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Heat and Mass Transfer in a Micropolar Fluid in the Presence of Thermal Radiation and Chemical Reaction

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Abstract

Unsteady heat and mass transfer by free convective micropolar fluid flow bounded by a semi- infinite porous plate in a rotating frame with radiation in the presence of chemical reaction is considered. The plate is assumed to oscillate in time with constant frequency so that the solutions of the boundary layer are the same oscillatory type. The governing system of partial differential equations is transformed to dimensionless equations using dimensionless variables. The dimensionless equations are then solved analytically using perturbation technique. With the help of graphs, the effects of the various important parameters entering into the problem on the velocity, microrotation, temperature and concentration fields within the boundary layer are discussed. Also the effects of the pertinent parameters on the local skin friction coefficient and rates of heat and mass transfer in terms of the local Nusselt and Sherwood numbers are presented numerically in tabular form. The results show that the observed parameters have significance influence on the flow.

Keywords: Micropolar fluid, radiation, chemical reaction, perturbation technique, porous medium, heat and mass transfer, rotating frame.

1.0 Introduction

Micropolar fluids are referred to those fluids that contain micro-constituents that can undergo rotation which affect the hydrodynamics of the flow, this make them distinctly non-Newtonian in nature. The basic continuum theory for this class of fluids was originally formulated by Eringen [1]. The equations governing the flow of a micropolar fluid involve microrotation vector and gyration parameter in addition to the classical velocity vector field. Eringen's micropolar fluid theory has been employed to study a number of various flow situations such as the flow of low concentration suspension, liquid crystals, blood and turbulent shear flows. The theory may also be applied to explain the flow of colloidal solutions, fluids with additives and many other situations.

The effects of radiation on unsteady free convection flow and heat transfer problem have become more important industrially. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for design of reliable equipments, nuclear plants, gas turbines and various propulsion devices or aircraft, missiles, satellites and space vehicles. Based on these applications, Cogley et. al [2] showed that in the optically thin limit, the fluid does not absorb it's own emitted radiation but the fluid does absorb radiation emitted by the boundaries. Hossain and Takhar[3] have considered the radiation effects on mixed convection boundary layer flow of an optically dense viscous incompressible fluid along a vertical plate with uniform surface temperature. Makinde [4] examined the transient free convection interaction with thermal radiation of an absorbing emitting fluid along moving vertical permeable plate. Satter and Hamid [5] investigated the unsteady free convection

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interaction with thermal radiation of an absorbing emitting plate. Rahman and Satter [6] studied transient convective flow of micropolar fluid past a continuous moving porous plate in the presence of radiation. Heat and mass transfer effects on unsteady magneto hydrodynamics free convection flow near a moving vertical plate embedded in a porous medium was presented by Das and Jana [7]. Olajuwon [8] examined convection heat and mass transfer in a hydromagnetic flow of a second grade fluid past a semi-infinite stretching sheet in the presence of thermal radiation and thermo diffusion. Haque et al [9] studied micropolar fluid behavior on steady magneto hydrodynamics free convection flow and mass transfer through a porous medium with heat and mass fluxes. Soret and dufour effects on mixed convection in a non- Darcy porous medium saturated with micropolar fluid was studied by Srinivascharya [10]. Rebhi [11] studied unsteady natural convection heat and mass transfer of micropolar fluid over a vertical surface with constant heat flux. The governing equations were solved numerically using McCormack's technique and effects of various parameters were investigated on the flow. Eldabe and Ouaf [12] solved the problem of heat and mass transfer in a hydromagnetic flow of a micropolar fluid past a stretching surface with ohmic heating and viscous dissipation using the Chebyshev finite difference method. Keelson and Desseaux [13] studied the effect of surface conditions on the flow of a micropolar fluid driven by a porous stretching surface. The governing equations were solved numerically. Sunil et al [14] studied the effect of rotation on a layer of micropolar ferromagnetic fluid heated from below saturating a porous medium. The resulted non-linear coupled differential equations from the transformation were solved using finite-difference method. Mahmoud [15] investigated thermal radiation effect on magneto hydrodynamic flow of a micropolar fluid over a stretching surface with variable thermal conductivity. The solution was obtained numerically by iterative, Runge-Kuta order-four method. Magdy [16] studied unsteady free convection flow of an incompressible electrically conducting micropolar fluid, bounded by an infinite vertical plane surface of constant temperature with thermal relaxation including heat sources. The governing equations were solved using Laplace transformation. The inversion of the Laplace transforms was carried out with a numerical method. Bayomi et al [17] consider magneto hydrodynamic flow of a micropolar fluid along a vertical semi-infinite permeable plate in the presence of wall suction or injection effects and heat generation or absorption. The obtained self-similar equation were solved numerically by an efficient implicit, iterative, infinite-difference method. Reena and Rana [18] investigated double-diffusive convection in a micropolar fluid layer heated and soluted from below a saturating porous medium. A linear stability analysis theory and normal mode analysis method was used.

Many processes involving chemical reaction take place in numerous industrial applications, e.g polymer production, manufacturing of ceramics or glassware, and food processing. Chambre and Young [19] analyzed the diffusion of chemically reactive species in a laminar boundary layer flow. Kandasamy et al [20] studied the nonlinear MHD flow, with heat and mass transfer characteristics, of an incompressible, viscous, electrically conducting, Boussinesq fluid on a vertical stretching surface with chemical reaction and thermal stratification effects. Chaudhary [21] investigated the effects of chemical reactions on magneto hydrodynamic micropolar fluid flow past a vertical plate in slip-flow regime. Heat and mass transfer effects on the unsteady flow of a micropolar fluid through a porous medium bounded by a semi-infinite vertical plate in a slip flow regime was studied taking into account a homogeneous chemical reaction of the first order. Modather et al[22] studied MHD heat and mass transfer oscillatory flow of a micropolar fluid over a vertical permeable plate in a porous medium. Bakr[23] presented an analysis on magneto hydrodynamics free convection and mass transfer adjacent to moving vertical plate for micropolar fluid in a rotating frame of reference in the presence of heat generation/absorption and a chemical reaction.

Rotating flows of unsteady free convection non-Newtonian fluids have many applications in geophysics, turbo machinery and many other fields. This present model have applications in the dialysis of blood in artificial kidney, blood flow in the capillaries, flow in blood oxygenation, design of filters, the porous pipe design, e.t.c. Motivated by these applications and previous work done, we studied the effects of radiation and chemical reaction on unsteady free convection heat and mass transfer flow of a micropolar fluid past a vertical porous plate in a rotating frame of reference with suction. It is assumed that the plate is embedded in a uniform porous medium and oscillates in time with a constant frequency. The governing equations are solved analytically using perturbation method. Different values of physical parameters are tabulated and discussed numerically and graphically.

2.0 Mathematical Formulation

Consider three dimensional unsteady free convection flow of an incompressible micropolar fluid past a semi –infinite vertical permeable moving plate embedded in a uniform porous medium in the presence of thermal and concentration buoyancy effects with thermal radiation and chemical reaction. The x-axis is taken along the plate in the upward direction and z-axis is normal to it as shown in Fig.1

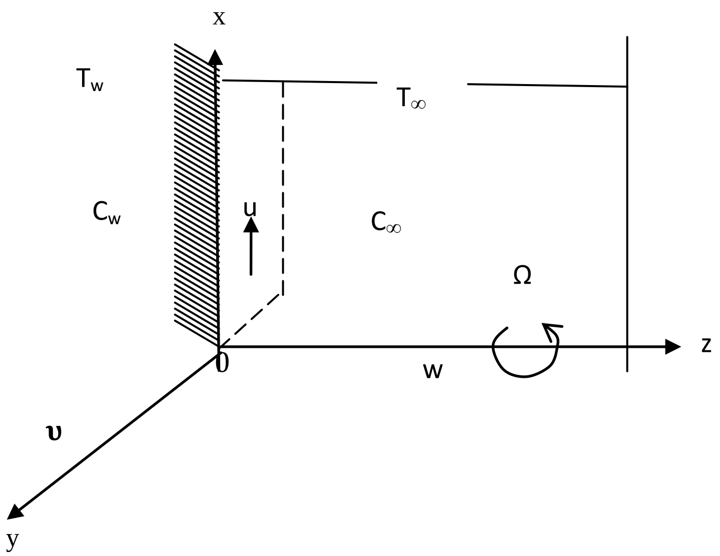

Fig 1: Physical model

The surface of the plate is held at a constant heat flux q_w, and the mass flux of a certain constituent in the solution that saturated the porous medium is held at m_w near the surface. Due to the semi-infinite plane surface assumption, the flow variables are functions of z and t only . The flow is assumed to be in the x-direction which is taken along the plate. Also it is assumed that the whole system rotate with a constant frame **Ω** in a micropolar fluid about z-axis. The radiation heat flux in x-direction is considered negligible in comparison with that in the z-direction

Under these assumptions, the governing equation of the flow are:

$$\frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial z} - 2\Omega\upsilon = (\nu + \nu_r)\frac{\partial^2 u}{\partial z^2} + g\beta_T(T - T_\infty) + g\beta_c(C - C_\infty) - \frac{\nu u}{k} - \nu_r\frac{\partial \varpi_2}{\partial z} \tag{2}$$

$$\frac{\partial \upsilon}{\partial t} + w\frac{\partial \upsilon}{\partial z} + 2\Omega u = (\nu + \nu_r)\frac{\partial^2 \upsilon}{\partial z^2} - \frac{\nu\upsilon}{k} + \nu_r\frac{\partial \varpi_1}{\partial z} \tag{3}$$

$$\frac{\partial \varpi_1}{\partial t} + w\frac{\partial \varpi_1}{\partial z} = \frac{\Lambda\Lambda}{\rho j}\frac{\partial^2 \varpi_1}{\partial z^2} \tag{4}$$

$$\frac{\partial \varpi_2}{\partial t} + w\frac{\partial \varpi_2}{\partial z} = \frac{\Lambda\Lambda}{\rho j}\frac{\partial^2 \varpi_2}{\partial z^2} \tag{5}$$

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial z^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial z} \tag{6}$$

$$\frac{\partial C}{\partial t} + w\frac{\partial C}{\partial z} = D_m\frac{\partial^2 C}{\partial z^2} + Rd(c - c_\infty) \tag{7}$$

Where u, υ and w are velocity components along x, y, and z-axis respectively. ϖ_1 and ϖ_2 are microrotation components along x and y-axis respectively. $\beta_{T,}$ βc are the coefficient of thermal expansion and concentration expansion, ρ is the density of the fluid, ν is the kinematic viscosity,ν_r is the kinematic microrotation viscosity, k is the permeability of porous medium,σ is the electrical conductivity of the fluid, Λ is the spin gradient velocity, j is the micro-inertia density, g is the acceleration due to gravity, T is the temperature of the fluid in the boundary layer while T_∞ is the temperature far from the surface, κ is the thermal conductivity of the mediun, C_p is the specific heat at constant pressure p, q_r is the radiation heat flux, C is the concentration of the solute while C_∞ is the concentration of the solute far from the surface, D_m is the molecular diffusivity and Rd is the rate of chemical reaction.

The boundary conditions are given by

$$u = \upsilon = 0,\ \varpi_1 = \varpi_2 = 0,\ T = T_\infty,\ C = C_\infty,\ for\ t \le 0 \tag{8}$$

$$\left.\begin{aligned} &u = U_r\left[1 + \frac{\varepsilon}{2}(e^{int} + e^{-int})\right], \upsilon = 0,\ \varpi_1 = -\frac{1}{2}\frac{\partial \upsilon}{\partial z},\ \varpi_2 = \frac{1}{2}\frac{\partial u}{\partial z}, \\ &-\kappa\frac{\partial T}{\partial z} = q_{w,} - D_m\frac{\partial C}{\partial z} = M_w\ \ at\ z = 0 \qquad t>0 \\ &\text{and} \\ &u = \upsilon = 0,\ \varpi_1 = \varpi_2 = 0,\ T = T_\infty,\ C = C_\infty\ as\ z \to \infty \end{aligned}\right\} \tag{9}$$

Where U_r is the uniform reference velocity and ε is the smallest constant quantity. The oscillatory plate velocity assumed in equation (9) is based on the suggestion proposed by Ganapathy[24].
From equation (1), we have

$$w = -w_0 \tag{10}$$

Where w_0 represents the normal velocity at the plate which is positive for suction and negative for blowing.Following Rosseland approximation (Brewstar[25]) the radiative heat flux q_r is modeled as

$$q_r = \frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial z} \tag{11}$$

Where σ* is the Stefan-Boltzman constant and k* is the mean absorption coefficient. Assuming that the difference in temperature within the flow are such that T^4 can be expressed as a linear combination of the temperature, we expand T^4 in Taylor's series about T_∞ as follows:

$$T^4 = T_\infty^4 + 4T_\infty^3(T - T_\infty) + 6T_\infty^2(T - T_\infty)^2 + - - - \tag{12}$$

and neglecting higher order terms beyond the first degree in $(T\text{-}T_\infty)$, we have

$$T^4 \approx -3T_\infty^4 + 4T_\infty^3 T \tag{13}$$

Differentiating equation (11) with respect to z and using equation(13) to obtain

$$\frac{\partial q_r}{\partial z} = \frac{-16T_\infty^3\sigma^*}{3k*}\frac{\partial^2 T}{\partial z^2} \tag{14}$$

Let us introduce the following dimensionless variables:

$$\left.\begin{aligned} & u' = \frac{u}{U_r}, \upsilon' = \frac{\nu}{U_r}, z' = \frac{zU_r}{\nu}, t' = \frac{tU_r^2}{\nu}, n' = \frac{n\nu}{U_r^2} \\ & \varpi_1' = \frac{\varpi_1\nu}{U_r^2}, \varpi_2' = \frac{\varpi_2\nu}{U_r^2}, \theta = \frac{\kappa(T - T_\infty)}{q_w}, \phi = \frac{D_m(C - C_\infty)}{M_w} \end{aligned}\right\} \tag{15}$$

Substituting equation (15) into equations (1) – (7) and dropping primes yield the following dimensionless equations:

$$\frac{\partial u}{\partial t} - S\frac{\partial u}{\partial z} - R\upsilon = (1+\lambda)\frac{\partial^2 u}{\partial z^2} + Gr\theta + Gm\phi - \frac{1}{\chi}u - \lambda\frac{\partial \varpi_2}{\partial z} \tag{16}$$

$$\frac{\partial \upsilon}{\partial t} - S\frac{\partial \upsilon}{\partial z} + Ru = (1+\lambda)\frac{\partial^2 \upsilon}{\partial z^2} - \frac{1}{\chi}\upsilon + \lambda\frac{\partial \varpi_1}{\partial z} \tag{17}$$

$$\frac{\partial \varpi_1}{\partial t} - S\frac{\partial \varpi_1}{\partial z} = K\frac{\partial^2 \varpi_1}{\partial z^2} \tag{18}$$

$$\frac{\partial \varpi_2}{\partial t} - S\frac{\partial \varpi_2}{\partial z} = K\frac{\partial^2 \varpi_2}{\partial z^2} \tag{19}$$

$$\frac{\partial \theta}{\partial t} - S\frac{\partial \theta}{\partial z} = \frac{1}{Pr}\left(1 + \frac{4Nr}{3}\right)\frac{\partial^2 \theta}{\partial z^2} \tag{20}$$

$$\frac{\partial \phi}{\partial t} - S\frac{\partial \phi}{\partial z} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial z^2} + \gamma\phi \tag{21}$$

Where

$R = \frac{2\Omega\nu}{U_r^2}$ is the rotational parameter

$Pr = \frac{\mu\rho C_p}{\kappa}$ is the Prandtl number

$Sc = \frac{\nu}{Dm}$ is the Schmidt number

$Gr = \frac{\nu g\beta_T q_w}{kU_r^3}$ is the Grashof number

$Gm = \frac{\nu g\beta_c M_w}{DmU_r^3}$ is the modified Grashof number

$Nr = \frac{4T_\infty^3\sigma*}{\kappa k*}$ is the heat radiation parameter

$\chi = \frac{kU_r^2}{\nu^2}$ is the permeability of the porous medium parameter

$S = \frac{w_0}{U_r}$ is the suction parameter

$K = \frac{\Lambda\Lambda}{\mu j}$ is the dimensionless material parameter

$\lambda = \frac{\nu_r}{\nu}$ is the viscosity ratio

γ ɣ $= \frac{\nu Rd}{U_r^2}$ is the chemical reaction parameter.

Also the boundary conditions become

$$u = \upsilon = 0, \varpi_1 = \varpi_2 = 0, \theta = 0, \ \phi = 0, t \le 0 \tag{22}$$

$$u = 1 + \frac{\varepsilon}{2}(e^{int} + e^{-int}), \upsilon = 0$$

$$\varpi_1 = -\frac{1}{2}\frac{\partial \upsilon}{\partial z}, \varpi_2 = \frac{1}{2}\frac{\partial u}{\partial z}$$

$$\theta' = -1, \phi' = -1 \ \ at \ z = 0 \tag{23}$$

and

$$u = \upsilon = 0, \varpi_1 = \varpi_2 = 0, \theta = \phi = 0 \ as \ z \to \infty$$

We simplify equation(16) - (21) further by putting the fluid velocity and angular velocity in the complex form as

$V = u + i\upsilon$, $w = \varpi_1 + i\varpi_2$ and have the following:

$$\frac{\partial V}{\partial t} - S\frac{\partial V}{\partial z} + iRV = (1+\lambda)\frac{\partial^2 V}{\partial z^2} - \frac{1}{\chi}V + Gr\theta + Gm\phi + i\lambda\frac{\partial w}{\partial z} \tag{24}$$

$$\frac{\partial w}{\partial t} - S\frac{\partial w}{\partial z} = K\frac{\partial^2 w}{\partial z^2} \tag{25}$$

$$\frac{\partial \theta}{\partial t} - S\frac{\partial \theta}{\partial z} = \frac{1}{Pr}(1 + \frac{4Nr}{3})\frac{\partial^2 \theta}{\partial z^2} \tag{26}$$

$$\frac{\partial \phi}{\partial t} - S\frac{\partial \phi}{\partial z} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial z^2} + ɣ\phi \tag{27}$$

The associated boundary conditions (22) and (23) becomes:

$$V = 0, w = 0, \theta = 0, \phi = 0, for \ t \le 0 \tag{28}$$

$$V = 1 + \frac{\varepsilon}{2}(e^{int} + e^{-int}), \upsilon = 0$$

$$w = \frac{i}{2}\frac{\partial V}{\partial z}, \theta' = -1, \phi' = -1 \ at \ z = 0 \qquad t > 0 \tag{29}$$

and

$$V = 0, w = 0, \theta = 0, \phi = 0 \ as \ z \to \infty$$

3.0 Method of Solution

To find the solution of the above system of partial differential equations (24) – (27) in the neighborhood of the plate under the above boundary conditions (28), (29), we assume a perturbation of the form:

$$V(z,t) = V_0(z) + \frac{\varepsilon}{2}[e^{int}V_1(z) + e^{-int}V_2(z)] \tag{30}$$

$$w(z,t) = w_0(z) + \frac{\varepsilon}{2}[e^{int}w_1(z) + e^{-int}w_2(z)] \tag{31}$$

$$\theta(z,t) = \theta_0(z) + \frac{\varepsilon}{2}[e^{int}\theta_1(z) + e^{-int}\theta_2(z)] \quad (32)$$
$$\phi(z,t) = \phi_0(z) + \frac{\varepsilon}{2}[e^{int}\phi_1(z) + e^{-int}\phi_2(z)] \quad (33)$$

From equations (30) – (33), we have

$$\frac{\partial V}{\partial z} = V_0' + \frac{\varepsilon}{2}e^{int}V_1' + \frac{\varepsilon}{2}e^{-int}V_2' \quad (34)$$

$$\frac{\partial w}{\partial z} = w_0' + \frac{\varepsilon}{2}e^{int}w_1' + \frac{\varepsilon}{2}e^{-int}w_2' \quad (35)$$

$$\frac{\partial \theta}{\partial z} = \theta_0' + \frac{\varepsilon}{2}e^{int}\theta_1' + \frac{\varepsilon}{2}e^{-int}\theta_2' \quad (36)$$

$$\frac{\partial \phi}{\partial z} = \phi_0' + \frac{\varepsilon}{2}e^{int}\phi_1' + \frac{\varepsilon}{2}e^{-int}\phi_2' \quad (37)$$

$$\frac{\partial^2 V}{\partial z^2} = V_0'' + \frac{\varepsilon}{2}e^{int}V_1'' + \frac{\varepsilon}{2}e^{-int}V_2'' \quad (38)$$

$$\frac{\partial^2 w}{\partial z^2} = w_0'' + \frac{\varepsilon}{2}e^{int}w_1'' + \frac{\varepsilon}{2}e^{-int}w_2'' \quad (39)$$

$$\frac{\partial^2 \theta}{\partial z^2} = \theta_0'' + \frac{\varepsilon}{2}e^{int}\theta_1'' + \frac{\varepsilon}{2}e^{-int}\theta_2'' \quad (40)$$

$$\frac{\partial^2 \phi}{\partial z^2} = \phi_0'' + \frac{\varepsilon}{2}e^{int}\phi_1'' + \frac{\varepsilon}{2}e^{-int}\phi_2'' \quad (41)$$

$$\frac{\partial V}{\partial t} = \frac{in\varepsilon}{2}e^{int}V_1 - \frac{in\varepsilon}{2}e^{-int}V_2 \quad (42)$$

$$\frac{\partial w}{\partial t} = \frac{in\varepsilon}{2}e^{int}w_1 - \frac{in\varepsilon}{2}e^{-int}w_2 \quad (43)$$

$$\frac{\partial \theta}{\partial t} = \frac{in\varepsilon}{2}e^{int}\theta_1 - \frac{in\varepsilon}{2}e^{-int}\theta_2 \quad (44)$$

$$\frac{\partial \phi}{\partial t} = \frac{in\varepsilon}{2}e^{int}\phi_1 - \frac{in\varepsilon}{2}e^{-int}\phi_2 \quad (45)$$

Substitute equations (30) – (45) into equations (24) - (29), equating the harmonic and non-harmonic terms and neglecting the higher order terms of $O(\varepsilon^2)$ to obtain the following set of equations:

$$(1+\lambda)V_0'' + SV_0' - b_1V_0 + Gr\theta_0 + Gm\phi_0 + i\lambda w_0' = 0 \quad (46)$$

$$Kw_0'' + Sw_0' = 0 \quad (47)$$

$$(3+4Nr)\theta_0'' + 3SPr\theta_0' = 0 \quad (48)$$

$$\phi_0'' + SSc\phi_0' + Sc\gamma\phi_0 \quad (49)$$

$$(1+\lambda)V_1'' + SV_1' - b_2V_1 + Gr\theta_1 + Gm\phi_1 + i\lambda w_1' \quad (50)$$

$$Kw_1'' + Sw_1' - inw_1 = 0 \quad (51)$$

$$(3+4Nr)\theta_1'' + 3SPr\theta_1' - 3inPr\theta_1 = 0 \quad (52)$$

$$\phi_1'' + SSc\phi_1' - Sc(in-\gamma)\phi_0 \quad (53)$$

$$(1+\lambda)V_2'' + SV_2' - b_3V_2 + Gr\theta_2 + Gm\phi_2 + i\lambda w_2' = 0 \quad (54)$$

$$Kw_2'' + Sw_2' + inw_2 = 0 \quad (55)$$

$$(3+4Nr)\theta_2'' + 3SPr\theta_1' + 3inPr\theta_2 \quad (56)$$

$$\phi_2'' + SSc\phi_2' + Sc(in + ɣ)\phi_2 \tag{57}$$

Where the prime denote differentiation with respect to z and $b_1 = iR + \frac{1}{\chi}$, $b_2 = i(R+n) + \frac{1}{\chi}$ and $b_3 = i(R-n) + \frac{1}{\chi}$

The corresponding boundary conditions can be written as

$$\left.\begin{array}{l} V_0 = V_1 = V_2 = 1, w_0 = \frac{i}{2}V_0', w_1 = \frac{i}{2}V_1', w_2 = \frac{i}{2}V_2' \\ \theta_0' = -1, \phi_0' = -1 \; at \; z = 0 \end{array}\right\} \tag{58}$$

and

$$V_0 = V_1 = V_2 = 0, w_0 = w_1 = w_2 = 0, \theta_0 = \theta_1 = \theta_2 = 0$$

$$\phi_0 = \phi_1 = \phi_2 = 0 \; as \; z \to \infty \tag{59}$$

Solving equations (46) – (57) with equation (58) and (59) yield the expression for velocity, microrotation, temperature and concentration as follows:

$$V = B_1 e^{-a_3 z} + B_2 e^{-a_1 z} + B_3 e^{-a_2 z} + B_4 e^{-\frac{S}{K}z} + \frac{\varepsilon}{2}\{(B_5 e^{-a_5 z} + B_6 e^{-a_4 z})e^{int} + (B_7 e^{-a_7 z} + B_8 e^{-a_6 z})e^{-int}\} \tag{60}$$

$$w = C_1 e^{-\frac{S}{K}z} + \frac{\varepsilon}{2}\{C_2 e^{-a_4 z} e^{int} + C_3 e^{-a_6 z} e^{-int}\} \tag{61}$$

$$\theta = \frac{1}{a_1} e^{-a_1 z} \tag{62}$$

$$\phi = \frac{1}{a_2} e^{-a_2 z} \tag{63}$$

Where

$$a_1 = \frac{3SPr}{3+Nr}$$

$$a_2 = \frac{SSc + \sqrt{S^2 Sc^2 - 4Scɣ}}{2}$$

$$a_3 = \frac{S + \sqrt{S^2 + 4(1+\lambda)b_1}}{2(1+\lambda)}$$

$$a_4 = \frac{S + \sqrt{S^2 + 4Kin}}{2K}$$

$$a_5 = \frac{S + \sqrt{S^2 + 4(1+\lambda)b_2}}{2(1+\lambda)}$$

$$a_6 = \frac{S + \sqrt{S^2 - 4Kin}}{2K}$$

$$a_7 = \frac{S + \sqrt{S^2 + 4(1+\lambda)b_3}}{2(1+\lambda)}$$

$$B_1 = 1 - (B_2 + B_3 + B_4)$$

$$B_2 = \frac{-Gr}{a_1((1+\lambda)a_1^2 - Sa_1 - b_1}$$

$$B_3 = \frac{-Gm}{a_2(1+\lambda)a_2^2 - Sa_2 - b_1}$$

$$B_4 = \frac{i\lambda SC_1 K}{(1+\lambda)S^2 - KS^2 - K^2 b_1}$$

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$$B_5 = 1 - B_6$$
$$B_6 = \frac{\lambda i a_4 C_2}{(1+\lambda)a_4^2 - Sa_4 - b_2}$$

$$B_7 = 1 - B_8$$

$$B_8 = \frac{i\lambda a_6 C_3}{(1+\lambda)a_6^2 - Sa_6 - b_3}$$

$$C_1 = \frac{-i(a_3B_1 + a_1B_2 + a_2B_3)((1+\lambda)S^2 - KS^2 - K^2b_1)}{2((1+\lambda)S^2 - KS^2 - K^2b_1) - \lambda S^2}$$

$$C_2 = -\frac{ia_5\lambda B_5((1+\lambda)a_4^2 - Sa_4 - b_2)}{2\left((1+\lambda)a_4^2 - Sa_4 - b_2\right) - \lambda a_4^2}$$

$$C_3 = -\frac{ia_7B_7((1+\lambda)a_6^2 - Sa_6 - b_3)}{2\left((1+\lambda)a_6^2 - Sa_6 - b_3\right) - \lambda a_6^2}$$

For practical engineering applications, quantities of interest include skin-friction coefficient, couple stress coefficient, Nusselt number and Sherwood number.

The local skin-friction coefficient is given by

$$C_f = \frac{\tau_w|z=0}{\rho U_r^2} = \{1 + \lambda\left(1 + \frac{i}{2}\right)\}\frac{\partial V}{\partial z}$$

$$= -\left\{1 + \lambda\left(1 + \frac{i}{2}\right)\right\}[a_3B_1 + a_1B_2 + a_2B_3 + \frac{S}{K}B_4 + \frac{\varepsilon}{2}\{(a_5B_5 + a_4B_6)e^{int} + (a_7B_7 + a_6B_8)e^{-int}\}] \quad (64)$$

The couple stress coefficient at the wall C_w is given by

$$C_w = \frac{\partial w}{\partial z}|_{z=0} = -\{\frac{S}{K}C_1 + \frac{\varepsilon}{2}(a_1C_2e^{int} + a_6C_3e^{-int})\} \quad (65)$$

The local Nusselt number Nu is given by

$$Nu = -\frac{x(\frac{\partial T}{\partial x})_{z=0}}{K(T_w - T_\infty)} = {Re_x}/{\theta(0)} \quad (66)$$

Where $Re_x = \frac{U_r x}{\nu}$ is the local Reynolds number

$$\text{Thus } \frac{Nu}{Re_x} = {1}/{\theta(0)} \quad (67)$$

Also the local Sherwood number Sh is given by

$$Sh = \frac{x(\frac{\partial C}{\partial x})_{z=0}}{Dm(C_w - C_\infty)} = \frac{Re_x}{\phi(0)}$$

$$\text{Thus} \quad \frac{Sh}{Re_x} = \frac{1}{\phi(0)} \quad (68)$$

4.0 Results and Discussion

We have formulated the effect of radiation and chemical reaction on free convection heat and mass transfer flow of an incompressible micropolar fluid along a semi-infinite vertical permeable moving plate embedded in a porous medium in a rotating frame of reference. The numerical calculation for distribution of the translational velocity, microrotation, temperature and concentration across the boundary layer for different values of of the parameters are carried out. For the purpose our computation, we have chosen $nt = \pi/2$, Pr=0.71, Sc=0.16, Gm=5, Gr=10, ε=0.01 while γ, S, R, λ and Nr and are varied over range as shown in the figures.

Fig.2 shows the behavior of velocity profile with different values of chemical reaction parameter γ. It was observed that Velocity profile decreases with increase in chemical reaction parameter. Fig.3 shows the translational velocity across the boundary layer for different values of radiation parameter Nr. It is depicted that velocity increase with the increase in the radiation parameter and as a result, the momentum boundary layer thickness increases. Fig.4 shows the influence of the suction parameter on the translational velocity across the boundary layer. The results indicate that with increase in the parameter S, the velocity profile decreases within the boundary layer region. Thus the effect of increasing the values of the suction parameter is to decrease the momentum boundary layer thickness. Fig.5 Shows the effect of λ on the translational velocity. It is observed that the velocity profile decrease with an increase in viscosity ratio parameter. Fig.6 displays the translational velocity profile against z for different values of R. It is observed that an increase in rotational parameter R leads to decreasing velocity profile and so decrease momentum boundary layer thickness.

Fig.7 shows the effect of chemical reaction parameter on microrotation profiles. The result indicate that an increase in chemical reaction parameter leads to an increase in microrotation profile. Fig.8 displays microrotation distribution across the boundary layer for different values of radiation parameter. It is observed that microrotation profiles increase with the increase in the radiation parameter and as a result, the momentum boundary layer thickness increases. The influence of the suction parameter S on microrotation distribution across the boundary layer is shown in Fig.9. The result indicate that with increase in the parameter S, microrotation profile decreases. Thus the effect of increasing the values of the suction parameter S is to decrease the momentum boundary layer thickness. Microrotation profile against z for different values of rotational parameter R is displayed in fig.10. It is observed that an increase in R leads to increase in microrotation profile and so increase the momentum boundary layer thickness.

Fig.11 shows the variation of concentration distribution across the boundary layer for different values of chemical reaction parameter. The figure indicates that the concentration of the fluid increases with increase in the chemical reaction parameter. The effect of suction parameter on concentration profiles is presented in Fig.12 and it can be seen from the figure that the concentration of the fluid decreases as the suction parameter increases. Fig13. Show the behavior of concentration profiles with different values of radiation parameter Nr. The result indicate that an increase in the thermal radiation parameter Nr tends to increase the concentration distribution of the fluid.

Fig.14 illustrates the variation of the suction parameter S on the temperature profiles.It is observed that the temperature profile decreases as the suction parameter increases which indicate that the thickness of the boundary layer is reduced for higher value of suction parameter. Fig15 shows temperature profiles with different values of radiation parameters. The temperature profiles increases as the radiation parameter increases and so increase the thermal boundary layer thicknes.

Table 1 depicts the effect of λ, γ, S, R and Nr on the skin friction coefficient C_f. It is observed that the local skin friction coefficient C_f increases as the viscosity ratio parameter λ and radiation parameter Nr increases while it decreases as chemical reaction parameter γ, suction parameter S and rotational parameter increases.Table2 shows the effect of γ , S , R , and Nr on couple stress coefficient. It is also observed that the couple stress coefficient increases as R and Nr increases while it decreases as γ and S increases.Table3 indicate the effects of S and Nr on Nusselt number. It is found that the Nusselt number increases as S increases while it decreases as Nr increases. Table4 analyzed the effects of γ and S on Sherwood number. It is also found that Sherwood number increases as suction parameter increases while it decreases as chemical reaction parameter increases. These results are in excellent agreement with Modather [22] and Bakr [23].

5.0 Conclusion

The problem of unsteady free convection heat and mass transfer flow of an incompressible chemically reacting micropolar fluid past a semi-infinite vertical permeable moving plate embedded in a uniform porous medium with suction in the presence of thermal radiation was studied. The resulting partial differential equations which describe the problem, are transformed to dimensionless equations using dimensioless variables. The equations are then solved analytically by using perturbation technique. The results are discussed through graphs and tables for different values of parameters entering into the problem. Following conclusions can be drawn from the results obtained:

The translational velocity decreases with increase in the value of chemical reaction parameter, suction parameter ,rotational parameter and viscosity ratio while it increases with an increasing radiation parameter.

The magnitude of microrotation decreases with an increasing of suction parameter. Hence the momentum boundary layer thickness is reduced but the reverse effect is seen with an increase in the value of γ, S and R.

The temperature profile increases with an increasing value of radiation parameter whereas the effect is opposite for suction parameter. Thus the boundary layer thickness increases for higher values of the radiation parameter.

The concentration of the fluid increases with an increasing chemical reaction parameter and radiation parameter but the effect is reverse for suction parameter.

The results indicate that increasing the chemical reaction parameter produces a decreasing effect on the skin friction coefficient and the couple stress coefficient.

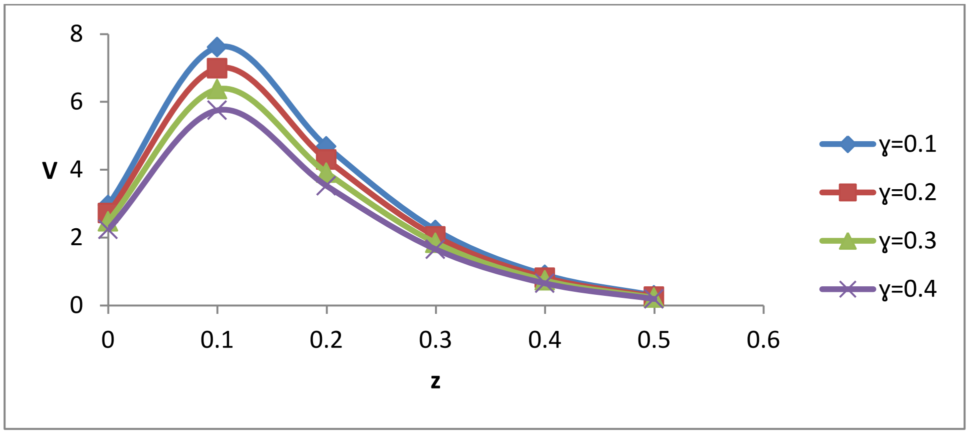

Fig.2. Velocity profiles for various values of γ with S=1.0, R=0.2, Nr=0.5, λ=0.1

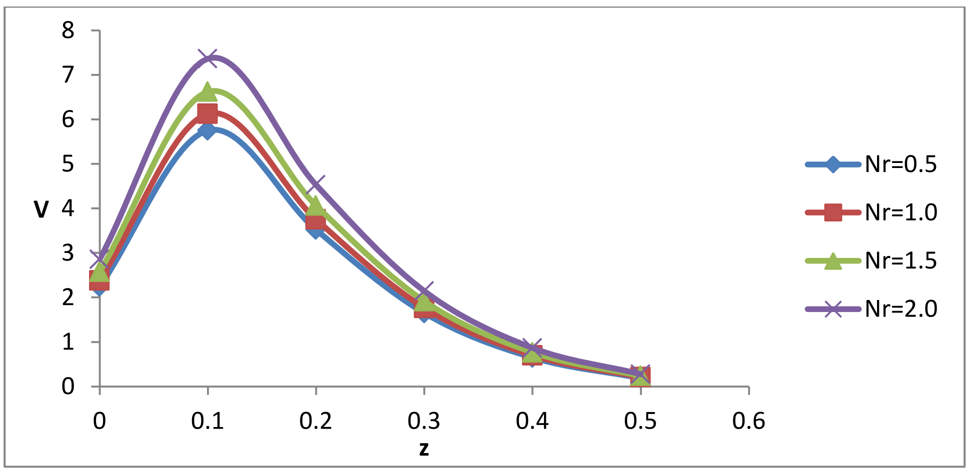

Fig.3.Velocity profiles for various values of Nr S=1.0, R=0.2, γ =0.1, λ=0.1

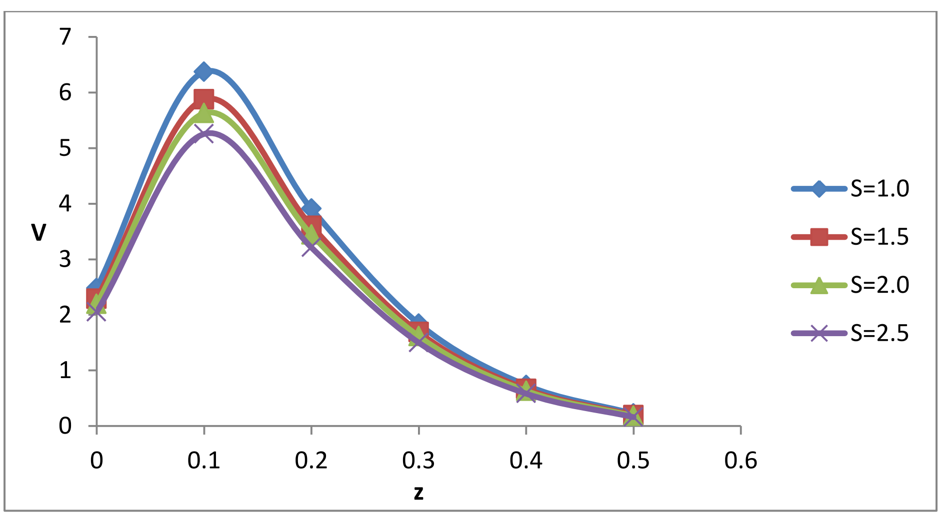

Fig.4.Velocity profiles for various values of S with R=0.2, λ=0.1, γ =0.1, Nr=0.5

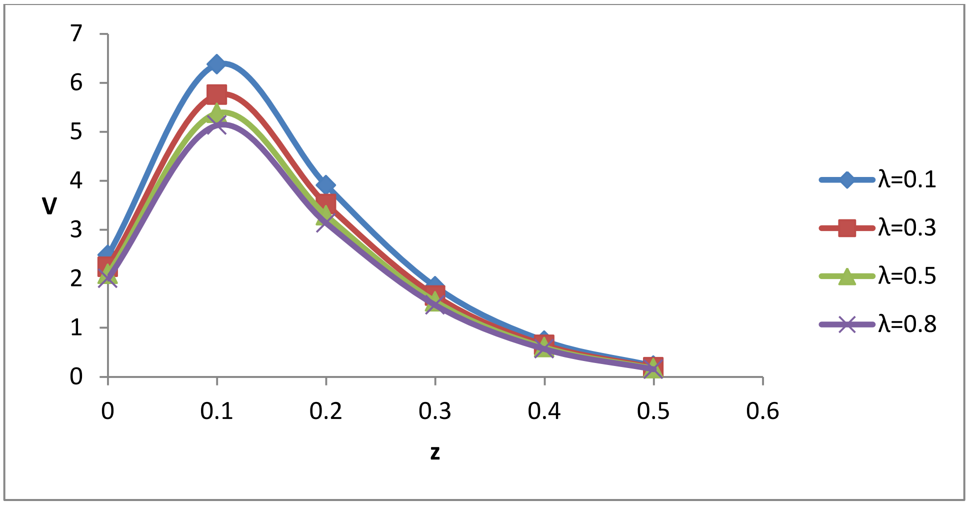

Fig.5. Velocity profiles for various values of λ with S=1.0, R=0.2, Nr=0.5, γ =0.1

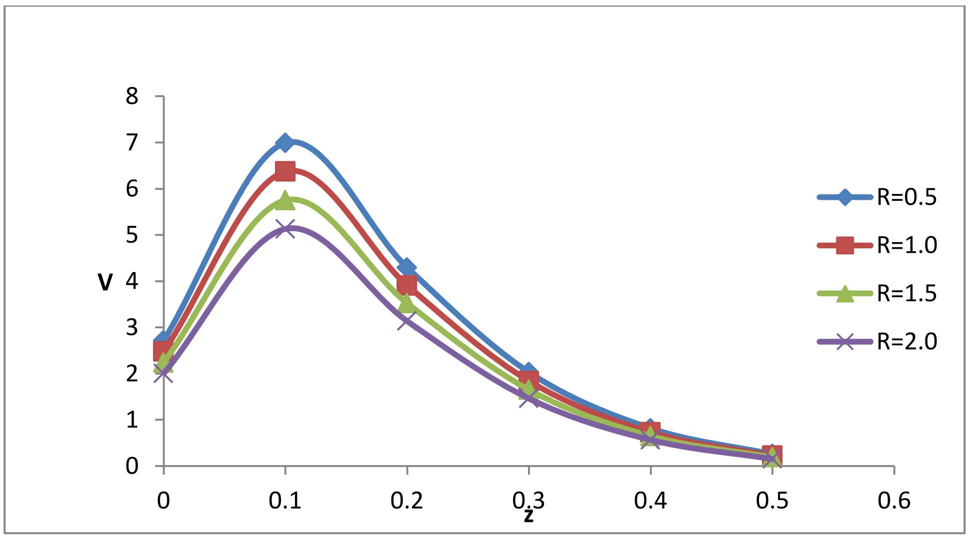

Fig.6. Velocity profiles for various values of R with S=1.0, Nr=0.5, γ =0.1, λ=0.1

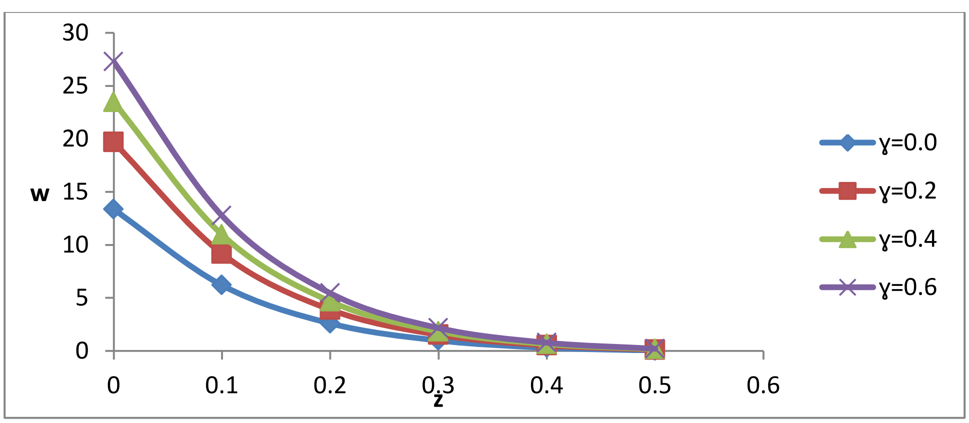

Fig.7. Microrotation profiles for various values of γ with S=1.0, Nr=0.5, R=0.2, λ=0.1

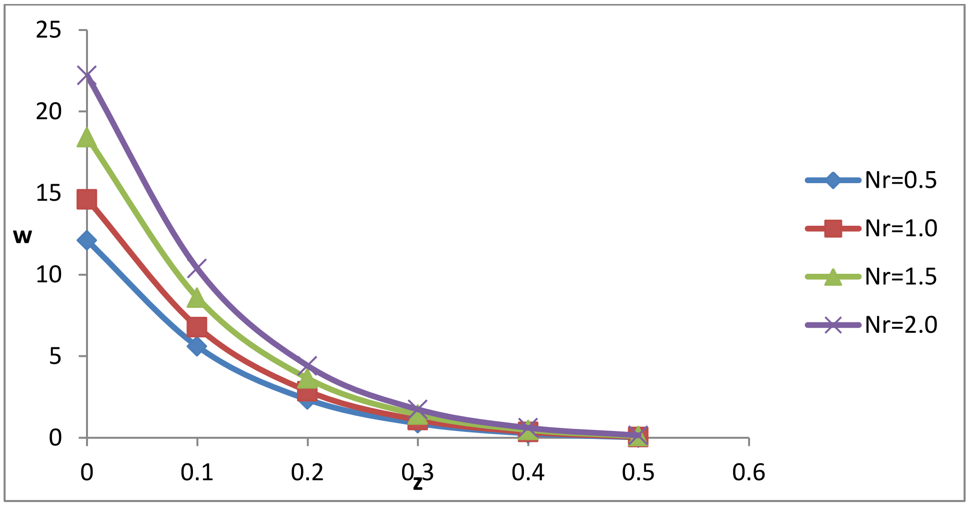

Fig.8.Microrotation profiles for various values of Nr with S=1.0, R=0.2, λ=0.1 γ =0.1

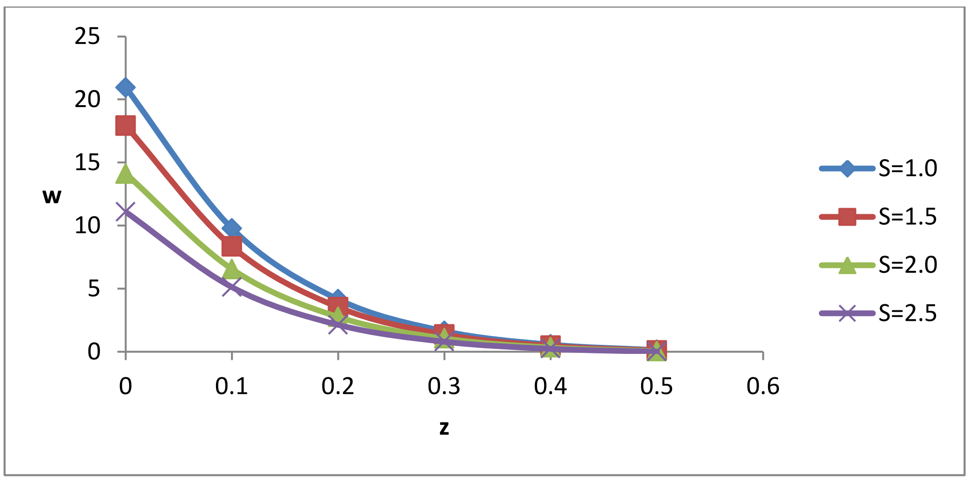

Fig.9. Microrotation profiles for various values of S with R=0.2, Nr=0.5, γ =0.1, λ=0.1

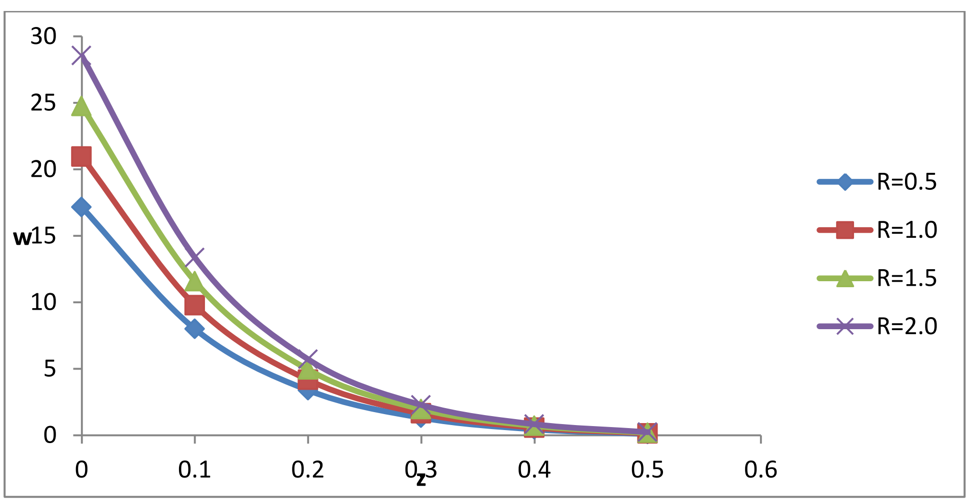

Fig.10. Microrotation profiles for various values of R with S=1.0. Nr=0.5, λ=0.1, γ =0.

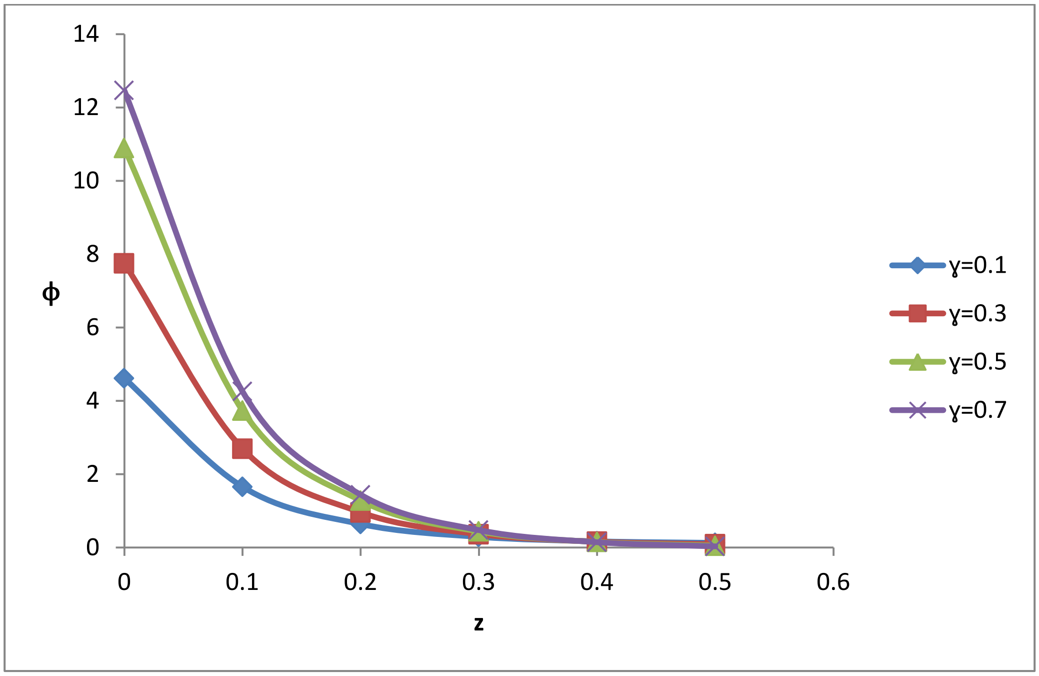

Fig.11. Concentration profiles for various values of γ with S=1.0, Nr=0.5, R=0.2, λ=0.1

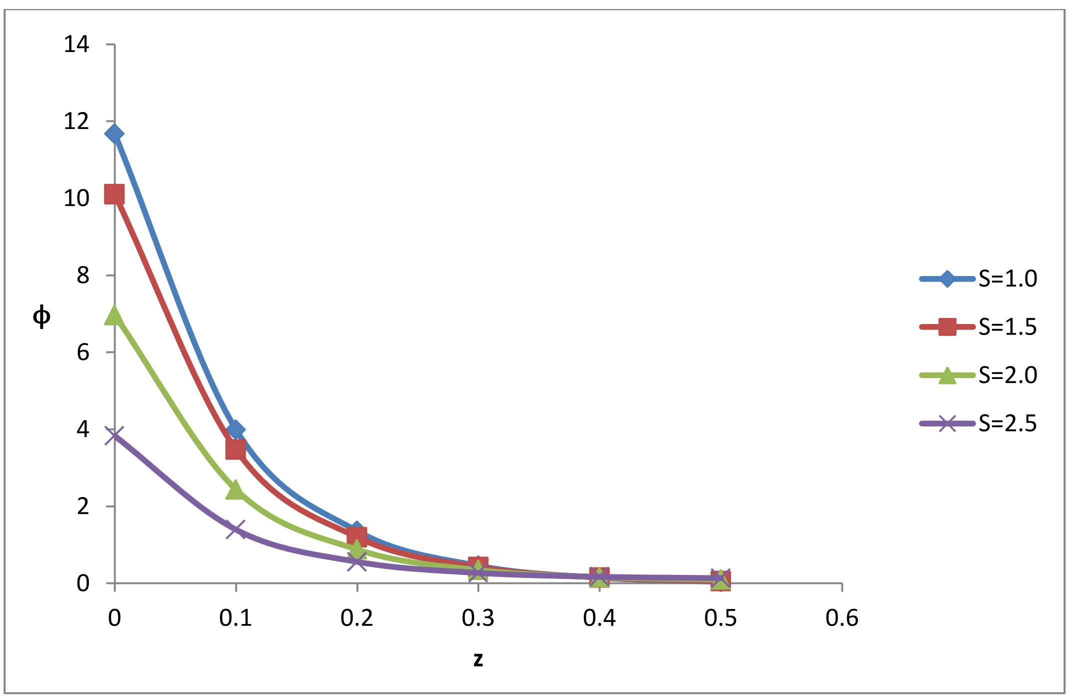

Fig.12. Concentration profiles for various values of S with Nr=0.5, R=0.2, λ=0.1, γ =0.1

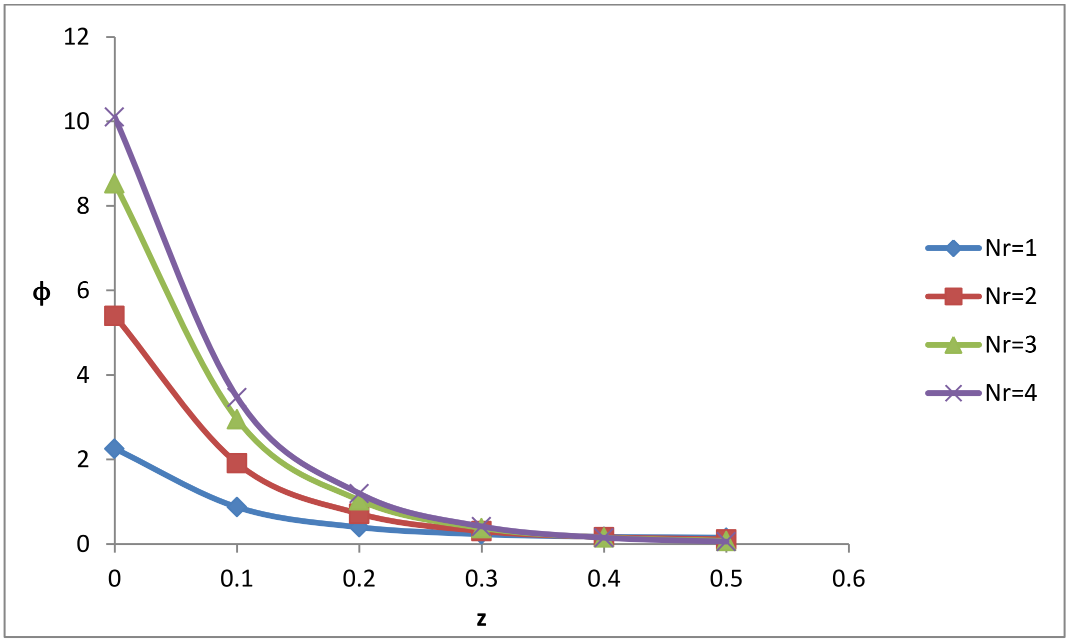

Fig.13. Concentration profiles for various values of Nr with S=1.0, R=0.2, γ =0.1, λ=0.1

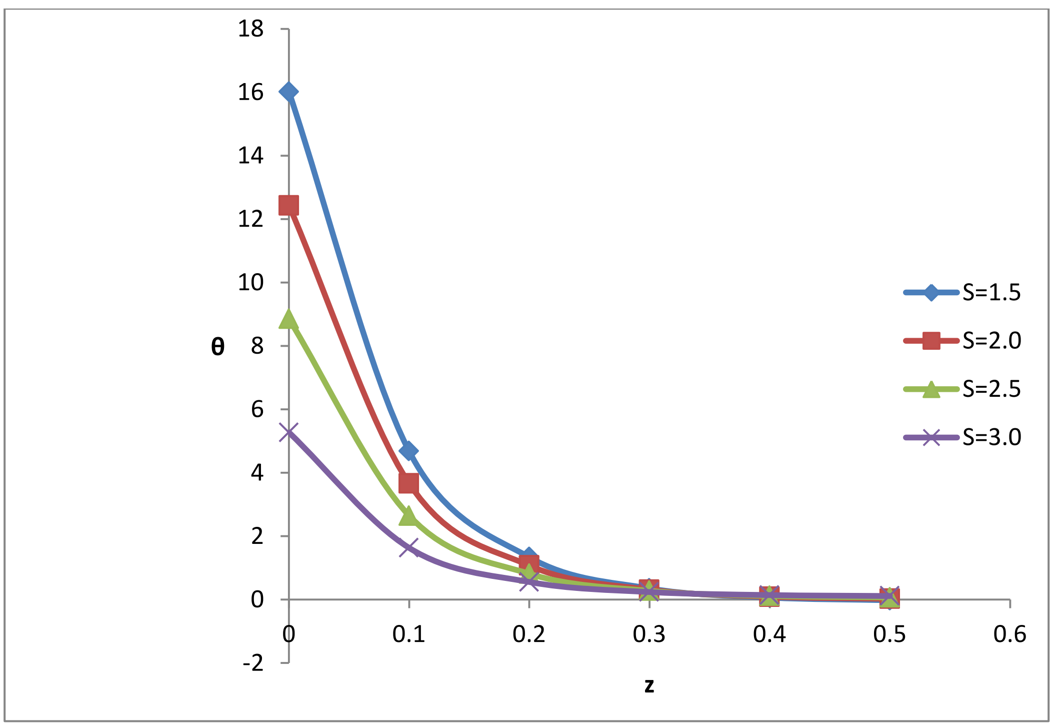

Fig.14. Temperature profiles for various values of S with Nr=0.5, R=0.2, λ=0.1, γ =0.1

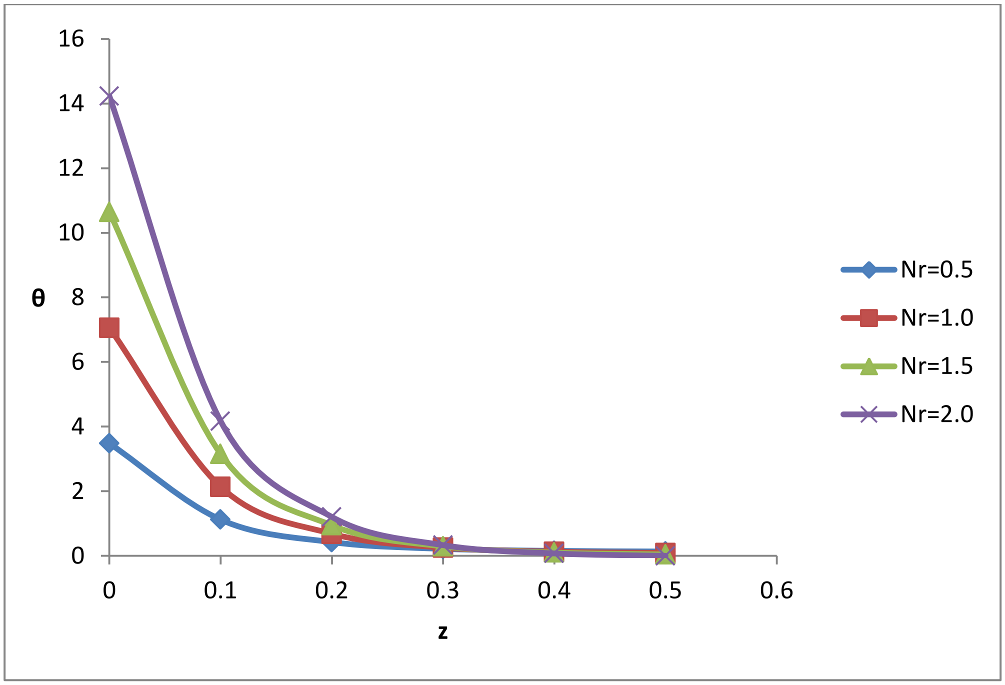

Fig.15. Temperature profiles for different values of Nr with S = 1.0, R = 0.5, γ = 0.1, λ = 0.1

TABLE 1
Effect of various parameter on Cf with χ=0.5, Sc=0.16, Pr=0.71, K=0.1.

λ	γ	S	R	Nr	C_f
0.2	0.1	2.5	0.2	0.5	33.0956
0.4	0.1	2.5	0.2	0.5	35.1056
0.8	0.1	2.5	0.2	0.5	37.4702
0.2	0.2	2.5	0.2	0.5	30.1564
0.2	0.3	2.5	0.2	0.5	27.3448
0.2	0.4	2.5	0.2	0.5	22.8698
0.2	0.1	4.0	0.2	0.5	13.7352
0.2	0.1	5.0	0.2	0.5	6.5498
0.2	0.1	6.0	0.2	0.5	2.7387
0.2	0.1	2.5	0.5	0.5	30.6296
0.2	0.1	2.5	0.8	0.5	26.5168
0.2	0.1	2.5	1.0	0.5	23.3301
0.2	0.1	2.5	0.2	1.0	40.0369
0.2	0.1	2.5	0.2	1.5	47.8606
0.2	0.1	2.5	0.2	2.0	56.3802

TABLE2

Effect of various parameter on –Cw with χ=0.5, Sc=0.16, Pr=0.71, K=0.1, λ=0.2

γ	S	R	Nr	-Cw
0.2	2.5	0.2	0.5	11.6020
0.3	2.5	0.2	0.5	9.7668
0.4	2.5	0.2	0.5	7.9241
0.1	4.0	0.2	0.5	2.5323
0.1	5.0	0.2	0.5	1.3205
0.1	6.0	0.2	0.5	0.7212
0.1	2.5	0.5	0.5	10.9402
0.1	2.5	0.8	0.5	19.1674
0.1	2.5	1.0	0.5	22.9899
0.1	2.5	0.2	1.0	11.0445
0.1	2.5	0.2	1.5	18.1138
0.1	2.5	0.2	2.0	27.0417

TABLE3

Effect of S and Nr on Nu/Rey with χ=0.5,=0, Sc=0.16, Pr=0.71, λ=0.2, γ =0.1, R=0.2, K=0.1

S	Nr	Nu/Rey
4.0	0.5	1.5014
5.0	0.5	2.1073
6.0	0.5	2.3250
2.5	1.0	0.5364
2.5	1.5	0.3675
2.5	2.0	0.2505

TABLE 4

Effect of γ and S on Sh/Rey with Sc=0.16, χ=0.5, Pr=0.71, λ=0.2, Nr=0.5, K=0.1 and R=0.2

γ	S	Sh/Rey
0.2	2.5	0.1636
0.3	2.5	0.0132
0.4	2.5	0.0045
0.1	4.0	0.2126
0.1	5.0	0.3114
0.1	6.0	0.4165

Reference.

[1]. A.C Eringen, Theory of micropolar fluids *J. math.mech. 16, pp.1-18.1966*

[2]. Cogley A.C., Vincent W.E., Gilles S.E.Differential approximation for radiation in a non-gray gas near equilibrium. *AIAAJ. 6:551-553,1968.*

[3]. Hossain MA, Takhar Hs. Radiation effect on mixed convection along a vertical plate with uniform surface temperature, *Heat mass transfer 31:243-248, 1996*

[4]. Makinde OD. Free convection flow with thermal radiation and mass transfer past a moving vertical porous plate. Int. comm.Heat mass transfer, 25:289-295, 2005

[5]. Satter MD A, Hamid MD K. Unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate. *Jour. Math. Phys. Sci. 30:25-37, 1996*

[6]. Rahman, M.A. and Sattar, M.A. Transient convective flow of micropolar fluid past a continuously moving vertical porous plate in the presence of radiation, *International Journal of Applied Mechanics and Engineering,* vol.12, no. 2 pp.497-513, 2007.

[7]. Das K, Jana S.Heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in a porous medium. *Bull. Soc. Banja luka 17:15-32,2010*

[8]. Olajuwon. B.I. Convection heat and mass transfer in a hydromagnetic flow of a second grade fluid in the presence of thermal radiation and thermal diffusion. *Int. commu. Heat and mass 38:377-382,2011.*

[9]. Haque Md Z, Alam Md M, Ferdows M, Postelnicu A. Micropolar fluid behaviors on steady MHD free convection flow and mass transfer with constant heat and mass fluxes, joule heating and viscous dissipation. J.King Saud Univ. Engg.Sci doi:10.1016/j.jksues.2011.02.003, 2011

[10]. Srinivasachanya.D., Ramreddy. Ch.,Soret and Dufour effect on mixed convection in a non-Darcy porous medium saturated with micropolar fluid, *Non-analysis modelling and control, vol.16,No.1, 100-115,2011.*

[11]. Rehbi, A.D., Tariq, A.A. Benbella, A.S. Mahoud, A.A. Unsteady natural convection heat transfer of micropolar fluid over a vertical surface with constant Heat flux, *Turkish J. Eng. Env. Sci.* Vol. 31, pp. 225-233 ,2007.

[12]. Eldabe, N.T. and Ouat, M.E., Chebyshew finite difference method for heat and mass transfer in hydromagnetic flow of a micropolar fluid past a stretching surface with Ohmic heating and viscous dissipation, *Appl. Math. Comput.*, 177, pp. 561-571, 2006.

[13]. Keelson, N.A., Desseaux, A. Effects of surface condition on flow of a micropolar fluid driven by a porous stretching sheet, *Int. J. Eng. Sci*, 39, pp. 1881-1897, 2001.

[14]. Sunil, A. Sharma, A. Bharti, P.K. and Shandi, R.G. Effect of rotation on a layer of micropolar ferromagnetic fluid heated from below saturating a porous medium, *International Journal of Engineering science,* vol. 44 no. 11-12, pp. 683-698, 2006.

[15]. Mahmoud, M.A.A., Thermal radiation effects on MHD flow of a micropolar fluid over a stretching surface with variable thermal conductivity, *Physical A.* 375, pp. 401-410, 2007.

[16]. Magdy, A.C., Free convection flow of conducting micropolar fluid with thermal relaxation including heat sources, *Journal of Applied Mathematics,* Vol. 2, 70.4 pp.271-292,2005

[17]. Bayomi, M. Chamkha, A.S. Khedr, E.M. MHD flow of a micropolar fluid past a stretched permeable surface with heat generation or absorption, *Non-analysis, modeling and control,* vol. 14, no. 1, pp. 27-40, 2009.

[18]. Reena, Rana U.S., Linear stability of thermo solutal convection in a micropolar fluid saturating a porous medium, *International Journal of Application and Applied mathematics,* vol. 4, no. 1 pp.62-87

[19]. Chambre P.L and Yonng J.D The diffusion of chemically reactive species in a laminar boundary later fow. *Phys. Fluids, 1, 48 – 54, 1958*.

[20]. Kandasamy R., Periasamy K. and Sivaagnana K. Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and stratification effects. *Int. j. of heat and mass transfer*, *48*, *4557*,*2005*.

[21]. Chaudhary, R.C. and Abhay K.J . Effect of chemical reaction on MHD micropolar fluid flow past a vertical plate in slip-flow regime. *Appl. Math. Mech. Engl. Ed. 29(9): 117-1194, 2008*

[22]. Modather M, Rashad A.M, Chamkha A.J. Study of MHD heat and mass transfer oscillatory flow of a micropolar fluid over a vertical permeable plate in a porous medium. *Turkis J. eng. Env. Sci. vol33, 245-257, 2009*

[23]. Bakr AA, EFFects of chemical reaction on MHD free convection and mass transfer flow of a micropolar fluid with oscillatory plate velocity and constant heat source in a rotating frame of reference. *Commun nonlinear Sci Numer. Simul.16:698-710,2011*

[24]. Ganapathy R. A note on oscillatory coquette flow in a rotating system. *ASME. J. Appl. Mech. 61: 208 – 209, 1994.*

[25]. Brewster, M. Q., Thermal radiation transfer properties. *John Wiley and sons,12:6-9, 1972*