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Erratum: Generation of Modified Sinusoidal Waves Using Operational Integrators

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Some equations and symbols in this paper did not appear properly in the vol. 17 issue of the Journal of NAMP. The entire article is therefore reproduced below as it ought to appear in pages 207 – 214 (Vol. 17).

Abstract

The production of modified sinusoidal waves is currently a field of active research even in already developed countries as it combines the ease of production associated with square waves and avoids the low energy efficiency associated with true sinusoidal waves. This paper discusses the production of Modified Sinusoidal Waves using electronic wave shaping circuits that performs the process of mathematical integration.

The particular method of interest starts with a square wave as the input signal and a sine wave as the output. Derivations are first discussed and then circuit elements are used to implement equations derived from the analysis. Finally a comparison is made between the modified sinusoidal wave generated using this method and a true sine wave.

Keywords: - Asymptotic techniques, Maxwell fluid, constantly accelerating plates and velocity fields.

1.0 Introduction

Production of alternating current form direct current is still a challenge for Scientist and Engineers even in developed countries. This is because the easy to produce square and triangular waves, possesses sharp edges that greatly reduces the kind of appliances that can use them. One the other hand, true sinusoidal waves are more difficult to produce, their wave generating circuits have lower energy efficiency and circuit components used in these circuits are very expensive and are not readily available especially in developing countries like ours.

However, there exists one method of generating sine – like waveforms that do not posses sharp edges. Its production involves the use of electronic wave shaping circuits to convert easy to produce wave forms into sine – like wave forms. These circuits have energy efficiencies close to that of square and triangular waves and also avoid other disadvantages like unavailability and high cost of circuit components. Because this type of modified sinusoidal wave avoids the sharp edges associated with square and triangular waves, it can be used by all power circuits that use the true sine wave.

2.0 Theoretical Analysis

This is a method of producing modified sinusoidal waves I developed as a result of my interest in electronic inverters. Here a square wave input is the starting point. With respect to the production of square waves, various easy to implement method exist some of which are the Bipolar Junction Transistor, Operational Amplifier, Logic Gate, 4069UB, 555 and 7404 Integrated Circuit – Astable Multivibrators to mention a few.

The equation of a Square Wave having positive and negative voltage swings could be expressed as:

$$V_{sq} = \pm|V_{sqmax}| \tag{2.1}$$

Where V_{sqmax} is the maximum amplitude of the square wave as shown in the Figure 1.

The equation of a triangular wave can be gotten by integrating the equation of a square wave with respect to time that is:

$$V_{tg} = \int(\pm|V_{sqmax}|)dt \quad \Rightarrow V_{tg} = \pm|V_{sqmax}|t \tag{2.2}$$

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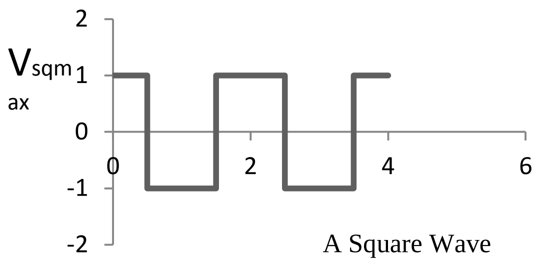


Figure 1

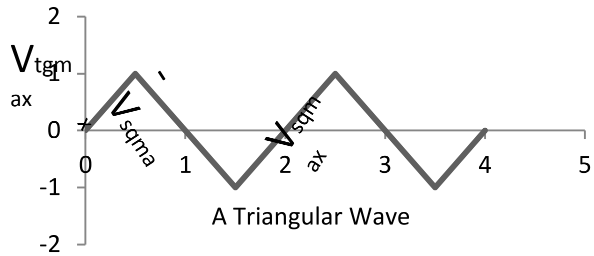


Figure 2.

Equation (2) is that of a straight line having series of positive and negative slopes of $\pm|V_{sqmax}|$ with positive and negative voltage swings like the square wave that was integrated, as seen in Figure 2.
The equation of a sine – like looking wave can then be gotten by integrating the equation of a triangular wave with respect to time that is:

$$V_{sn} = \int\left(\pm|V_{sqmax}|t\right)dt \quad \Rightarrow V_{sn} = \pm\frac{|V_{sqmax}|}{2}t^2 \qquad (2.3)$$

Equation 3 is an equation of a parabola (a quadratic in terms of t^2) where $+\frac{|V_{sqmax}|}{2}t^2$ is upward facing parabola and $-\frac{|V_{sqmax}|}{2}t^2$ is a downward facing parabola as shown in figure 3 (a and b respectively).

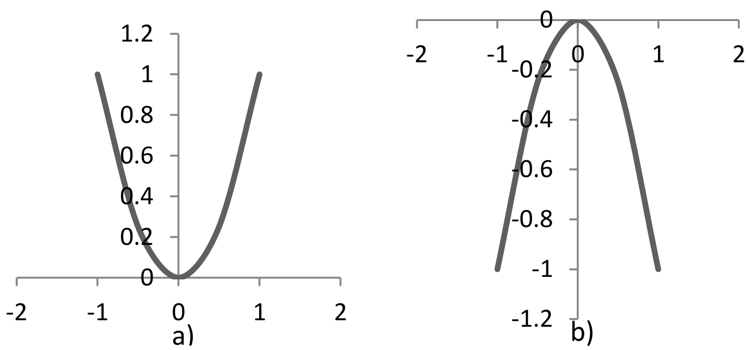


Fig 3. Upward and Downward Facing

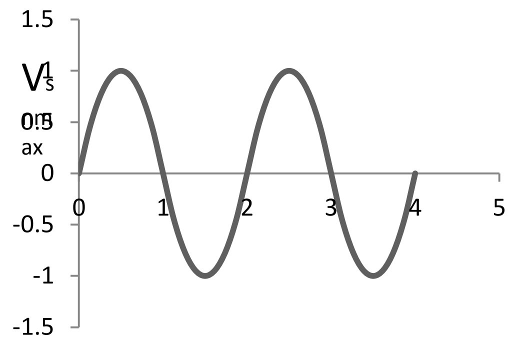


Fig. 4 A modified sine wave

When the parabolas above are compared to a sinusoidal wave, an upward facing parabola corresponds to the part of a sinusoidal wave below the horizontal axis while the downward facing parabola corresponds to the part above the horizontal axis. If parabolas are to be joined together to form a sine – like looking wave they must have the following corrections.

$$V_{sn} = \frac{|V_{sqmax}|}{2}t^2 - |V_{snmax}| \qquad (2.4a)$$

$$V_{sn} = -\frac{|V_{sqmax}|}{2}t^2 + |V_{snmax}| \qquad (2.4b)$$

Where V_{snmax} is the maximum voltage amplitude of the sinusoidal wave. The derived sinusoidal wave is shown in the Fig 4.
However because the modified sine wave is gotten by integrating a triangular wave and not by joining already produced upward and downward facing parabolas the last two equations only help to put the derivation process in the right perspective. The basic process is that of integration and the origin of the last two terms is from the nature of the triangular wave that was integrated. These equations are relevant for theoretical analysis and are very important in a case where already produced parabolas are joined together.
Thus it could be said that a relationship derived through the mathematical process of integration is used by the modified sinusoidal wave oscillator. This could be expressed as:

$$V_{sn} = \pm Kt^2 \qquad (2.5)$$

Where K is a constant

2.1 Operational Integrators

Considering the above explanation, all that is needed to produce a sine wave is a square wave oscillator and at least two integrators. It is worthy to note that triple and higher number of integrations also avoid the sharp edges associated with square and triangular waves. If a constant is integrated with respect to time, the result is in form of time multiplied by a constant; this implies the integrated output should change uniformly with time. An RC circuit whose output is approximately equal to the integral of the input signal over a limited interval of time is shown in figure 5 below:

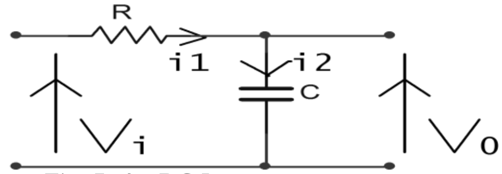


Fig. 5 An RC Integrator

From figure 5 $i_1 = -\frac{V_i}{R}$ and $i_2 = \frac{dQ}{dt}$

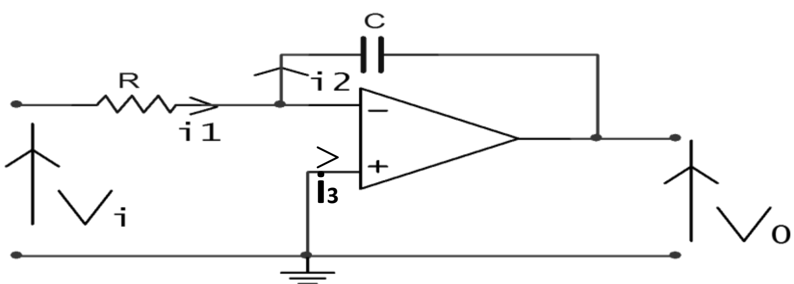
but: $Q = CV_0$

$\Rightarrow i_2 = C\frac{dV_0}{dt}$ if $i_1 = i_2$ then:

$-\frac{V_i}{R} = C\frac{dV_0}{dt}$

$$V_i = -RC\frac{dV_0}{dt} \tag{2.6}$$

In Figure 6 i_1 is only approximately equal to i_2 if the time settling constant RC is large as compared to the period of the input signal. To ensure that this is case, an operational amplifier is used and because of its high input impedance, it ensures i_1 is approximately equal to i_2. This circuit is shown in Figure 7.



An Operational Integrator

Figure 7

It should be noticed that the amplifier is used in inversion mode and from the above diagram $i_1 = i_2 + i_3$ but because of the amplifier high input impedance, $i_3 \cong 0$ which implies that $i_1 \cong i_2$. Thus:

$i_1 = -\left[\frac{V_i - V_a}{R}\right]$ and $i_2 = C\frac{dV}{dt}$

Where $V = V_0 - V_a$ and V_a is the input voltage of the amplifier. Since $i_1 \cong i_2$ it follow that:

$$-\left[\frac{V_i - V_a}{R}\right] = C\frac{d}{dt}(V_0 - V_a)$$

$$\Rightarrow V_i - V_a = -RC\frac{d}{dt}(V_0 - V_a)$$

introducing the amplifier's gain A by writing $V_0 = -AV_a$

that is $V_a = -\frac{V_0}{A}$ yeilds:

$$V_i + \frac{V_0}{A} = -RC\frac{d}{dt}\left[V_0\left(1 + \frac{1}{A}\right)\right] - \frac{V_0}{A} \text{ thus:}$$

$$V_i = -RC\frac{d}{dt}\left[V_0\left(1+\frac{1}{A}\right)\right] - \frac{V_0}{A} \tag{2.7}$$

If it's assumed that the amplifiers gain A approaches infinity then equation (2.7) reduces to equation (6):

$$V_i = -RC\frac{dV_0}{dt}$$

$$dV_0 = -\frac{1}{RC}V_i dt$$

$$V_0 = -\frac{1}{RC}\int V_i dt \tag{2.8}$$

Equation (2.8) shows that the output voltage is proportional to the integral of the input voltage and is valid for amplifiers with finite input impedance provided that;

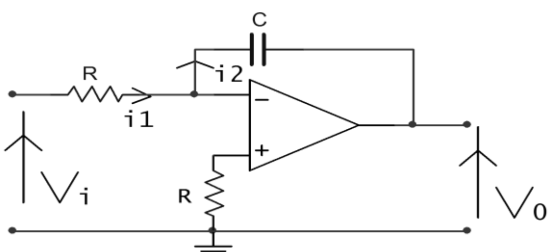
1. The input signal frequency:

$$\omega \gg \frac{\omega_c}{A} \text{ and } \omega_c = \frac{1}{RC} \tag{2.9}$$

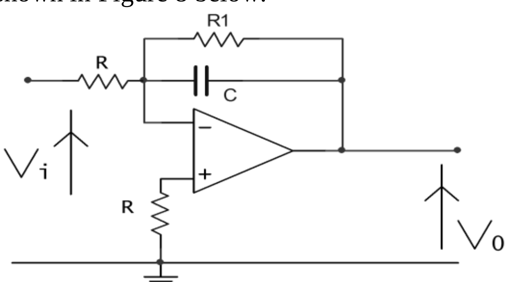
2. The amplifier input impedance:

$$\left|Z_i \gg \frac{1}{\omega AC}\right| \tag{2.10}$$

In practically working integrators circuits, to minimize the offset due to bias current, another resistor of equal value to R is connected between the ground and the non-inverting input. To minimize drift in the integrator, the chosen amplifier must possess very low bias and offset currents, an amplifier with FET input stage would be most suitable. A very large capacitance C also helps because for a given bias current the rate of build-up of potential across the capacitor would be less. A further refinement that is often used when integration of DC components is not required is the placing of a large value resistor in parallel with the integrating capacitor. These final circuits are shown in Figure 8 below.



a) Operational Integrator used when integration of DC is desired



b) Operational Integrator used when integration of DC is not desired

Fig. 8(a & b)

2.2 Modified Sine Wave Oscillator Circuit Diagram And Analysis

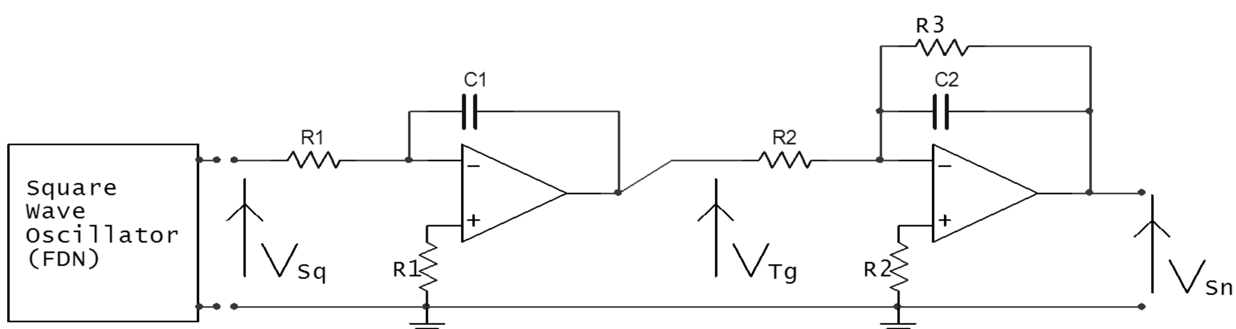


Fig. 9 A Modified Sine Wave Electronic Oscillator

Above is the Modified Sine Wave Electronic Oscillator which is explained thus. Aside providing the required square wave to be integrated, the square wave oscillator also functions as the frequency determining network as the frequency of its square wave is theoretically equal to that of the resulting sine wave. For practically working integrators it has been shown that:

$$V_0 = -\frac{1}{RC}\int V_i dt \tag{2.8}$$

For the modified sinusoidal wave oscillator shown in Figure 9 the output voltage is given as:

$$V_{msn} = -\frac{1}{R_2C_2}\int\left(-\frac{1}{R_1C_1}\int V_{sq}dt\right)dt$$

$$V_{sq} = \pm|V_{sqmax}|$$

$$\Rightarrow V_{msn} = -\frac{1}{R_2C_2}\int\left(-\frac{1}{R_1C_1}\int \pm|V_{sqmax}|dt\right)dt$$

$$V_{msn} = -\frac{1}{R_2C_2}\int \mp\frac{V_{sqmax}}{R_1C_1}tdt$$

$$V_{msn} = \pm\frac{1}{2}\left(\frac{V_{sqmax}}{R_2C_2R_1C_1}\right)t^2 \qquad (2.11)$$

The above calculation takes into account the inverting property of both amplifiers and the answer only remained unchanged as a result of the double stage inversion of both amplifiers.

TABLE 1

t	t^2	$\lvert V_{snmax}\rvert$	$t^2 - \lvert V_{snmax}\rvert$	$\left[\frac{t+1}{2}\right]+1$	$-t^2 + \lvert V_{snmax}\rvert$	$\left[\frac{t+1}{2}\right]$	θ	$\sin\theta$
-1.0000	1.0000	1.0000			0.0000	0.0000	0.0000	0.0000
-0.7500	0.5625	1.0000			0.4375	0.1250	22.50	0.3827
-0.5000	0.2500	1.0000			0.7500	0.2500	45.00	0.7071
-0.2500	0.0625	1.0000			0.9375	0.3750	67.50	0.9239
-0.0000	0.0000	1.0000			1.0000	0.5000	90.00	1.0000
0.2500	0.0625	1.0000			0.9375	0.6250	112.50	0.9239
0.5000	0.2500	1.0000			0.7500	0.7500	135.00	0.7071
0.7500	0.5625	1.0000			0.4375	0.8750	157.50	0.3827
1.0000	1.0000	1.0000	0.0000		0.0000	1.0000	180.00	0.0000
-1.0000				1.0000				
-0.7500	0.5625	1.0000	-0.4375	1.1250			202.50	-0.3827
-0.5000	0.2500	1.0000	-0.7500	1.2500			225.00	-0.7071
-0.2500	0.0625	1.0000	-0.9375	1.3750			247.50	-0.9239
0.0000	0.0000	1.0000	-1.0000	1.5000			270.00	-1.0000
0.2500	0.0625	1.0000	-0.9375	1.6250			292.50	-0.9239
0.5000	0.2500	1.0000	-0.7500	1.7500			315.00	-0.7071
0.7500	0.5625	1.0000	-0.4375	1.8750			337.50	-0.3827
1.0000	1.0000	1.0000	0.0000	2.0000			360.00	0.0000

Comparing equation 11with equation 2.5 gives a value of K for the circuit as:

$$K = \frac{1}{2}\left(\frac{V_{sqmax}}{R_2C_2R_1C_1}\right) \qquad (2.12)$$

Figure 10 is a comparison between the modified sine wave described by equation 2.5 and the true sine wave. Both graphs are drawn for values of voltages V falling in the range of $-1 \leq V \leq 1$, they both have the same frequency and are drawn for an interval of one period.

Upon close examination of Table 1 and Figure 10 it can be seen that despite the vertical exaggeration both curves follow each other quite closely. Since the area under the graph is directly proportional to the power delivered by both waves, it can be concluded from Figure 10 that the modified sinusoidal wave delivers slightly more power. However its most desirable property is the very close resemblance it shares with a true sine wave. This property also makes double integration more favorable over triple and higher number of integrations.

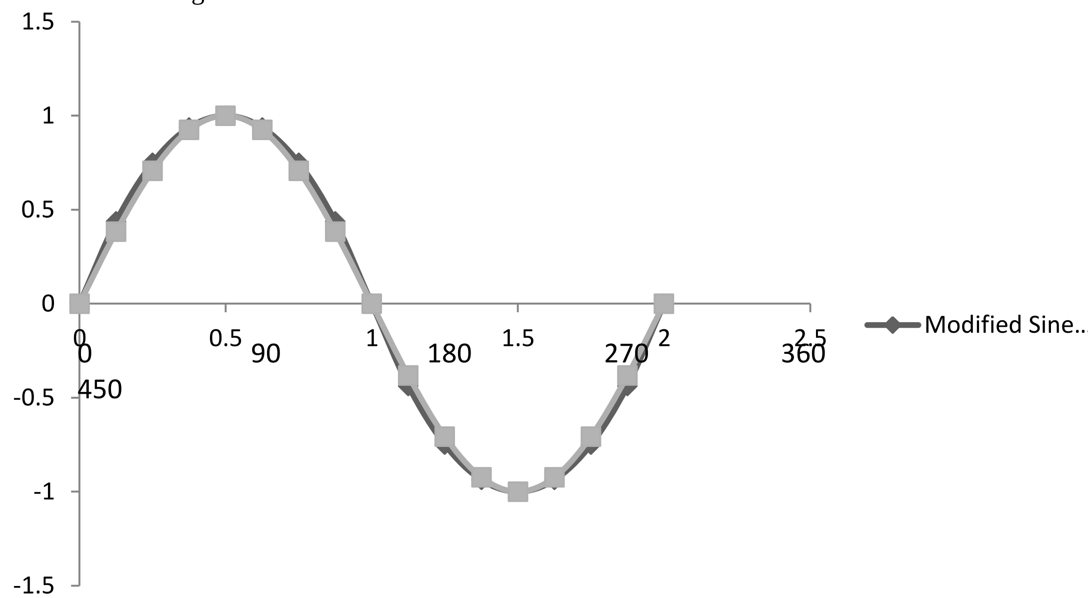


Figure 10

3.00 Experimental Work

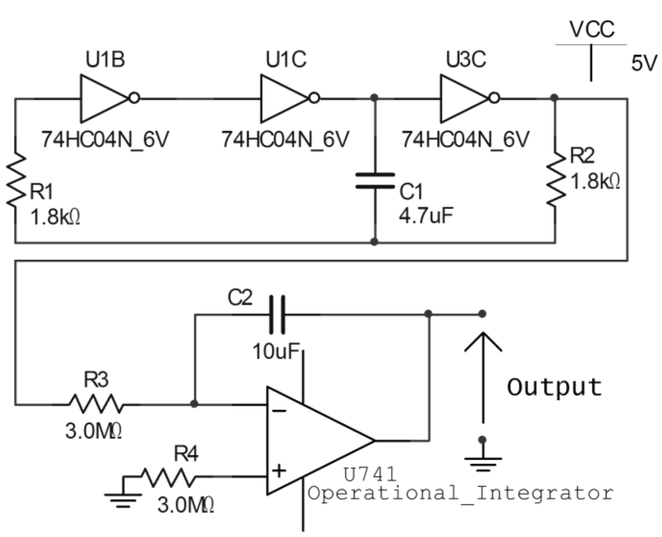


Fig. 11 A Practically working Modified Sinusiodal Wave Oscillator

Figure 11 is a practically working modified sinusoidal wave electronic oscillator. The U741 integrated Circuit is an operational integrator available as a unit. Thus the above circuit is a combination of an ordinary RC integrator (designed to satisfy the conditions in equation 2.9 and 2.10) and an operational integrator whose internal circuitry looks like that of Figure 8(b). The circuit above the two integrators serves as the square wave generating circuit. The square wave oscillator is a logic gate astable multivibrator (three gate version) whose frequency is given by the equation:

$$F = \frac{0.46}{R_2 C_1} \tag{2.13}$$

This corresponds to a theoretical value of $54.37\,Hertz$. The closeness of the square wave shape to perfection is essential if the above circuit is to function properly. Electronically generated square waves are not perfect because capacitors used in their circuits charges exponentially rather than instantaneously. Figure 11 produces square waves that are the closest to perfection to the best of my knowledge.

4.00 Results

After the switch was turned on the only observed anomaly for both the practically constructed circuit and the simulated circuit was that regular circuit oscillation started after about 0.60 seconds . This is as a result of the internal properties of the logic gates used in the square wave oscillator; in practice a delay circuit would be placed between the modified sine wave oscillator and the output. Figure 12 shows the oscillator in simulation environment it also shows the display of an oscilloscope and a frequency counter connected to it.

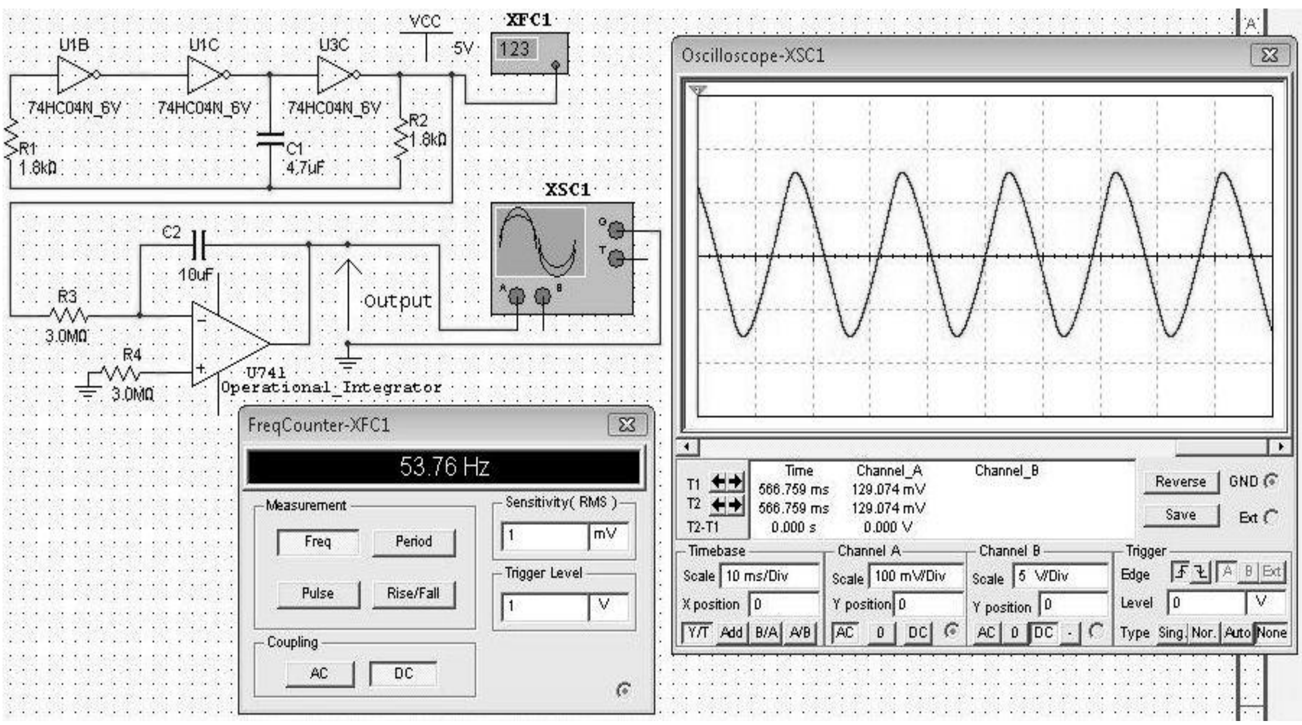


Fig 12 The Modified Sinusiodal Wave Oscillator in Simullation Environment

5.00 Discussions

The behavior of the circuit was very similar when the practically constructed circuit was tested and when it was tested using Electronic Workbench Simulation Software. The only observed difference was that the frequency of the practical circuit was 53.37Hertz while the simulated circuit had a frequency of 53.76Hertz as seen in Figure 12. On both occasions, the frequency reading was very close to the theoretical value of 54.37Hertz and the observed difference is as a result of deviations of circuit elements from their exact value and the presence of wires and junction resistance as the larger their values the lower the observed frequency. The frequency problem can be solved by replacing resistor R_2 in Figure 12 with a 1.5 kilo ohm resistor and a variable resistor of 0.5 kilo ohm connected in series so that the circuit will have an oscillation frequency range of 48.94Hertz to 65.25Hretz.

The above is a very interesting result because it shows that sine – like looking waves can be produced using compact circuit elements. The closeness of the shape and phase of the wave generated by this circuit has been shown by table 1 and Figure 10. As a result this circuit possesses very important application in inverters. This is because for inverters that are currently manufactured, the size of a square wave inverter as compared to that of a true sine wave inverter of the same capacity, has a ratio that is close to 1:30, but with this circuit a sine wave inverter of the same capacity as a square wave inverter would be approximately the same size and cost. This would then lead to the eradication of square wave inverters and other types of modified sine waves inverters that do not produce sine – like looking waves in favor of this circuit, whose wave forms (as in the case of this circuit) might even supply more power for the same value of voltage.

The method by which this circuit can be used to convert low voltage DC power can be completed in two steps. The first being the conversion of the low voltage DC power to a high voltage DC source and the second step is the conversion of the high voltage DC source to an AC waveform using this circuit. Another method to complete the desired outcome would be to first convert the low voltage DC power to AC using this circuit, and then use a transformer to boost the voltage to any desired value.

6.0 Conclusion

The success of this work is perfectly in line with the ultimate aim of science where thoughts are developed into postulates and then into laws and theories before the manufacture of devices, equipments and instruments applying these laws and theories. My interest in the generation of AC from DC sources is as a result of my belief that it can serve as an alternative source of power supply especially in African Countries where electric power supply is erratic and there exist DC sources like solar energy that can adequately meet their power supply needs.

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