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Response under a Moving Load of an Elastically Supported Euler- Bernoulli Beam on Pre-Stressed and Variable Elastic Foundation.

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Abstract

The dynamic response under a partially distributed moving load of an elastically supported Euler- Bernoulli Beam on pre-stressed and variable elastic foundation was investigated. The governing partial differential equations were analyzed for both moving force and moving mass in order to determine the behavior of the system under consideration. The analytical method in terms of series solution and numerical method were used for the governing equation. It was observed that the response amplitude of the moving mass increase as mass of the load M increases. It was also found that the response amplitude due to the moving mass for pre-stressed is greater than that due to moving force.

Keywords: pre-stressed, Euler-Bernoulli Beam, partially distributed, moving load, moving force, elastic foundation.

1.0 Introduction

Beam with various shapes and materials are important structural elements. They are widely used in modern engineering and science. In the recent years all areas of transport have experienced great advances characterized by increasing higher speeds and weight of vehicles. As a result, structures and media over or in which the vehicles move have been subjected to vibrations and dynamic stresses far larger than ever before. Many researchers have studied vibration of elastic and inelastic structures under the action of moving loads for many years, and effort are still being made to carry out more investigation that deal with various aspect of the problem [1-18].The more practical cases when velocities at which these loads move are no longer constants but vary with the time have received little attention. This may be as a result of the complex space-Time dependencies inherent in such problems specifically, even when the inertia effect of the moving load are neglected analytical solutions involving integral transforms are both intractable and cumbersome. However, such practical problem as acceleration and braking of automobile on roadways and high way bridges. Taking off and landing of aircrafts on runway and braking and acceleration forces in the calculation of rails and railway bridges in which motion is not uniform but a function of time have intensified the need for the study of the behavior of structures under the action of loads moving with variable velocity. In spite of all the published work, there seems to be very little literature concerned with the pre stressed beams (beams which do experience compression when no external load is applied i.e. artificial creation of stresses in structure before loading) of any type. This problem has some practical application they are commonly incorporated in the design aero planes. Advances in technology have accelerated the utilization of such pre-stressed structural element. In general an aircraft is subjected to a wide range of temperature variation during flight which may cause considerable tensile or compressive pre-stressed in the beams when they are fixed in the plane direction. Emailzadeh and Ghorashi [8] worked on the vibration analysis of beams traversed by uniform partially distributed moving mass using analytical-numerical method. They discovered that the inertia effect of the distributed moving mass is of importance in the dynamic behavior of the structure. The critical speeds of the moving load were also calculated for the mid-span of the beam. The length of the distributed moving mass was also found to affect the dynamic response. Dada [3] studied uniform distributed moving masses vibration

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for Euler Bernoulli beam on elastic foundation. He reduced partial differential equation governing the beam to ordinary differential equation and then expressed as a system of linear equations by finite differences schemes. The analysis is valid for Euler Bernoulli beam with various boundary conditions. However, simply supported boundary conditions were used as an illustrated example. The numerical results are presented in graphical forms and the limiting cases compared well with known existing results. The numerical analysis shows that the foundation stiffness and loads distribution have significant effects on the dynamic deflection of the beam. Oni and Omolofe [4] investigates the dynamics behavior of non-uniform Bernoulli Euler beam subjected to concentrated loads traveling at variable velocities. The solution technique is based on the generalized Galerkin method and the use of the generating function of the Bessel function types. The results show that, for all the illustrative examples considered, for the same natural frequency, the critical speed for the system, consisting of a non-uniform beam traversed by a force moving at a non-uniform velocity is greater than that of the corresponding moving mass problem. It was also found that, for fixed axial forces, an increase in foundation moduli reduces the response amplitudes of the dynamical system, furthermore, it was shown that the traverse-displacement amplitude of a clamped- clamped non-uniform Euler Bernoulli beam traversed by a moving load at variable velocities is lower than of the cantilever. The response amplitude of the same dynamical systems which is simply supported is higher than those which consist of clamped – clamped or clamped free (cantilever) end conditions. Finally, an increase in the value of foundation moduli and axial force reduce the critical speed for all variant of the boundary conditions. Recently Oni and Awodola [2] considered the dynamic response under a moving load of an elastically supported non-prismatic Bernoulli Euler beam resting on variable elastic foundation. The technique was based on the Generalized Galerkin's method and the struble's asymptotic technique. The results show that response amplitude of the elastically supported non-prismatic Bernoulli Euler beam decrease as the foundation modulli K increases. Also, the displacement of an elastically supported non-prismatic Bernoulli Euler beam resting on a variable elastic foundation for fixed value of K decrease as the pre-stress N increases. The results again show that the critical speed for the moving mass problem is reached earlier than that for the moving force problem for the illustrative examples considered. More recent Akinpelu [9] studied the Response of viscously Damped Euler Bernoulli Beam to uniform partially Distributed moving loads. Her approach involves using analytical method in terms of series solution and numerical method was used for the governing equation. It was found in her result that the response amplitude of the moving force problem with non-initial stress increase as mass of the load M increases.

This Research work presents a new method for dynamic response under a partially distributed moving load of an elastically supported Bernoulli Euler beam on pre-stressed and variable elastic foundation. The usual assumption of constant moment of inertia is taken to be variable moment of inertia.

2.0 The Governing Equation:

The problem of the dynamic response under a partially distributed moving load of an elastically supported Euler Bernoulli beam on pre-stressed and variable elastic foundation is governed by the fourth order partial differential equation given by

$$\left.\begin{array}{l}\dfrac{\partial^2}{\partial x^2}\left[EI(x)\dfrac{\partial^2 U(x,t)}{\partial x^2}\right]+\mu(x)\dfrac{\partial^2 U(x,t)}{\partial t^2}-N\dfrac{\partial^2 U(x,t)}{\partial x^2}+K(x)U(x,t)+M\left[\dfrac{\partial^2}{\partial t^2}+2V\dfrac{\partial^2}{\partial x\partial t}+V^2\dfrac{\partial^2}{\partial x^2}\right]\\ U(x,t)\left[H\left(x-\xi+\dfrac{\varepsilon}{2}\right)-H\left(x-\xi+\dfrac{\varepsilon}{2}\right)=Mg\left[H\left(x-\xi+\dfrac{\varepsilon}{2}\right)-H\left(x-\xi-\dfrac{\varepsilon}{2}\right)\right]\right.\end{array}\right\} \tag{1}$$

Subject to the boundary and initial conditions

$$\left.\begin{array}{l}U(0,t)=0=U'(0,t\\ U(L,t)=0=U''(L,t)\\ U(x,0)=\dot{U}(x,0)\end{array}\right\} \tag{2}$$

Where

- E is the young's modulus
- U(x, t) is the transverse displacement
- K(x) is the variable elastic foundation
- $\mu(x)$ is the variable mass per unit length of the beam
- I(x) is the variable moment of inertia,
- N(x) is the pre-stress and
- (x, t) are respectively spatial and time coordinates.

$$\left.\begin{aligned} K(x) &= K(4x-3x^2+x^3) \\ I(x) &= I_O\left(1+\sin\frac{\pi x}{L}\right)^3 \\ N(x) &= N_O\left(1+\sin\frac{\pi x}{L}\right) \end{aligned}\right\} \tag{3}$$

Where K is foundation modulus, I_0 and N_0 are taken to be constants
Substituting equation (3) into equation (1), to obtain

$$\left.\begin{aligned} & EI_O\frac{\partial^2}{\partial x^2}\left[\left(1+Sin\frac{\varepsilon}{2}\right)^3\frac{\partial^2 U(x,t)}{\partial x^2}\right]+\mu_o\left(1+Sin\frac{\varepsilon}{2}\right)\frac{\partial^2 U(x,t)}{\partial t^2}-N_O\frac{\partial^2 U(x,t)}{\partial x^2}+K\left(4x-3x^2+x^3\right)U(x,t) \\ & +M\left[\frac{\partial^2}{\partial t^2}+2V\frac{\partial^2}{\partial x\partial t}+V^2\frac{\partial^2}{\partial x^2}\right]U(x,t)H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)=MgH\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right) \end{aligned}\right\}-(4)$$

On further simplification of equation (4) yields

$$\left.\begin{aligned} & \left(10+15\,Sin\frac{\pi x}{L}-6Cos\frac{2\pi x}{L}-Sin\frac{3\pi x}{L}\right)\frac{\partial^4 U(x,t)}{\partial x^4}+\left(\frac{30\pi}{L}Cos\frac{\pi x}{L}+\frac{24\pi}{L}Sin\frac{2\pi x}{L}-\frac{6\pi}{L}Cos\frac{3\pi x}{L}\right)\frac{\partial^3 U(x,t)}{\partial x^3}+ \\ & \left(\frac{24\pi^2}{L^2}Cos\frac{2\pi x}{L}-\frac{15\pi}{L^2}Sin\frac{3\pi x}{L}+\frac{9\pi^2}{L^2}Sin\frac{\pi x}{L}\right)\frac{\partial^2 U(x,t)}{\partial x^2}+\mu_o\left(1+Sin\frac{\pi x}{L}\right)\frac{\partial^2 U(x,t)}{\partial t^2}-N\frac{\partial^2 U(x,t)}{\partial x^2} \\ & +K\left(4x-3x^2+x^3\right)U(x,t)=\left[Mg-M\left(\frac{\partial^2}{\partial t^2}+2V\frac{\partial^2}{\partial x\partial t}+V^2\frac{\partial^2}{\partial x^2}\right)U(x,t)\right] \end{aligned}\right\}--(5)$$

3.0 Method of Solution

Evidently, a closed form solution of the differential equation (5) does not exist. A series solution of the form equation (6) is assumed

$$\mathrm{U(x,t)}=\sum_{n=1}^{\infty}X_n(x)T_n(t) \tag{6}$$

Where $T_n(t)$ is chosen such that desired boundary condition is satisfied.
Substituting equation (6) into equation (5) and multiplying both sides of the equation by $X_k(x)$ and then integrating it along the length of the beam

$$\left.\begin{aligned} & EI_O\int_O^L\left(\left[\sum_{n=1}^{\infty}X_n^{iv}(x)T_n(t)\left(10+15\,Sin\frac{\pi x}{L}-6Cos\frac{2\pi x}{L}-Sin\frac{3\pi x}{L}\right)\sum_{n=1}^{\infty}X_n^{iii}(x)T_n(t)\right.\right. \\ & \left(\frac{30\pi}{L}Cos\frac{\pi x}{L}+\frac{24\pi}{L}Sin\frac{2\pi x}{L}-\frac{6\pi}{L}Cos\frac{3\pi x}{L}\right)+\sum_{n=1}^{\infty}X_n^{ii}(x)T_n(t) \\ & \left.\left(\frac{24\pi^2}{L^2}Cos\frac{2\pi x}{L}-\frac{15\pi^2}{L^2}Sin\frac{3\pi x}{L}+\frac{9\pi^2}{L^2}Sin\frac{\pi x}{L}\right)\right]+\mu_0\left(1+Sin\frac{\pi x}{L}\right)\sum_{n=1}^{\infty}X_n(x)\ddot{T}_n(t)-N\sum_{n=1}^{\infty}X_n^{ii}(x)T_n(t) \\ & +K\left(4x-3x^2+x^3\right)\sum_{n=1}^{\infty}X_n(x)T_n(t)'+M\sum_{n=1}^{\infty}X_n(x)\ddot{T}_n(t)H\left(x-\varepsilon+\frac{\varepsilon}{2}\right)-M\sum_{n=1}^{\infty}X_n(x)\ddot{T}_n(t)H\left(x-\varepsilon-\frac{\varepsilon}{2}\right) \\ & +2MV\sum_{n=1}^{\infty}X_n^{i}(x)\dot{T}_n(t)H\left(x-\varepsilon+\frac{\varepsilon}{2}\right)-2MV\sum_{n=1}^{\infty}X_n^{i}(x)\dot{T}_n(t)H\left(x-\varepsilon-\frac{\varepsilon}{2}\right)+MV^2\sum_{n=1}^{\infty}X_n^{ii}(x)\ddot{T}_n^{ii}(t)H\left(x-\varepsilon+\frac{\varepsilon}{2}\right) \\ & -MV^2\sum_{n=1}^{\infty}X_n^{ii}(x)\ddot{T}_n^{ii}(t)H\left(x-\varepsilon-\frac{\varepsilon}{2}\right)X_K(x)dx=\int_0^L Mg\left\{H\left(x-\varepsilon+\frac{\varepsilon}{2}\right)-H\left(x-\varepsilon-\frac{\varepsilon}{2}\right)\right\}X_K(x)dx \end{aligned}\right\}(7)$$

By applying orthogonality condition

$$\int_O^L X_n(x)X_k(x)dx = \left.\begin{matrix} 0, n \neq k \\ \alpha, n = k \end{matrix}\right\} \tag{8}$$

And α is a constant, equation (7) become

$$\left.\begin{matrix} \ddot{T}_n(t)[P_{11} + P_{12} + P_{17} - P_{18}] + \dot{T}_n(t)[P_{19} - P_{20}] + T_n(t)\, P_1 + P_2 - P_3 - P_4 + P_5 + P_6 - P_7 + P_8 - P_9 + P_{10} - \\ P_{13} + P_{14} + P_{15} + P_{16} + P_{21} - P_{22}] = P_{23} - P_{24} \end{matrix}\right\} (9)$$

where

$$\left.\begin{matrix}
P_1 = \frac{10EI_O}{4}\int_O^L X_n^{iv}(x)\,X_k(x)dx, P_{13} = N\int_O^L X_n^{II}(x)\,X_k(x) \\
P_2 = \frac{15EI_O}{4}\int_O^L X_n^{iv}(x)\,X_k(x)\sin\left(\frac{\pi x}{L}\right)dx, P_{14} = K\int_O^L X_n(x)\,X_k(x)4xdx \\
P_3 = \frac{6EI_O}{4}\int_O^L X_n^{iv}(x)\,X_k(x)\cos\left(\frac{2\pi x}{L}\right)dx, P_{15} = K\int_O^L X_n(x)\,X_k(x)3x^2\,dx \\
P_4 = \frac{EI_O}{4}\int_O^L X_n^{iv}(x)\,X_k(x)\sin\left(\frac{2\pi x}{L}\right)dx, P_{16} = K\int_O^L X_n(x)\,X_k(x)x^3\,dx \\
P_5 = \frac{30EI_O}{4}\int_O^L X_n^{III}(x)\,X_k(x)\frac{\pi}{L}\cos\left(\frac{\pi x}{L}\right)dx, P_{17} = M\int_O^L X_n(x)\,X_k(x)\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)\right]dx \\
P_6 = \frac{24EI_O}{4}\int_O^L X_n^{III}(x)\,X_k(x)\frac{\pi}{L}\sin\left(\frac{2\pi x}{L}\right)dx, P_{18} = M\int_O^L X_n(x)\,X_k(x)\left[H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]dx \\
P_7 = \frac{6EI_O}{4}\int_O^L X_n^{III}(x)\,X_k(x)\frac{\pi}{L}\cos\left(\frac{3\pi x}{L}\right)dx, P_{19} = 2mv\int_O^L X_n^1(x)\,X_k(x)\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)\right]dx \\
P_8 = \frac{24EI_O}{4}\int_O^L X_n^{II}(x)\,X_k(x)\frac{\pi^2}{L^2}\cos\left(\frac{2\pi x}{L}\right)dx, P_{20} = 2mv\int_O^L X_n^1(x)\,X_k(x)\left[H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]dx \\
P_9 = \frac{15EI_O}{4}\int_O^L X_n^{II}(x)\,X_k(x)\frac{\pi^2}{L^2}\sin\left(\frac{\pi x}{L}\right)dx, P_{21} = 2mv^2\int_O^L X_n^{II}(x)\,X_k(x)\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)\right]dx \\
P_{10} = \frac{9EI_O}{4}\int_O^L X_n^{II}(x)\,X_k(x)\frac{\pi^2}{L^2}\sin\left(\frac{3\pi x}{L}\right)dx, P_{22} = 2mv^2\int_O^L X_n^{II}(x)\,X_k(x)\left[H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]dx \\
P_{11} = N_O\int_O^L X_n(x)\,X_k(x)dx, P_{23} = mg\int_O^L X_n(x)\,X_k(x)\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)\right]dx \\
P_{12} = N_O\int_O^L X_n(x)\,X_k(x)\sin\left(\frac{\pi x}{L}\right)dx, P_{24} = mg\int_O^L X_n(x)\,X_k(x)\left[H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]dx
\end{matrix}\right\} ------(10)$$

For the general solution of the equation (9) and for the purpose of the solution we consider mass (m) traveling with constant velocity (V). Solutions for the greater number of mass can be obtained in the same manner.

Thereafter, the following special case for the equation (9) follows

(a) Moving force

If we neglect the inertia term, we have the classical case of the moving force, under the assumption of equation (9), we have

$$\left.\begin{aligned}&\ddot{T}_n(t)[P_{11} + P_{12} +] + T_n(t)\,P_1 + P_2 - P_3 - P_4 + P_5 + P_6 - P_7 + P_8 - P_9 + P_{10} - \\ &P_{13} + P_{14} + P_{15} + P_{16}] = P_{23} - P_{24}\end{aligned}\right\} \tag{11}$$

(b) Moving mass

If we consider all the inertia terms we have classical case of moving mass under the assumption of equation (9), we have

$$\left.\begin{aligned}&\ddot{T}_n(t)[P_{11} + P_{12} + P_{17} - P_{18}] + \dot{T}_n(t)[P_{19} - P_{20}] + T_n(t)\,P_1 + P_2 - P_3 - P_4 + P_5 + P_6 - P_7 + P_8 - P_9 + P_{10} - \\ &P_{13} + P_{14} + P_{15} + P_{16} + P_{21} - P_{22}] = P_{23} - P_{24}\end{aligned}\right\} \tag{12}$$

5.0 Numerical Analysis and Discussion of Result

To illustrate the foregoing analysis, The uniform Bernoulli – Euler beam of length 10m were considered, M = 3kg, 6kg, 9kgm, N = 75, E = 2.07 x 10”19 N/m^2, $\varepsilon = 0.1$

I = 1.04x10^{-6}m^4, v = 3.3m/s, g = 10ms^{-2} pi= 3.142, n = 1, 2, 3 --- the results were shown graphically below for the various value of foundation moduli and masses.

Table 1 and Figure 1 display the deflection against time of moving mass at different values of pre – stressed. The graph shows that, the response deflection of the beam decrease as the value of the pre-stressed increases. And the maximum deflection was attained at pre-stressed equal 0.Table 2 and Figure 2 display the deflection against time of moving force at different values of pre-stressed. The graph shows that, the response deflection of the beam decrease as the value of the pre-stressed increases. And the maximum deflection was attained at pre-stressed equal 0.Table 3 and Figure 3 shows the comparison of moving mass and moving force for pre-stressed. The graph shown that moving mass is greater than that of moving force. Table 4 and Figure 4 display moving mass at different values of Mass for pre-stressed. The graph shows that the response deflection of the beam increase as the value of the mass increases. Table 5 and Figure 5 show moving force at different values of Mass for pre-stressed. The graph shows that the response deflection of the beam increases as the value of the mass increases. Table 6 and Figure 6 display the deflection against time of moving mass at different values of foundation moduli k. The graph shows that, the response deflection of the beam decreases as the value of the foundation moduli k increases. Table 7 and Figure 7 display the deflection against time of moving force at different values of foundation moduli k. The graph shows that, the response deflection of the beam decreases as the value of the foundation moduli k increases. Table 8 and Figure 8 show the comparison of moving mass and moving force for simple uniform Bernoulli beam on a variable elastic foundation and the graph shows that moving mass is greater than that of moving force. Table 9 and Figure 9 display moving mass at different values of Mass for foundation moduli k. The graph shows that the response deflection of the beam increases as the value of the mass M increases. Table 10 and Figure 10 display moving force at different values of Mass for foundation moduli k. The graph shows that the response deflection of the beam increases as the value of the mass M increases.

Table 1

Moving Mass at different values of N				
s/n	t(sec)	W(x,t), at N=0	W(x,t), at N=10000	W(x,t), at N=20000
1	0	0.00E+00	0.00E+00	0.00E+00
2	0.001	7.25E-08	7.18E-08	7.11E-08
3	0.002	5.19E-08	5.20E-08	5.20E-08
4	0.003	5.88E-09	7.55E-09	9.15E-09
5	0.004	8.55E-08	8.28E-08	8.03E-08
6	0.005	2.93E-08	3.07E-08	3.21E-08
7	0.006	2.19E-08	2.37E-08	2.54E-08
8	0.007	8.76E-08	8.30E-08	7.88E-08
9	0.008	1.06E-08	1.49E-08	1.87E-08
10	0.009	4.38E-08	4.35E-08	4.33E-08
11	0.01	7.82E-08	7.32E-08	6.90E-08

Figure 1: Displacement response of moving mass for simple uniform Bernoulli beam on variable elastic foundation for different values of Pre - stress N

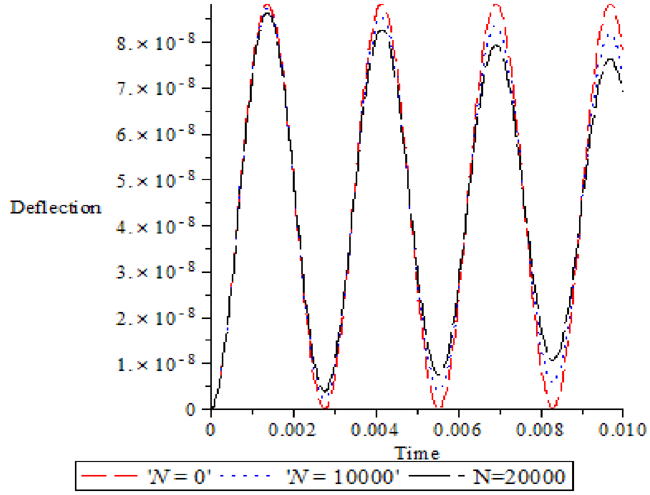


Table2

Moving Force at different values of N				
s/n	t(sec)	W(x,t), at N=0	W(x,t), at N=10000	W(x,t), at N=20000
1	0	0.00E+00	0.00E+00	0.00E+00
2	0.001	3.62E-08	3.59E-08	3.55E-08
3	0.002	2.59E-08	2.60E-08	2.60E-08
4	0.003	2.94E-09	3.78E-09	4.58E-09
5	0.004	4.28E-08	4.14E-08	4.01E-08
6	0.005	1.46E-08	1.54E-08	1.60E-08
7	0.006	1.09E-08	1.19E-08	1.27E-08
8	0.007	4.38E-08	4.15E-08	3.94E-08
9	0.008	5.31E-09	7.46E-09	9.35E-09
10	0.009	2.19E-08	2.18E-08	2.17E-08
11	0.01	3.91E-08	3.66E-08	3.45E-08

Figure 2: Displacement response of moving force for simple uniform Bernoulli beam on variable elastic foundation for different values of Pre - stress N

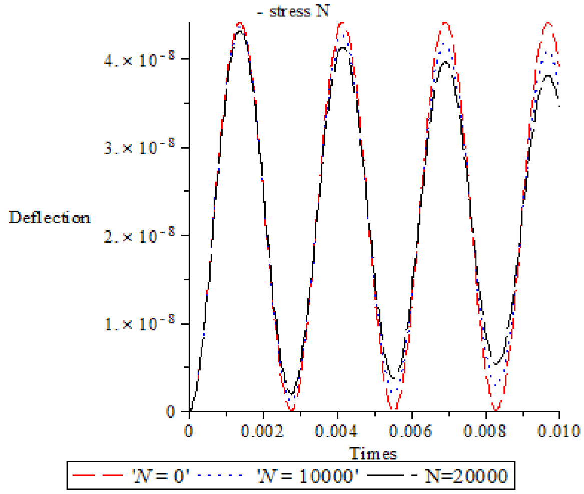


Table3

Comparison of moving mass and moving Force for Pre-stress N			
s/n	t(sec)	moving mass at N=10000	moving mass at N=10000
1	0	0.00E+00	0.00E+00
2	0.001	7.18E-08	3.59E-08
3	0.002	5.20E-08	2.60E-08
4	0.003	7.55E-09	3.78E-09
5	0.004	8.28E-08	4.14E-08
6	0.005	3.07E-08	1.54E-08
7	0.006	2.37E-08	1.19E-08
8	0.007	8.30E-08	4.15E-08
9	0.008	1.49E-08	7.46E-09
10	0.009	4.35E-08	2.18E-08
11	0.01	7.32E-08	3.66E-08

Figure 3: Comparison of moving mass and moving force

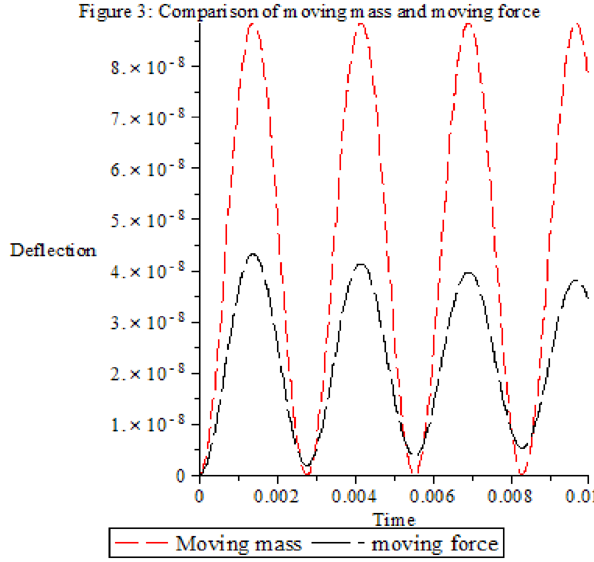


Table 4

Moving Mass at different masses for Pre-stress N				
s/n	t(sec)	W(x,t), at M=3kg	W(x,t), at M=6kg	W(x,t), at M=9kg
1	0	0.00E+00	0.00E+00	0.00E+00
2	0.001	7.25E-08	1.44E-07	2.13E-07
3	0.002	5.19E-08	1.04E-07	1.56E-07
4	0.003	5.88E-09	1.52E-08	2.76E-08
5	0.004	8.55E-08	1.66E-07	2.41E-07
6	0.005	2.93E-08	6.15E-08	9.63E-08
7	0.006	2.19E-08	4.75E-08	9.63E-08
8	0.007	8.76E-08	1.66E-07	2.36E-07
9	0.008	1.06E-08	3.00E-08	5.64E-08
10	0.009	4.38E-08	8.70E-08	1.30E-07
11	0.01	7.82E-08	1.46E-07	2.07E-07

Figure 4: Displacement response of moving mass for simple uniform Bernoulli beam on variable elastic foundation for different values of mass M

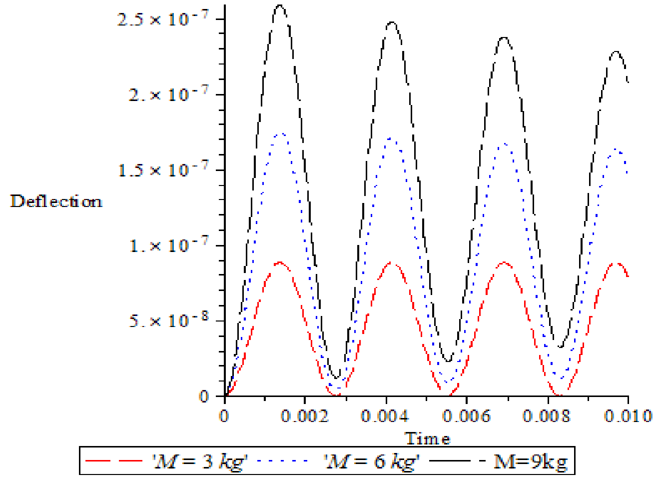


Table 5

Moving Force at different masses for Pre-stress N				
s/n	t(sec)	W(x,t), at M=3kg	W(x,t), at M=6kg	W(x,t), at M=9kg
1	0	0.00E+00	0.00E+00	0.00E+00
2	0.001	3.62E-08	7.18E-08	1.07E-07
3	0.002	2.59E-08	5.20E-08	7.80E-08
4	0.003	2.94E-09	7.58E-09	1.38E-08
5	0.004	4.28E-08	7.58E-09	1.20E-07
6	0.005	1.46E-08	3.07E-08	4.82E-08
7	0.006	1.09E-08	2.37E-08	3.81E-08
8	0.007	4.38E-08	8.29E-08	1.18E-07
9	0.008	5.31E-09	1.50E-08	2.82E-08
10	0.009	2.19E-08	4.35E-08	6.50E-08
11	0.01	3.91E-08	7.31E-08	1.03E-07

Figure 5: Displacement response of moving force for simple uniform Bernoulli beam on variable elastic foundation for different values of mass M

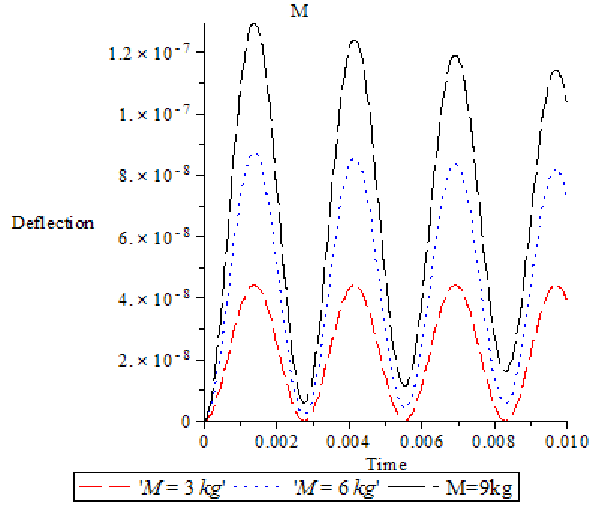


Table 6

Moving Mass at different value of foundation moduli K				
s/n	t(sec)	W(x,t), at K=10000	W(x,t), at K=20000	W(x,t), at K=30000
1	0	0.00E+00	0.00E+00	0.00E+00
2	0.01	7.62E-08	2.59E-09	3.26E-08
3	0.02	1.51E-07	9.98E-09	4.62E-08
4	0.03	7.41E-08	2.12E-08	5.67E-09
5	0.04	1.10E-10	3.46E-08	1.57E-08
6	0.05	7.83E-08	4.85E-08	5.03E-08

Figure 6: Displacement response of moving mass for simple uniform Bernoulli beam on variable elastic foundation for different values of K

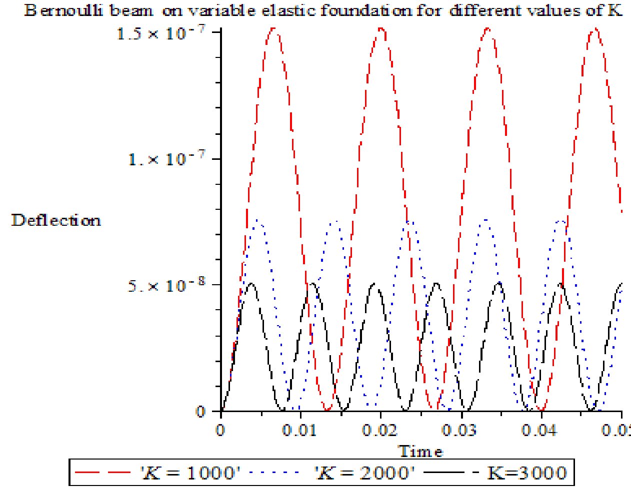


Table 7

Moving Force at different value of foundation moduli K				
s/n	t(sec)	W(x,t), at K=10000	W(x,t), at K=20000	W(x,t), at K=30000
1	0	0.00E+00	0.00E+00	0.00E+00
2	0.01	3.81E-08	1.29E-09	1.63E-08
3	0.02	7.57E-08	4.99E-09	2.31E-08
4	0.03	3.71E-08	1.06E-08	2.83E-09
5	0.04	5.48E-11	1.73E-08	7.87E-09
6	0.05	3.92E-08	2.43E-08	2.52E-08

Figure 7: Displacement response of moving force for simple uniform Bernoulli beam on variable elastic foundation for different values of K

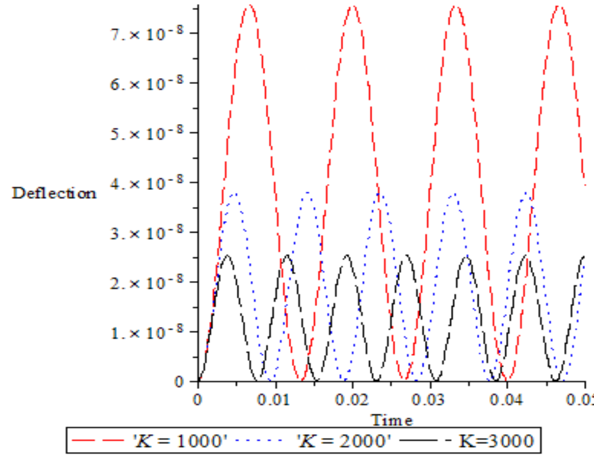


Table 8

Comparison of moving mass and moving Force			
s/n	t(sec)	moving mass at K=10000	moving force at K=10000
1	0	0.00E+00	0.00E+00
2	0.01	7.62E-08	3.81E-08
3	0.02	1.51E-07	7.57E-08
4	0.03	7.41E-08	3.71E-08
5	0.04	1.10E-10	5.48E-11
6	0.05	7.83E-08	3.92E-08

Figure 8: Comparison of moving mass and moving force for simple uniform Bernoulli beam on variable elastic foundation

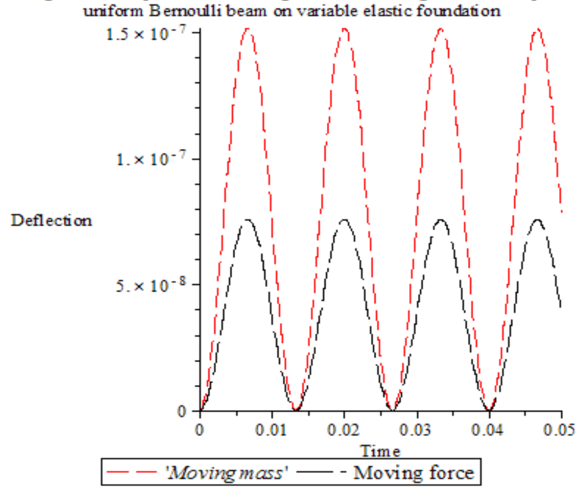


Table 9

Moving Mass at different masses for Foundation moduli K				
s/n	t(sec)	W(x,t), at M=3kg	W(x,t), at M=6kg	W(x,t), at M=9kg
1	0	0.00E+00	0.00E+00	0.00E+00
2	0.01	7.62E-08	1.55E-07	2.37E-07
3	0.02	1.51E-07	3.02E-07	4.53E-07
4	0.03	7.41E-08	1.40E-07	1.97E-07
5	0.04	1.10E-10	1.12E-09	4.28E-09
6	0.05	7.83E-08	1.71E-07	2.77E-07

Figure 9: Displacement response of moving mass for simple uniform Bernoulli beam on variable elastic foundation for different values of mass M

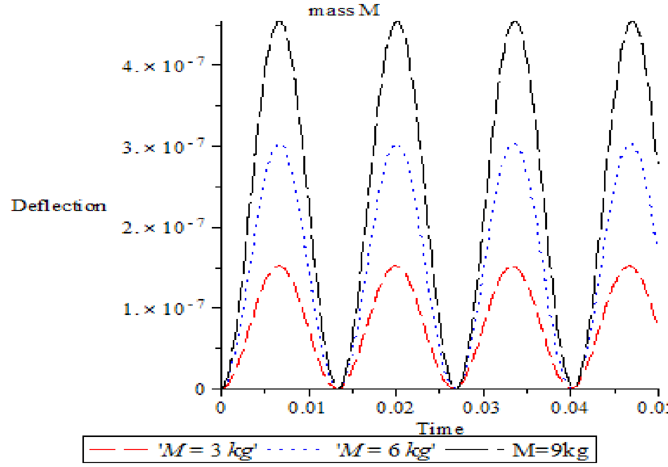
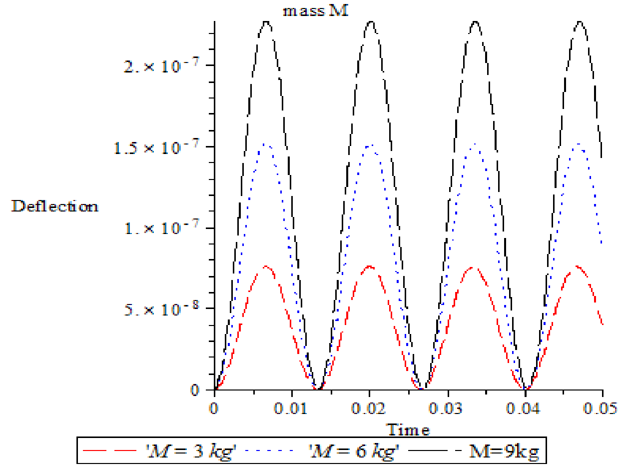


Table 10

Moving Force at different masses for Foundation moduli K				
s/n	t(sec)	W(x,t), at M=3kg	W(x,t), at M=6kg	W(x,t), at M=9kg
1	0	0.00E+00	0.00E+00	0.00E+00
2	0.01	3.81E-08	7.77E-08	7.77E-08
3	0.02	7.57E-08	1.51E-07	1.51E-07
4	0.03	3.71E-08	6.99E-08	6.99E-08
5	0.04	5.48E-11	5.62E-10	5.62E-10
6	0.05	3.92E-08	8.54E-08	8.54E-08

Figure 10: Displacement response of moving froce for simple uniform Bernoulli beam on variable elastic foundation for different values of mass M



APPENDIX I: maple code for linearly varying distributed moving load.

> $restart;$

> $w_1 := m_1 \cdot g :$

> $w_2 := m_2 \cdot g :$

> $a_1 := v \cdot t - \frac{d}{2} :$

> $a_2 := v \cdot t + \frac{d}{2} :$

> $P_1 := \int_0^L w_1 \cdot \sin\left(\frac{k \cdot \mathrm{pi} \cdot x}{L}\right) \mathrm{d}x :$

> $P_2 := \frac{(w_2 - w_1)}{d} \int_0^L (x - a_1) \sin\left(\frac{k \cdot \mathrm{pi} \cdot x}{L}\right) \mathrm{d}x :$

> $P_3 := \int_0^L -w_2 \cdot \sin\left(\frac{k \cdot \mathrm{pi} \cdot x}{L}\right) \mathrm{d}x :$

> $P_4 := \frac{(-w_2 - w_1)}{d} \int_0^L (x - a_1) \sin\left(\frac{k \cdot \mathrm{pi} \cdot x}{L}\right) \mathrm{d}x :$

> $Q4 := \frac{1}{\alpha \cdot \mu} (P_1 + P_2 + P_3 + P_4) :$

> $equ1 := diff(T(t), t, t) + 2 \cdot \omega_b \cdot diff(T(t), t) + \omega_n^2 \cdot T(t) = Q4 :$

> $equ2 := evalf(subs(m_1 = 3, m_2 = 6, \alpha = 1, \mu = 70, k = 1, n = 1, g = 10, v = 3.3, L = 15, \mathrm{pi} = 180, x = 1, \omega_1 = 0.08201461400, \omega_b = 2.0, d = 0.01, equ1)) :$

> $X := evalf(rhs(dsolve(\{equ2, T(0) = 0, \mathrm{D}(T)(0) = 0\}, T(t)))) :$

> $D1 := evalf\left(\sin\left(\frac{180}{15}\right) \cdot X\right) :$

> $evalf(subs(t = 3.0, D1)) :$

```
> plot(D1, t = 0 ..3, labels = ["time","Deflection"], legend
      = [deflection at ω_b = 2]) :

> equ2 := evalf( subs( m_1 = 3, m_2 = 6, α = 1, μ = 70, k = 1, n = 1, g = 10, ι
      = 3.3, L = 15, pi = 180 , x = 1, ω_1 = 0.08201461400, ω_b = 4.0, d
      = 0.01, equ1)) :

> X := evalf(rhs(dsolve({equ2, T(0) = 0, D(T)(0) = 0}, T(t)))) :
```

$$> D2 := evalf\left(\sin\left(\frac{180}{15}\right)\cdot X\right) :$$

```
> evalf(subs(t = 3.0, D2)) :

> plot(D2, t = 0 ..3, labels = ["time","Deflection"], legend
      = [deflection at ω_b = 4]) :

> equ2 := evalf( subs( m_1 = 3, m_2 = 6, α = 1, μ = 70, k = 1, n = 1, g = 10, ι
      = 3.3, L = 15, pi = 180 , x = 1, ω_1 = 0.08201461400, ω_b = 6.0, d
      = 0.01, equ1)) :

> X := evalf(rhs(dsolve({equ2, T(0) = 0, D(T)(0) = 0}, T(t)))) :
```

$$> D3 := evalf\left(\sin\left(\frac{180}{15}\right)\cdot X\right) :$$

```
> evalf(subs(t = 3.0, D3)) :
> plot(D3, t = 0 ..3, labels = ["time","Deflection"], legend
      = [deflection at ω_b = 6]) :

> plot([D1, D2, D3], t = 0 ..3, linestyle = [dash, dot, longdash] , color
      = [red, blue, black], legend = ["ω_b = 2.0","ω_b = 4.0", "ω_b = 6.0"]) :
```

APPENDIX II: **maple code for linearly moving load.**

```
> restart;
```

$$> w_1 := m_1\cdot g :$$

$$> w_2 := m_2\cdot g :$$

$$> a_1 := v\cdot t - \frac{d}{2} :$$

$$> a_2 := v\cdot t + \frac{d}{2} :$$

$$> P_1 := \int_0^L w_1\cdot\sin\left(\frac{k\cdot \mathrm{pi}\cdot x}{L}\right)\mathrm{d}x :$$

$$> P_2 := \frac{(w_2 - w_1)}{d}\int_0^L (x - a_1)\sin\left(\frac{k\cdot \mathrm{pi}\cdot x}{L}\right)\mathrm{d}x :$$

$$> P_3 := \int_0^L -w_2\cdot\sin\left(\frac{k\cdot \mathrm{pi}\cdot x}{L}\right)\mathrm{d}x :$$

$> P_4 := \frac{(-w_2 - w_1)}{d} \int_0^L (x - a_1)\sin\left(\frac{k \cdot \mathrm{pi} \cdot x}{L}\right) \mathrm{d}x :$

$> Q4 := \frac{1}{\alpha \cdot \mu}(P_1) :$

$> equ1 := diff(T(t), t, t) + 2 \cdot \omega_b \cdot diff(T(t), t) + \omega_n^2 \cdot T(t) = Q4 :$

$> equ2 := evalf\Big(subs\Big(m_1 = 3, m_2 = 6, \alpha = 1, \mu = 70, k = 1, n = 1,\ g = 10,$
$= 3.3, L = 15, \mathrm{pi} = 180\ , x = 1, \omega_1 = 0.08201461400, \omega_b = 2.0, d$
$= 0.01, equ1\Big)\Big) :$

$> X := evalf(rhs(dsolve(\{equ2, T(0) = 0, \mathrm{D}(T)(0) = 0\}, T(t)))) :$

$> D1 := evalf\left(\sin\left(\frac{180}{15}\right) \cdot X\right) :$

$> evalf(subs(t = 2.2, D1)) :$

$> plot\Big(D1, t = 0\,..3, labels = [\text{"time"},\text{"Deflection"}], legend$
$= \left[deflection\ at\ \omega_b = 2\right]\Big) :$

$> equ2 := evalf\Big(subs\Big(m_1 = 3, m_2 = 6, \alpha = 1, \mu = 70, k = 1, n = 1,\ g = 10,$
$= 3.3, L = 15, \mathrm{pi} = 180\ , x = 1, \omega_1 = 0.08201461400, \omega_b = 4.0, d$
$= 0.01, equ1\Big)\Big) :$

$> X := evalf(rhs(dsolve(\{equ2, T(0) = 0, \mathrm{D}(T)(0) = 0\}, T(t)))) :$

$> D2 := evalf\left(\sin\left(\frac{180}{15}\right) \cdot X\right) :$

$> evalf(subs(t = 2.2, D2)) :$

$> plot\Big(D2, t = 0\,..3, labels = [\text{"time"},\text{"Deflection"}], legend$
$= \left[deflection\ at\ \omega_b = 4\right]\Big) :$

$> equ2 := evalf\Big(subs\Big(m_1 = 3, m_2 = 6, \alpha = 1, \mu = 70, k = 1, n = 1,\ g = 10,$
$= 3.3, L = 15, \mathrm{pi} = 180\ , x = 1, \omega_1 = 0.08201461400, \omega_b = 6.0, d$
$= 0.01, equ1\Big)\Big) :$

$> X := evalf(rhs(dsolve(\{equ2, T(0) = 0, \mathrm{D}(T)(0) = 0\}, T(t)))) :$

$> D3 := evalf\left(\sin\left(\frac{180}{15}\right) \cdot X\right) :$

$> plot\Big(D3, t = 0\,..3, labels = [\text{"time"},\text{"Deflection"}], legend$
$= \left[deflection\ at\ \omega_b = 6\right]\Big) :$

$> plot\Big([D1, D2, D3], t = 0\,..3, linestyle = [dash, dot, longdash]\ , color$
$= [red, blue, black], legend = \left[\text{"}\omega_b = 2.0\text{"},\text{"}\omega_b = 4.0\text{"}, \text{"}\omega_b = 6.0\text{"}\right]\Big) :$

APPENDIX III: **maple code for concentrated moving load.**

> *restart*;

>

$$A := \frac{M\cdot g}{L}\cdot\int_0^L \sin\left(\frac{k\cdot\mathrm{pi}\cdot x}{L}\right)\mathrm{d}x + \frac{2\cdot M\cdot g}{L}\cdot\sum_{n=1}^{2}\cos\left(\frac{n\cdot\mathrm{pi}\cdot v\cdot t}{L}\right)$$
$$\int_0^L \sin\left(\frac{k\cdot\mathrm{pi}\cdot x}{L}\right)\cos\left(\frac{n\cdot\mathrm{pi}\cdot x}{L}\right)\mathrm{d}x :$$

> $$B := M\cdot\left(\frac{1}{L} + \frac{2}{L}\cdot\sum_{n=1}^{2}\cos\left(\frac{n\cdot\mathrm{pi}\cdot v\cdot t}{L}\right)\cos\left(\frac{n\cdot\mathrm{pi}\cdot x}{L}\right)\right) :$$

>

$$C := \frac{2\cdot M\cdot v}{L}\cdot\left(\frac{n\cdot\mathrm{pi}}{L}\right)\cdot\int_0^L \cos\left(\frac{n\cdot\mathrm{pi}\cdot x}{L}\right)\sin\left(\frac{k\cdot\mathrm{pi}\cdot x}{L}\right)\mathrm{d}x + \frac{2\cdot M\cdot v}{L}$$
$$\cdot\left(\frac{n\cdot\mathrm{pi}}{L}\right)\cdot\sum_{n=1}^{2}\cos\left(\frac{n\cdot\mathrm{pi}\cdot v\cdot t}{L}\right)$$
$$\int_0^L \cos\left(\frac{n\cdot\mathrm{pi}\cdot x}{L}\right)\sin\left(\frac{k\cdot\mathrm{pi}\cdot x}{L}\right)\cos\left(\frac{n\cdot\mathrm{pi}\cdot x}{L}\right)\mathrm{d}x :$$

$$D1 := \frac{-1\cdot M\cdot v^2}{L}\cdot\left(\frac{n\cdot\mathrm{pi}}{L}\right)^2\cdot\int_0^L \sin\left(\frac{n\cdot\mathrm{pi}\cdot x}{L}\right)\sin\left(\frac{k\cdot\mathrm{pi}\cdot x}{L}\right)\mathrm{d}x$$
$$-\frac{M\cdot v^2}{L}\cdot\left(\frac{n\cdot\mathrm{pi}}{L}\right)^2\cdot\sum_{n=1}^{2}\cos\left(\frac{n\cdot\mathrm{pi}\cdot v\cdot t}{L}\right)$$
$$\int_0^L \sin\left(\frac{n\cdot\mathrm{pi}\cdot x}{L}\right)\sin\left(\frac{k\cdot\mathrm{pi}\cdot x}{L}\right)\cos\left(\frac{n\cdot\mathrm{pi}\cdot x}{L}\right)\mathrm{d}x :$$

>

$$equ := diff\left(T_n(t), t, t\right)\cdot(\alpha\cdot\mu + B) + diff\left(T_n(t), t\right)\cdot\left(2\cdot\omega_b\cdot\alpha\cdot\mu \quad C\right)$$
$$+ T_n(t)\cdot\left(\omega_n^2\cdot\alpha\cdot\mu + D1\right) = A :$$

> $equ1 := evalf\Big(subs\big(M = 3, \alpha = 1, \mu = 70, k = 5, n = 1, g = 10, v = 3.3, L = 15, \mathrm{pi} = 3.142, x = 1, \omega_1 = 0.08201461400, \omega_b = 2.0, equ\big)\Big) :$

> $equ2 := evalf\Big(subs\big(M = 3, \alpha = 1, \mu = 70, k = 5, n = 1, g = 10, v = 3.3, L = 15, \mathrm{pi} = 3.142, x = 1, \omega_1 = 0.08201461400, \omega_b = 4.0, equ\big)\Big) :$

> $equ3 := evalf\Big(subs\big(M = 3, \alpha = 1, \mu = 70, k = 5, n = 1, g = 10, v = 3.3, L = 15, \mathrm{pi} = 3.142, x = 1, \omega_1 = 0.08201461400, \omega_b = 6.0, equ\big)\Big) :$

> $equ4 := evalf\Big(subs\big(M = 6, \alpha = 1, \mu = 70, k = 5, n = 1, g = 10, v = 3.3, L = 15, \mathrm{pi} = 3.142, x = 1, \omega_1 = 0.08201461400, \omega_b = 2.0, equ\big)\Big) :$

> $equ5 := evalf\Big(subs\big(M = 9, \alpha = 1, \mu = 70, k = 5, n = 1, g = 10, v = 3.3, L = 15, \mathrm{pi} = 3.142, x = 1, \omega_1 = 0.08201461400, \omega_b = 2.0, equ\big)\Big) :$

APPENDIX IV: maple code for moving mass.

> $restart;$

> $w_1 := m_1 \cdot g :$

> $w_2 := m_2 \cdot g :$

> $a_1 := v \cdot t - \frac{d}{2} :$

> $a_2 := v \cdot t + \frac{d}{2} :$

> $P_1 := \int_0^L w_1 \cdot \sin\left(\frac{k \cdot \mathrm{pi} \cdot x}{L}\right) \mathrm{d}x :$

> $P_2 := \frac{(w_2 - w_1)}{d} \int_0^L (x - a_1) \sin\left(\frac{k \cdot \mathrm{pi} \cdot x}{L}\right) \mathrm{d}x :$

> $P_3 := \int_0^L -w_2 \cdot \sin\left(\frac{k \cdot \mathrm{pi} \cdot x}{L}\right) \mathrm{d}x :$

> $P_4 := \frac{(-w_2 - w_1)}{d} \int_0^L (x - a_1) \sin\left(\frac{k \cdot \mathrm{pi} \cdot x}{L}\right) \mathrm{d}x :$

> $Q4 := \frac{1}{\alpha \cdot \mu} (P_1 + P_2 + P_3 + P_4) :$

> $equ1 := diff(T(t), t, t) + 2 \cdot \omega_b \cdot diff(T(t), t) + \omega_n^2 \cdot T(t) = w_1 + w_2 :$

> $equ2 := evalf\left(subs\left(m_1 = 3, m_2 = 6, \alpha = 1, \mu = 70, k = 1, n = 1, g = 10, \right.\right.$
$= 3.3, L = 15, \mathrm{pi} = 180 , x = 1, \omega_1 = 0.08201461400, \omega_b = 2.0, d$
$\left.\left. = 0.01, equ1 \right)\right) :$

> $X := evalf(rhs(dsolve(\{equ2, T(0) = 0, \mathrm{D}(T)(0) = 0\}, T(t)))) :$

> $D1 := evalf\left(\sin\left(\frac{180}{15}\right) \cdot X\right) :$

> $evalf(subs(t = 3.0, D1)) :$

> $plot\Big(D1, t = 0 ..3, labels = [\text{"time"},\text{"Deflection"}], legend$
$= \left[deflection\ at\ \omega_b = 2\right]\Big) :$

> $equ2 := evalf\left(subs\left(m_1 = 3, m_2 = 6, \alpha = 1, \mu = 70, k = 1, n = 1, g = 10, \right.\right.$
$= 3.3, L = 15, \mathrm{pi} = 180 , x = 1, \omega_1 = 0.08201461400, \omega_b = 4.0, d$
$\left.\left. = 0.01, equ1 \right)\right) :$

> $X := evalf(rhs(dsolve(\{equ2, T(0) = 0, \mathrm{D}(T)(0) = 0\}, T(t)))) :$

> $D2 := evalf\left(\sin\left(\frac{180}{15}\right) \cdot X\right) :$

> $evalf(subs(t = 2.9, D2)) :$

> $plot\Big(D2, t = 0 ..3, labels = [\text{"time"},\text{"Deflection"}], legend$
$= \left[deflection\ at\ \omega_b = 4\right]\Big) :$

> $equ2 := evalf\left(subs\left(m_1 = 3, m_2 = 6, \alpha = 1, \mu = 70, k = 1, n = 1, g = 10, \right.\right.$
$= 3.3, L = 15, \mathrm{pi} = 180 , x = 1, \omega_1 = 0.08201461400, \omega_b = 6.0, d$
$\left.\left. = 0.01, equ1 \right)\right) :$

> $X := evalf(rhs(dsolve(\{equ2, T(0)=0, \mathrm{D}(T)(0)=0\}, T(t)))):$

> $D3 := evalf\left(\sin\left(\frac{180}{15}\right)\cdot X\right):$

> $evalf(subs(t=3.0, D3)):$

> $plot\left(D3, t=0..3, labels=["time","Deflection"], legend = \left[deflection\ at\ \omega_b = 6\right]\right):$

> $plot\left([D1, D2, D3], t=0..3, linestyle=[dash, dot, longdash], color = [red, blue, black], legend = \left["\omega_b = 2.0","\omega_b = 4.0", "\omega_b = 6.0"\right]\right):$

APPENDIX V: maple code for moving mass

> $restart;$

> $w_1 := m_1 \cdot g:$

> $w_2 := m_2 \cdot g:$

> $a_1 := v\cdot t - \frac{d}{2}:$

> $a_2 := v\cdot t + \frac{d}{2}:$

> $P_1 := \int_0^L w_1 \cdot \sin\left(\frac{k\cdot \mathrm{pi}\cdot x}{L}\right) \mathrm{d}x:$

> $P_2 := \frac{(w_2 - w_1)}{d} \int_0^L (x - a_1)\sin\left(\frac{k\cdot \mathrm{pi}\cdot x}{L}\right) \mathrm{d}x:$

> $P_3 := \int_0^L -w_2 \cdot \sin\left(\frac{k\cdot \mathrm{pi}\cdot x}{L}\right) \mathrm{d}x:$

> $P_4 := \frac{(-w_2 - w_1)}{d} \int_0^L (x - a_1)\sin\left(\frac{k\cdot \mathrm{pi}\cdot x}{L}\right) \mathrm{d}x:$

> $Q4 := \frac{1}{\alpha\cdot\mu}(P_1 + P_2 + P_3 + P_4):$

> $equ1 := diff(T(t), t, t) + 2\cdot\omega_b\cdot diff(T(t), t) + \omega_n^2 \cdot T(t) = Q4:$

> $equ2 := evalf\left(subs\left(m_1 = 3, m_2 = 6, \alpha = 1, \mu = 70, k = 1, n = 1, g = 10, = 3.3, L = 15, \mathrm{pi} = 3.142\ , x = 1, \omega_1 = 0.08201461400, \omega_b = 2.0, d = 0.01, equ1\right)\right):$

> $X := evalf(rhs(dsolve(\{equ2, T(0)=0, \mathrm{D}(T)(0)=0\}, T(t)))):$

> $D1 := evalf\left(\sin\left(\frac{3.142}{15}\right)\cdot X\right):$

> $plot\left(D1, t=0..3, labels=["time","Deflection"], legend = \left[deflection\ at\ \omega_b = 2\right]\right):$

> $equ2 := evalf\left(subs\left(m_1 = 3, m_2 = 6, \alpha = 1, \mu = 70, k = 1, n = 1, g = 10, = 3.3, L = 15, \mathrm{pi} = 3.142, x = 1, \omega_1 = 0.08201461400, \omega_b = 4.0, d = 0.01, equ1\right)\right):$

> $X := evalf(rhs(dsolve(\{equ2, T(0)=0, \mathrm{D}(T)(0)=0\}, T(t)))):$

> $D2 := evalf\left(\sin\left(\frac{3.142}{15}\right)\cdot X\right):$

> $plot\left(D2, t = 0..3, labels = [\text{"time"},\text{"Deflection"}], legend = \left[deflection\ at\ \omega_b = 4\right]\right):$

> $equ2 := evalf\left(subs\left(m_1 = 3, m_2 = 6, \alpha = 1, \mu = 70, k = 1, n = 1, g = 10, \right.\right.$ $= 3.3, L = 15, \text{pi} = 3.142, x = 1, \omega_1 = 0.08201461400, \omega_b = 6.0, d = 0.01, equ1\left.\left.\right)\right):$

> $X := evalf(rhs(dsolve(\{equ2, T(0) = 0, \mathrm{D}(T)(0) = 0\}, T(t)))):$

> $D3 := evalf\left(\sin\left(\frac{3.142}{15}\right)\cdot X\right):$

> $plot\left(D3, t = 0..3, labels = [\text{"time"},\text{"Deflection"}], legend = \left[deflection\ at\ \omega_b = 6\right]\right):$

> $plot\left([D1, D2, D3], t = 0..3, labels = [\text{"Time"}, \text{"Deflection"}], linestyle = [dash, dot, longdash], color = [red, blue, black], legend = \left[\text{"}\omega_b = 2.0\text{"},\text{"}\omega_b = 4.0\text{"}, \text{"}\omega_b = 6.0\text{"}\right]\right):$

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