotic Lorenz system. We have illustrated numerically the effectiveness of the proposed method for the stabilization to the origin and bounded points; and also showed that the method could be used to track a trajectory. We remark that from our observation, the choice of the position of the control function in the state equations could significantly affect the control performance; and this should be properly chosen to ensure effective control performance. This could have remarkable consequences in hyperchaos synchronization with different synchronized states emerging, as we would show in our forthcoming paper.

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References

[1] Boccaletti, S., Grebogi, C; Lai, Y. C; Mancinv, H and Maza, D. The control of chaos; theory and applications. Physics reports **329** (2000) **103 – 197**.
[2] Strogatz, S. H. (2000): Non – linear Dynamics and chaos: with applications to Physics, Biology, Chemistry and Engineering, Persues Books publishing, LLC, USA.
[3] Tiengang, G., Guanrong, C., Zengqiang C. and Shijian, C. The generation and Circuit Implementation of a New Hyperchaos Based Upon Lorenz system. Physics letters A. **361** (2007) **78 – 86.**
[4] Gao, T., Chen, G., Chen, Z and Cang, S: The generation and circuit implementation of a new hyper chaos based upon Lorenz system. Phys. Lett. A **361** (2007) **78 - 86**.
[5] Njah, A. N. Tracking control and synchronization of the new hyperchaotic Liu system via backstepping techniques. Nonlinear Dynamics **61** (2010) **1–9.**
[6] Ott, E., Grebogi, C. and Yorke, J. A. in: D Campbell (Ed). Chaos / XAOC, Soviet – American Perceptive on non-linear science, American Institute of Physics, New York. (1990), **153 – 173.**
[7] Ott, E., Grebogi, C. And Yorke, J.A.: Controlling chaos. Phys. Rev. Lett. **64**, (1990) **1196–1199**.
[8] Pyragas, K.: Control of chaos via an unstable delayed feedback controller. Phys. Rev. Lett.**86**, (2001) **2265–2268**
[9] Pyragas K: Continuous control of chaos by self-controlling feedback. PhysLett A **170** (1992) **421**
[10] Pyragas, V. and Pyragas, K: Delayed feedback control of periodic orbits without torsion in non-autonomous chaotic systems - Theory and experiment.Phys. Rev. 76, 026203 (2007).
[11] Fun, C. C. and Tung P. C. Controlling Chaos using Differential Geometric Method, Phys. Rev. Lett. **75** (1995) **2952–2955**
[12] Liaw, Y. M. and Tung, P. C., Application of the differential geometric method to control a noisy chaotic system via dither smoothing, Phys. Lett. A 239 (1998) **51-58**.

[13] Huijberts, H. J. C. and Nijmeijer, H. Nonlinear chaos control and synchronization. In: E. Schöll and H. G. Schuster (Eds) Handbook of chaos control. Wiley, 2008.

[14] Vincent, U. E. Recursive Active control for controlling chaos in Nonlinear Bloch equations. In: Proceeding of the 3rd International Physics and Control Conference (PHYSCON), Potsdam, Germany, 2007, page **1176.**

[15] Vincent, U. E. and Laoye, J. A. Synchronization and control of directed transports in inertia ratchets using active control. Phys. Lett. A. **363** (2007) **91- 95.**

[16] Yassen, M. T. Chaos Control of Chaotic dynamical systems using back stepping design. Chaos Solitons and Fractals **27** (2006) **537 – 548.**

[17] Chen G., Dong X. from chaos to order: perspectives, methodologies and applications. Singapore. World scientific; 1998

[18] Chen, G. Ueta, T. Yet another chaotic attractor Int. J. Bifurcat chaos (1999) 9: 1465.

[18] Ueta T, and Chen, G. Bifurcation Analysis of Chen's Attractor. Int. J. Bifurcat chaos **10** (2000) **1917**-.

[19] Chen, G. and Dong, X. On feedback control of chaotic continuous time systems. IEEE Trans Circ system **40** (1993) **591 - .**

[20] Yassen, M. T. Chaos Control of Chaotic dynamical systems using back stepping design. Chaos Solitons and Fractals **27** (2006) **537 – 548.**

[21] Agiza, H. N. Controlling Chaos for the Dynamical System of coupled Dynamos. Chaos, Solitons& Fractals **12** (2000) **341 -** .

[22] Laoye, J. A., Vincent, U. E. and Kareem, S.O.: Chaos control of 4-D chaotic systems using recursive backstepping nonlinear controller. Chaos Solitons Fractals **39**, (2009) **356–362**.

[23] Idowu, B.A., Vincent, U.E. and Njah, A.N.: Synchronization of chaos in non-identical parametrically excited systems.ChaosSolitons Fractals **39** (2009) **2322–2331.**

[23] Yu, Y. and Zhang, S. Controlling uncertain Lü system using backstepping design. Chaos Solitons Fractals **15** (2003) **897–902.**

[24] Zhang, H., Ma, X.K., Yang, Y. and Xu, C.D.: Generalized synchronization of hyperchaos and chaos using active backstepping design. Chin. Phys. **14** (2005) **86–94**.

[25] Vincent, U. E., Odunaike, R. K., Laoye, J. A. and A. A. Gbindinninuola. Adaptive backstepping control and synchronization of a modified and chaotic Van der Pol-Duffing oscillator, J. Control Theory Appl**9**(2011) **141–145**.

[26] Olusola, I. O., Vincent, U. E. and Otekola, O. BacksteppingTechnigue for chaos control in the Energy Resource System, Nigeria J. of Physics **21**(2009) **109-118**.

[27] Li, Y., Qiang, S., Zhuang, X. and Kaynak, O. Robust and Adaptive Backstepping Control for Nonlinear Systems Using RBF Neural Networks, IEEE Trans. Neural Networks **15** (2004) **693-701**.

[28] Mascolo, S., Grassi, G.: Controlling chaos via backstepping design. Phys. Rev. E **56** (1997) **6166–6169**.

[29] Kapitaniak, T. (1995): Controlling chaos. Theoretical and practical methods in non – Linear Dynamics. London. Academic press.

[30] Jang, M.J., Chen, C.L. and Chen, C. K.: Sliding mode control of hyperchaos in Rössler systems. Chaos Solitons Fractals **14** (2002) **1465–1476**.

[31] Lai, Y. C. and Grebogi, C, Controlling Chaos Phys Rev. E. **49** (1994) **1094-1098.**

[32] Lai, Y. C. and Grebogi, C., Controlling Chaos Phys. Rev. E. **47** (1993) **2357-2360.**

[33] Kuntsevich, B. F. and Pisarchik, A. N. Synchronization effects in a dual-wavelength class-B laser with modulated losses, Phys. Rev. E **64** (2001) **046221**.