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Exact Solution Of The Stochastic Edwards-Wilkinson Equation With Reduced Differential Transform Method

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Abstract

The Reduced Differential Transform Method (RDTM) was used to obtain exact analytical solutions to the Edwards-Wilkinson equation. The Edwards-Wilkinson equation which is a stochastic linear partial differential equation was used to investigate the growth (or erosion) of interfaces by particle deposition for bounded noise for initially sinusoidal and flat surfaces. Results from the analytical solutions obtained through RDTM with Mathematica package was compared to the finite difference solutions. The study shows the efficacy of the RDTM in obtaining analytical solutions to the Edwards-Wilkinson equation and is therefore a wonderful tool for solving linear partial differential equations analytically.

Keywords: RDTM, Edwards-Wilkinson equation, Surface growth, Particle deposition

1.0 Introduction

Stochastic partial differential equations appear in several different applications including the study of random evolution of systems (random interface growth, random evolution of surfaces, fluids subject to random forcing). They include the Edwards-Wilkinson, the Kadar-Parisi Zhang and the Cuerno-Barabasi equations.

Random deposition is the simplest growth model. The goal of differential equations in surface growth is to find the variation with time of the interface height $h(x,t)$ at any position x. The Edwards-Wilkinson (EW) equation is a stochastic equation of motion describing the most basic surface evolution of a growth model, consisting merely of diffusion and random particle deposition [1]. The EW equation is a linear stochastic PDE written in mathematical form as:

$$\frac{\partial h(x,t)}{\partial t} = \upsilon \nabla^2 h(x,t) + \eta(x,t) \qquad (1)$$

Here υ is sometimes called a 'surface tension' for the Laplacian term $\upsilon\nabla^2 h$ tends to smooth the interface. The Laplacian term smoothens by redistributing the irregularities on the interface while maintaining the average height unchanged. Thus, the surface tension acts as a conservative relaxation mechanism [2]. The PDE is stochastic because of the presence of $\eta(x,t)$, a Gaussian and white noise with the following properties:

$$\langle \eta(x,t) \rangle = 0$$
$$\langle \eta(x,t)\, \eta(x',t') \rangle = \Gamma^2 \delta(x-x')\delta(t-t')$$

This shows that the noise has zero configurational average and has no correlations in space and time. Often used in numerical simulations is 'bounded noise' in which $\eta = 1$ and $\eta = -1$ [3]. The EW equation is a keen area of interest in surface science [4-6]. The objective of this study is to obtain analytical solution to the stochastic EW equation using the reduced differential transform method (RDTM).

2.0 Reduced Differential Transform Method

The basic definitions of the Reduced Differential Transform Method (RDTM) are introduced as follows [7];

If the function $u(x,t)$ is analytic and differentiable continuously with respect to time t and space x in the domain of interest, then we introduce $U_k(x)$ such that,

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=0} \qquad (2)$$

Where the t-dimensional spectrum function$U_k(x)$is the transformed function of $u(x,t)$. The inverse differential transform of $U_k(x)$ is defined as follows;

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$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)\, t^k \tag{3}$$

Combining equation (2) and (3) we can write

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=0} t^k \tag{4}$$

From the above definitions, it can be seen that the concept of the RDTM is derived from the power series expansion. According to the Reduced Differential Transform Method and Table 1 [7-9] we can construct an iteration formula to obtain values of $U_1,\ U_2,\ U_3,\ U_4,\ U_5,\ etc.$

The first term of the transformed solution $U_0(x)$is just the initial condition at the time $t = 0$. Also, for second order partial differential equations, the derivative of the initial condition with respect to time gives $U_1(x)$.The inverse transformation of the set of values $\{U_k(x)\}_{k=0}^{n}$ gives approximate solution as

$$\overline{u_n}(x,t) = \sum_{k=0}^{n} U_k(x)\, t^k \tag{5}$$

where n is the order of the approximation. Therefore, the exact solution of the equation is given by;

$$u(x,t) = \lim_{n\to\infty} \overline{u_n}(x,t) \tag{6}$$

Table 1: Reduced differential transformation.

FUNCTIONAL FORM	**TRANSFORMED FORM**
$u(x,t)$	$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k u(x,t)}{\partial t^k}\right]_{t=0}$
$u(x,t) \pm v(x,t)$	$U_k(x) \pm V_k(x)$
$\alpha u(x,t)$	$\alpha U_k(x)$(α is a constant)
$x^m t^n$	$x^m \delta(k-n)$
$x^m t^n u(x,t)$	$x^m U(k-n)$
$u(x,t)\, v(x,t)$	$\sum_{r=0}^{k} U_r(x)\, V_{k-r}(x)$
$\frac{\partial^r}{\partial t^r} u(x,t)$	$\frac{(k+r)!}{k!} U_{k+r}(x)$
$\frac{\partial}{\partial x} u(x,t)$	$\frac{\partial}{\partial x} U_k(x)$
Nonlinear Function $N\big(u(x,t)\big) = F\big(u(x,t)\big)$	$N_k = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} F(U_0)\right]_{t=0}$

We intend to use the Edwards-Wilkinson equation to investigate the growth (or erosion) of interfaces by particle deposition for bounded noise in which η takes the values +1 and -1. The initial conditions including other necessary parameters are given below:

Initial condition 1: $\mathrm{h}(x, t=0) = \sin\left(\frac{\pi x}{l}\right)$, $\mathrm{h}(0,t) = \mathrm{h}(l,t) = \eta t$

with $\upsilon = 0, 0.3, 0.5$ $\quad \Delta x = 0.1\ \ \Delta t = 0.01$ for $0 \le x \le 5$ (7)

Initial condition 2: $\mathrm{h}(x, t=0) = \frac{1}{2}\left[1 - \sinh\left(\frac{x-a}{b}\right)\right]$, $\mathrm{h}(0,t) = \mathrm{h}(l,t) = \eta t + 0.5 - 0.5\sinh\left(\frac{x-a}{b}\right)e^{vt/b^2}$
where $\mathrm{a} = 6230.55$ and $\mathrm{b} = 7069.1362$ with $v = 0$ and 0.5 $\Delta x = 0.1$ $\Delta t = 0.01$ for $0 \le x \le 5$ (8)

3.0 Analytical Solution of the Edwards-Wilkinson Equation.

Recall the one-dimensional Edwards-Wilkinson equation given by equation (1)

$$\frac{\partial h(x,t)}{\partial t} = v\frac{\partial^2 h(x,t)}{\partial x^2} + \eta(x,t)$$

Using Table 1, we can transform each term of equation (1) as follows;

$$v\frac{\partial h(x,t)}{\partial x^2} = v\frac{\partial^2 H_k(x)}{\partial x^2} \tag{9}$$

$$\frac{\partial h(x,t)}{\partial t} = \frac{(k+1)!}{k!}H_{k+1}(x) \tag{10}$$

$$\eta(x,t) = \eta\,\delta(k) \quad \text{where } \delta(k)\begin{cases} 1 & \text{if k} = 0 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Substituting equations (9) – (11) into equation (1) we get the transformed form of the Edwards-Wilkinson equation.

$$\frac{(k+1)!}{k!}H_{k+1}(x) = v\frac{\partial^2 H_k(x)}{\partial x^2} + \eta\,\delta(k) \tag{12}$$

From the initial conditions we can write

$$H_0(x) = f(x)$$

Subsequent $H_k(x)$ values are obtained by the following set of equations.

$$\frac{1!}{0!}H_1(x) = v\frac{\partial^2 H_0(x)}{\partial x^2} + \eta\,\delta(0)$$
$$\frac{2!}{1!}H_2(x) = v\frac{\partial^2 H_1(x)}{\partial x^2} + \eta\,\delta(1)$$
$$\frac{3!}{2!}H_3(x) = v\frac{\partial^2 H_2(x)}{\partial x^2} + \eta\,\delta(2)$$
$$\frac{4!}{3!}H_4(x) = v\frac{\partial^2 H_3(x)}{\partial x^2} + \eta\,\delta(3)$$
$$\frac{5!}{4!}H_5(x) = v\frac{\partial^2 H_4(x)}{\partial x^2} + \eta\,\delta(4)$$

The solution is given by the inverse differential transform of the $H_k(x)$ values as follows:

$$h_n(x,t) = \sum_{k=0}^{n} H_k(x)\,t^k$$
$$= H_0(x) + H_1(x)t^1 + H_2(x)t^2 + H_3(x)t^3 + H_4(x)t^4 + H_5(x)t^5 + H_6(x)t^6 + \dots\dots \tag{13a}$$

For initial condition 1 we have;

$$H_0(x) = f(x) = \sin\left(\frac{\pi x}{l}\right)$$
$$H_1(x) = v\frac{\partial^2 H_0(x)}{\partial x^2} + \eta$$

$$H_1(x) = -\upsilon\left(\frac{\pi}{l}\right)^2 \sin\left(\frac{\pi x}{l}\right) + \eta$$

$$H_2(x) = \frac{1}{2}\left(\upsilon\frac{\partial^2 H_1(x)}{\partial x^2}\right)$$
$$H_2(x) = \frac{1}{2}\upsilon^2\left(\frac{\pi}{l}\right)^4 \sin\left(\frac{\pi x}{l}\right)$$

$$H_3(x) = \frac{1}{3}\left(\upsilon\frac{\partial^2 H_2(x)}{\partial x^2}\right)$$
$$H_3(x) = -\frac{1}{6}\upsilon^3\left(\frac{\pi}{l}\right)^6 \sin\left(\frac{\pi x}{l}\right)$$

$$H_4(x) = \frac{1}{4}\left(\upsilon\frac{\partial^2 H_3(x)}{\partial x^2}\right)$$
$$H_4(x) = \frac{1}{24}\upsilon^4\left(\frac{\pi}{l}\right)^8 \sin\left(\frac{\pi x}{l}\right)$$

$$H_5(x) = \frac{1}{5}\left(\upsilon\frac{\partial^2 H_4(x)}{\partial x^2}\right)$$
$$H_5(x) = -\frac{1}{120}\upsilon^5\left(\frac{\pi}{l}\right)^{10} \sin\left(\frac{\pi x}{l}\right)$$
$$H_0(x) = \sin\left(\frac{\pi x}{l}\right) = \frac{1}{0!}\upsilon^0\left(\frac{\pi}{l}\right)^{2\times 0} \sin\left(\frac{\pi x}{l}\right)$$
$$H_1(x) = -\upsilon\left(\frac{\pi}{l}\right)^2 \sin\left(\frac{\pi x}{l}\right) + \eta = -\frac{1}{1!}\upsilon^1\left(\frac{\pi}{l}\right)^{2\times 1} \sin\left(\frac{\pi x}{l}\right) + \eta$$
$$H_2(x) = \frac{1}{2}\upsilon^2\left(\frac{\pi}{l}\right)^4 \sin\left(\frac{\pi x}{l}\right) = \frac{1}{2!}\upsilon^2\left(\frac{\pi}{l}\right)^{2\times 2} \sin\left(\frac{\pi x}{l}\right)$$
$$H_3(x) = -\frac{1}{6}\upsilon^3\left(\frac{\pi}{l}\right)^6 \sin\left(\frac{\pi x}{l}\right) = -\frac{1}{3!}\upsilon^3\left(\frac{\pi}{l}\right)^{2\times 3} \sin\left(\frac{\pi x}{l}\right)$$
$$H_4(x) = \frac{1}{24}\upsilon^4\left(\frac{\pi}{l}\right)^8 \sin\left(\frac{\pi x}{l}\right) = \frac{1}{4!}\upsilon^4\left(\frac{\pi}{l}\right)^{2\times 4} \sin\left(\frac{\pi x}{l}\right)$$
$$H_5(x) = -\frac{1}{120}\upsilon^5\left(\frac{\pi}{l}\right)^{10} \sin\left(\frac{\pi x}{l}\right) = -\frac{1}{5!}\upsilon^5\left(\frac{\pi}{l}\right)^{2\times 5} \sin\left(\frac{\pi x}{l}\right)$$

Hence, a general value for $H_k(x)$ is written as

$$H_k(x) = (-1)^k\frac{1}{k!}\upsilon^k\left(\frac{\pi}{l}\right)^{2k} \sin\left(\frac{\pi x}{l}\right) + \eta\,\delta(k-1) \qquad (13b)$$

Therefore, the analytical solution to the Edwards-Wilkinson equation can be obtained using

$$\mathrm{h}(x,t) = \sum_{k=0}^{\infty} H_k(x)\,t^k \qquad (14)$$

$$\mathrm{h}(x,t) = \sum_{k=0}^{\infty}\left[(-1)^k\frac{1}{k!}\upsilon^k\left(\frac{\pi}{l}\right)^{2k} \sin\left(\frac{\pi x}{l}\right) + \eta\,\delta(k-1)\right]t^k$$

$$\mathrm{h}(x,t) = \sum_{k=0}^{\infty}\left[(-1)^k \frac{1}{k!} v^k \left(\frac{\pi}{l}\right)^{2k} \sin\left(\frac{\pi x}{l}\right)\right] t^k + \sum_{k=0}^{\infty} \eta\, \delta(k-1)\, t^k$$

$$\mathrm{h}(x,t) = \sin\left(\frac{\pi x}{l}\right) \sum_{k=0}^{\infty}(-1)^k \frac{1}{k!} v^k \left(\frac{\pi}{l}\right)^{2k} t^k + \eta \sum_{k=0}^{\infty} \delta(k-1)\, t^k$$

$$\mathrm{h}(x,t) = \sin\left(\frac{\pi x}{l}\right) \sum_{k=0}^{\infty}(-1)^k \frac{\left[v\left(\frac{\pi}{l}\right)^2 t\right]^k}{k!} + \eta \sum_{k=0}^{\infty} \delta(k-1)\, t^k \qquad (15)$$

Recall the power series expansion of $\exp(-x)$ given by

$$\exp(-x) = \sum_{k=0}^{\infty}(-1)^k \frac{x^k}{k!} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots\ldots..$$

$$\text{Then} \quad \sum_{k=0}^{\infty}(-1)^k \frac{\left[v\left(\frac{\pi}{l}\right)^2 t\right]^k}{k!} = \exp\left(-v\left(\frac{\pi}{l}\right)^2 t\right)$$

$$\delta(k-1)t^k = \begin{cases} t & \text{if } k = 1 \\ 0 & \text{otherwise} \end{cases} \qquad \text{Hence } \sum_{k=0}^{\infty} \delta(k-1)\, t^k = t$$

$$\mathrm{h}(x,t) = \sin\left(\frac{\pi x}{l}\right) \exp\left(-\frac{\pi^2 v t}{l^2}\right) + \eta t \qquad (16)$$

Equation (16) is the analytical of the Edwards-Wilkinson equation for initial condition 1.
For initial condition 2 we have;

$$H_0(x) = f(x) = \frac{1}{2}\left[1 - \sinh\left(\frac{x-a}{b}\right)\right]$$

$$H_1(x) = v \frac{\partial^2 H_0(x)}{\partial x^2} + \eta$$

$$H_1(x) = -\frac{v}{2b^2} \sinh\left(\frac{x-a}{b}\right) + \eta$$

$$H_2(x) = \frac{1}{2}\left(v \frac{\partial^2 H_1(x)}{\partial x^2}\right)$$

$$H_2(x) = -\frac{v^2}{4b^4} \sinh\left(\frac{x-a}{b}\right)$$

$$H_3(x) = \frac{1}{3}\left(v \frac{\partial^2 H_2(x)}{\partial x^2}\right)$$

$$H_3(x) = -\frac{v^3}{12b^6} \sinh\left(\frac{x-a}{b}\right)$$

$$H_4(x) = \frac{1}{4}\left(v \frac{\partial^2 H_3(x)}{\partial x^2}\right)$$

$$H_4(x) = -\frac{v^4}{48b^8}\sinh\left(\frac{x-a}{b}\right)$$

$$H_5(x) = \frac{1}{5}\left(v\frac{\partial^2 H_4(x)}{\partial x^2}\right)$$

$$H_5(x) = \frac{1}{5}\left(v\frac{\partial^2}{\partial x^2}\left[-\frac{v^4}{48b^8}\sinh\left(\frac{x-a}{b}\right)\right]\right)$$

$$H_5(x) = -\frac{v^5}{240b^{10}}\sinh\left(\frac{x-a}{b}\right)$$

$$H_0(x) = \frac{-1}{2}\sinh\left(\frac{x-a}{b}\right) + \frac{1}{2} = -\frac{v^0}{0!\times 2b^{2\times 0}}\sinh\left(\frac{x-a}{b}\right) + \frac{1}{2}$$

$$H_1(x) = -\frac{v}{2b^2}\sinh\left(\frac{x-a}{b}\right) + \eta = -\frac{v^1}{1!\times 2b^{2\times 1}}\sinh\left(\frac{x-a}{b}\right) + \eta$$

$$H_2(x) = -\frac{v^2}{4b^4}\sinh\left(\frac{x-a}{b}\right) = -\frac{v^2}{2!\times 2b^{2\times 2}}\sinh\left(\frac{x-a}{b}\right)$$

$$H_3(x) = -\frac{v^3}{12b^6}\sinh\left(\frac{x-a}{b}\right) = -\frac{v^3}{3!\times 2b^{2\times 3}}\sinh\left(\frac{x-a}{b}\right)$$

$$H_4(x) = -\frac{v^4}{48b^8}\sinh\left(\frac{x-a}{b}\right) = -\frac{v^4}{4!\times 2b^{2\times 4}}\sinh\left(\frac{x-a}{b}\right)$$

$$H_5(x) = -\frac{v^5}{240b^{10}}\sinh\left(\frac{x-a}{b}\right) = -\frac{v^5}{5!\times 2b^{2\times 5}}\sinh\left(\frac{x-a}{b}\right)$$

Hence, a general value for $H_k(x)$ is written as

$$H_k(x) = -\frac{v^k}{k!\,2b^{2k}}\sinh\left(\frac{x-a}{b}\right) + \eta\,\delta(k-1) + \frac{1}{2}\,\delta(k) \qquad (17)$$

$$\mathrm{h}(x,t) = \sum_{k=0}^{\infty} H_k(x)\,t^k$$

$$\mathrm{h}(x,t) = \sum_{k=0}^{\infty}\left[-\frac{v^k}{k!\,2b^{2k}}\sinh\left(\frac{x-a}{b}\right) + \eta\,\delta(k-1) + \frac{1}{2}\,\delta(k)\right]t^k$$

$$\mathrm{h}(x,t) = \sum_{k=0}^{\infty}\left[-\frac{v^k}{k!\,2b^{2k}}\sinh\left(\frac{x-a}{b}\right)\right]t^k + \sum_{k=0}^{\infty}\eta\,\delta(k-1)\,t^k + \sum_{k=0}^{\infty}\frac{1}{2}\,\delta(k)\,t^k$$

$$\mathrm{h}(x,t) = \sinh\left(\frac{x-a}{b}\right)\sum_{k=0}^{\infty} -\frac{v^k}{k!\,2b^{2k}}\,t^k + \eta\sum_{k=0}^{\infty}\delta(k-1)\,t^k + \frac{1}{2}\sum_{k=0}^{\infty}\delta(k)\,t^k$$

$$\mathrm{h}(x,t) = -\frac{1}{2}\sinh\left(\frac{x-a}{b}\right)\sum_{k=0}^{\infty}\frac{\left[{}^{vt}/_{b^2}\right]^k}{k!} + \eta\sum_{k=0}^{\infty}\delta(k-1)\,t^k + \frac{1}{2}\sum_{k=0}^{\infty}\delta(k)\,t^k$$

Recall the power series expansion of $\exp(x)$ given by

$$\exp(x) = \sum_{k=0}^{\infty}\frac{x^k}{k!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$$

$$\text{Then} \quad \sum_{k=0}^{\infty} \frac{\left[{}^{vt}/_{b^2}\right]^k}{k!} = \exp\left({}^{vt}/_{b^2}\right)$$

$$\delta(k-1)t^k = \begin{cases} t & \text{if } k = 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{Hence} \sum_{k=0}^{\infty} \delta(k-1)\, t^k = t$$

$$\delta(k)t^k = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{Hence} \sum_{k=0}^{\infty} \delta(k)\, t^k = 1$$

$$\mathrm{h}(x,t) = -\frac{1}{2}\sinh\left(\frac{x-a}{b}\right)\exp\left({}^{vt}/_{b^2}\right) + \eta t + \frac{1}{2} \qquad (18)$$

Equation (18) is the analytical of the Edwards-Wilkinson equation for initial condition2.

4.0 Discussion Of Results And Diagrams

The reduced differential transform method (RDTM) was used to obtain analytical solution to the Edwards-Wilkinson equation presented in equation (16) and (18) for initial condition 1 and 2 respectively. The implementation of the analytical solution is done with MATHEMATICA computer algebra system.

Figures 1 – 4 gives the RDTM and finite difference solution to the EW equation for the two initial conditions. The plots obtained for the RDTM solution were very identical to the corresponding finite difference plots. The performance of the RDTM is further analyzed by Table 2 which gives the solution to the EW equation obtained from finite difference and RDTM methods used at selected times and positions. From these tables, the values obtained from the RDTM were very close to the corresponding finite difference method values.

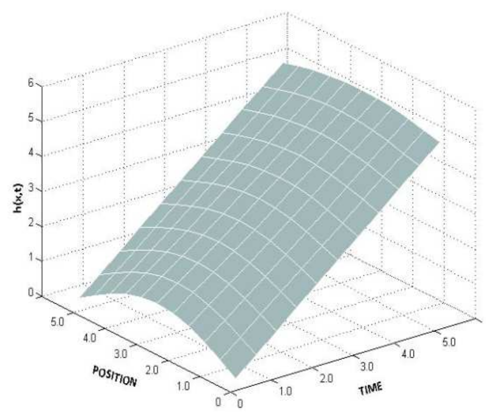

Figure 1: RDTM solution of the EW equation (initial condition 1) after 5000 time steps (5.0 seconds) for $\boldsymbol{v = 0.5}$ and $\boldsymbol{\eta = +1}$.

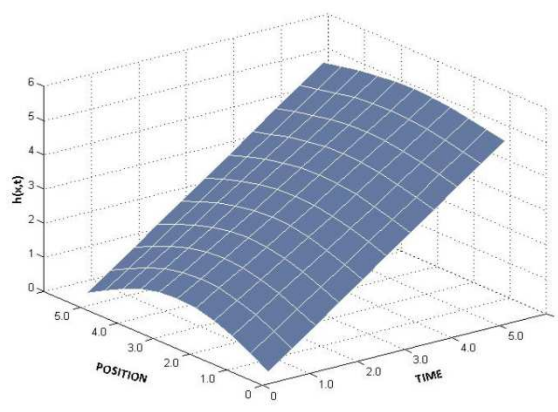

Figure 2: Finite difference solution of the EW equation (initial condition 1) after 5000 time steps (5.0 seconds) for $\boldsymbol{v = 0.5}$ and $\boldsymbol{\eta = +1}$.

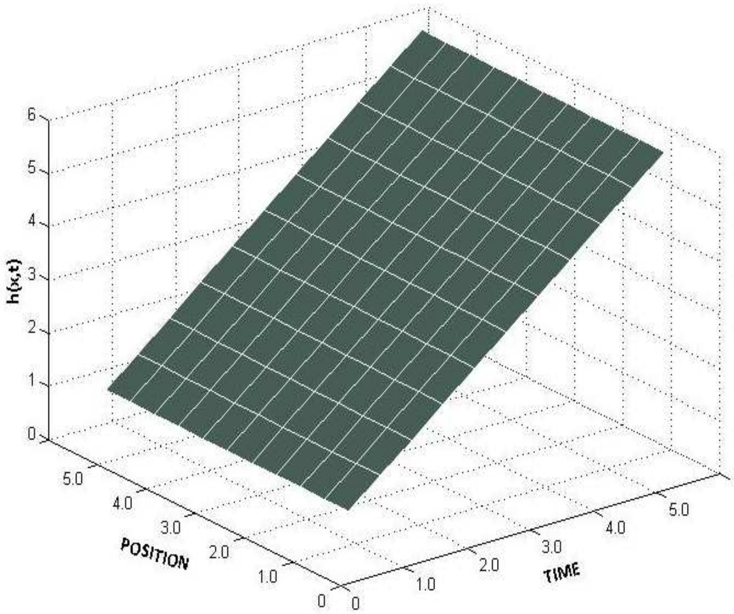

Figure 3: RDTM solution of the EW equation (initial condition 2) after 5000 time steps (5.0 seconds) for $\boldsymbol{\upsilon = 0.5}$ and $\boldsymbol{\eta = -1}$.

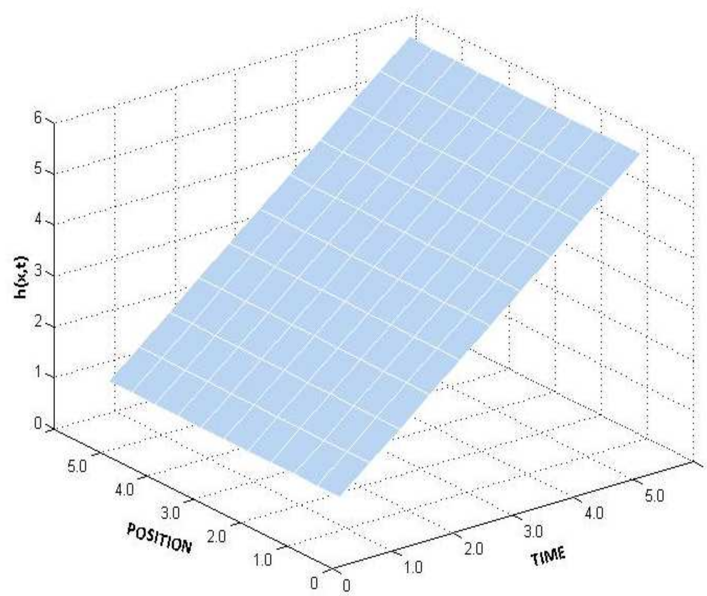

Figure 4: Finite difference solution of the EW equation (initial condition 2) after 5000 time steps (5.0 seconds) for $\boldsymbol{\upsilon = 0.5}$ and $\boldsymbol{\eta = +1}$.

Table 2: Comparison of the methods used for solving the Edwards-Wilkinson equation for ν = 0.5.

η	***TIME***	***POSITION***	***EXPLICIT***	***RDTM***
-1	***1.00***	***0.50***	*0.2878*	*0.2902*
		2.50	*0.9761*	*0.9840*
		4.00	*0.5655*	*0.5701*
	3.00	***0.50***	*0.2454*	*0.2527*
		2.50	*0.9282*	*0.9519*
		4.00	*0.5209*	*0.5348*
	4.50	***0.50***	*0.2136*	*0.2246*
		2.50	*0.8924*	*0.9279*
		4.00	*0.4874*	*0.5083*
1	***1.00***	***0.50***	*0.3278*	*0.3302*
		2.50	*1.0161*	*1.0240*
		4.00	*0.6055*	*0.6101*
	3.00	***0.50***	*0.3654*	*0.3727*
		2.50	*1.0482*	*1.0719*
		4.00	*0.6409*	*0.6548*
	4.50	***0.50***	*0.3936*	*0.4046*
		2.50	*1.0724*	*1.1079*
		4.00	*0.6674*	*0.6883*

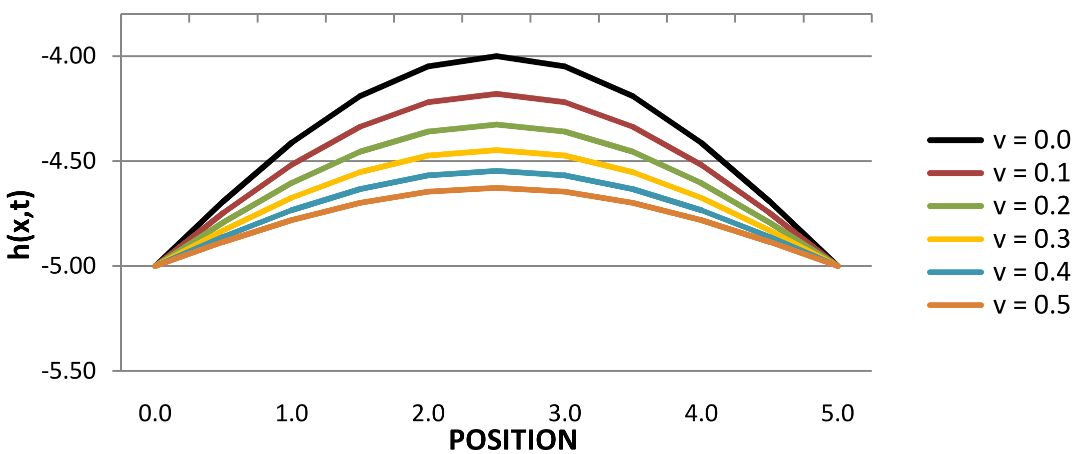

Figure 5: Effect of increasing surface tension υ on surface height for $\eta = -1$.

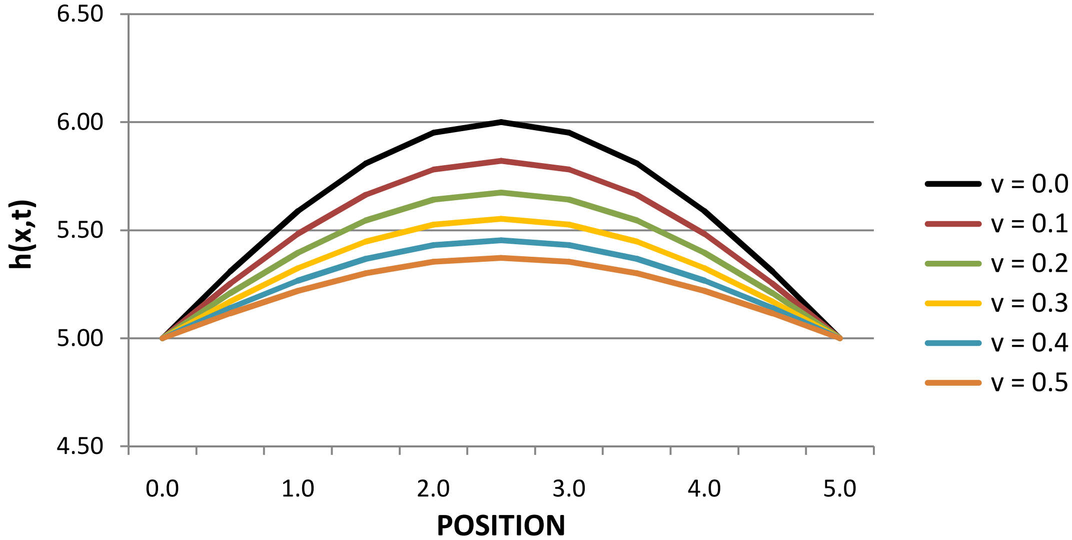

Figure 6: Effect of increasing surface tension υ on surface height for $\boldsymbol{\eta = +1}$.

Figure 5 and Figure 6 shows the effect of increasing surface tension υ on the surface for $\eta = -1$ and $\eta = +1$ respectively. Generally, an increase in υ reduces the surface height by redistributing the height in order to smoothen the surface. It is also observed that the surface relaxation term which tends to smooth the surface does not affect the flat surface initial condition (initial condition 2). This is obviously due to the already smooth nature of the flat surface.

5.0 Conclusion

In this study, the reduced differential transform method (RDTM) was used to obtain exact analytical solution to the Edwards-Wilkinson equation. Comparison between the analytical solution provided by RDTM and numerical finite difference solution of the Edwards-Wilkinson equation shows that the RDTM is a very powerful and effective tool for finding exact analytical solutions for non-linear partial differential equations. Computations required for the RDTM are executed using the MATHEMATICA package.

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