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An Approximate Analytical Investigation of Couette Flow In Composite Channel

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Abstract

An approximate analytical solution is presented for fully developed unsteady Couette flow between parallel plates partially filled with fluid saturated porous material and partially filled with clear fluid. The Brinkman-Forchheimer extension of Darcy equation is utilized to model the flow in a porous region. The two regions are coupled by equating the velocity and shear stress at the interface. The result obtained is compared to those obtained from an implicit finite-difference solution of the corresponding time dependent flow problem. It can be seen that the time dependent flow solution yields almost same steady state values as obtained by using the approximate analytical solution.

Keywords: Couette flow, composite channel, interface

Nomenclature

$H =$ total width of the channel, $y' =$ dimensional co-ordinate

$y =$ dimensionless co-ordinate, $\left(\dfrac{y'}{H}\right)$ $n =$ index

$C^* =$ inertia coefficient $C =$ dimensionless inertia coefficient, $\left(C^* \nu^{n-2} H^{\left[3-\frac{3n}{2}\right]}\right)$

$U_0 =$ motion of the channel wall at $y' = 0$ $B =$ dimensionless motion of the channel wall at $y' = 0$

$U_i^{'} =$ interface velocity at $y' = d'$ $U_i =$ dimensionless interface velocity at $y' = d'$, $\left(\dfrac{U_i^{'} H}{\nu}\right)$

$K =$ permeability of the porous medium $Da =$ Darcy number, $\left(\dfrac{K}{H^2}\right)$

$u' =$ velocity of the fluid $u =$ dimensionless velocity of the fluid, $\left(\dfrac{u'H}{\nu}\right)$

$t' =$ dimensional time $t =$ dimensionless time, $\left(\dfrac{t'\nu}{H^2}\right)$

$d' =$ distance of the interface from the plate $y' = 0$ $d =$ dimensionless distance of the interface from the plate $y' = 0$, $\left(\dfrac{d'}{H}\right)$

Greek symbols

$\nu_{eff} =$ effective kinematics viscosity of porous medium, $\nu =$ kinematics viscosity of fluid, $\gamma =$ ratio of kinematics viscosity

Subscripts

$f =$ fluid layer, $p =$ porous layer

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1.0 Introduction

Flow formation in composite channel partially filled with porous medium and partially filled with a clear fluid has many industrial, geophysical, biomedical, engineering, and environmental applications [1].These application include thermal insulation, crude oil extraction, solidification of castings. If a solidifying alloy does not have a eutectic composition, the frozen part of the casting is separated from the liquid part by a mushy zone, which can be viewed as a porous medium with variable permeability. In addition, the use of porous substrates to improve forced convection heat transfer in channels, which is considered as a composite of fluid and porous layers, finds applications in heat exchangers, electronic cooling, heat pipes, filtration and chemical reactions, etc. In these applications engineers avoid filling the entire channel with a solid matrix to reduce the pressure drop. Comprehensive literature survey concerning this subject is given in the monograph [2, 3].

An important step towards understanding fluid mechanics and heat transfer in the interface region was made in [4, 5]. In these references the non-Darcian effects are accounted for by using the Brinkman-Forchheimer extended Darcy equation for the flow in porous medium. In [5] for the first time the exact solution for the fully developed steady flow in the interface region, where the fluid layer is sandwiched between a semi-infinite porous body and an external impermeable boundary was presented.

One of the fundamental fluid flow situations in porous media is Couette flow which can occur, for example between two parallel plates, one of which is at rest, and the other is moving in its own plane with constant velocity. Since in the Couette flow velocity of the moving plate can be large, and viscous forces in the boundary layer near the moving plate can be significant, to obtain the correct analysis of the flow formation it can be important to account for non-Darcian effects, namely, for the inertial (Forchheimer) and for the viscous (Brinkman) effects [1]. Kuznetsov [6] analytically investigated the Couette flow in a composite channel partially filled with a porous medium and partially with clear fluid.

The aim of this work is to present approximate analytical solution for steady fully developed Couette flow in a composite channel partially filled with porous materials. The non-linear Brinkman-Forchheimer extension of Darcy equation is used to simulate momentum transfer in porous region.

2.0 Mathematical Model

The present study considers the fully developed laminar Couette flow of a viscous incompressible fluid between two infinitely long horizontal parallel plates containing fluid and porous layers. The x' -axis is taken along one of the plate while y' -axis is normal to it. The fluid is assumed to be Newtonian with uniform properties and the porous medium is isotropic and homogeneous. Using the above assumptions, the mathematical model representing the present physical situation in dimensionless form for fluid layer is

$$\frac{d^2u_f}{dy^2} = 0, \tag{1}$$

while for the porous layer in the dimensionless form is

$$\gamma\frac{d^2u_p}{dy^2} - \frac{u_p}{Da} - \frac{C}{\sqrt[4]{Da^n}}u_p{}^n = 0. \tag{2}$$

The first term in the left-hand side of Equation(2) is the Brinkman term, second is the Darcy and third is the Forchheimer term $(n = 2)$, hence the momentum transfer in the porous region is governed by steady Brinkman-Forchheimer extended Darcy model.

The boundary and matching conditions in dimensionless form are:

$$u_f = B, \text{ at } \quad y = 0;\ u_p = 0, \text{ at } \quad y = 1;$$

$$\begin{cases} u_f = u_p, \\ \gamma\dfrac{du_p}{dy} = \dfrac{du_f}{dy}, \end{cases} \quad y = d \tag{3}$$

The above equations have been rendered in dimensionless form by using the non-dimensional parameters defined in nomenclature.

3.0 Analytical Solutions

Using the assumption, $\alpha = u_p{}^{n-1}$ introduced in [7] in the equation (2), it becomes $\gamma\dfrac{d^2u_p}{dy^2} - \dfrac{u_p}{Da} - \dfrac{C\alpha}{\sqrt[4]{Da^n}}u_p = 0$ (4)

The solutions of Equation (1) and (4) under the condition (3) are

$$u_f = \frac{yX\gamma B}{(1.-X\gamma d)} + B, \tag{5}$$

$$u_p = \frac{BSinh(\lambda(y-1))}{Sinh(\lambda(d-1))(1.-X\gamma d)}, \tag{6}$$

where

$$X = \frac{\lambda Cosh(\lambda(d-1.))}{Sinh(\lambda(d-1.))},\ \lambda = \frac{1}{\sqrt{\gamma}}\sqrt{\frac{1}{Da} + \frac{C\alpha}{\sqrt[4]{Da^n}}}.$$

4.0 Numerical Solution

The analytical solution of the previous section is valid for steady state momentum transfer in composite parallel plate channel partially filled with fluid saturated porous material and partially filled with clear fluid. To explore the limits of validity of the analytical solution and to extend the investigation to time dependent momentum equation, numerical solution of the time dependent problem is obtained using implicit finite deference approach.

The governing conservation equations, for fluid and porous regions are formulated separately. For the fluid region the time-dependent momentum equation in dimensionless form is

$$\frac{\partial u_f}{\partial t} = \frac{d^2 u_f}{dy^2}, \tag{8}$$

while for the porous region respective equation is

$$\frac{\partial u_p}{\partial t} = \gamma\frac{\partial^2 u_p}{\partial y^2} - \frac{u_p}{Da} - \frac{C}{\sqrt[4]{Da^n}} u_p{}^n = 0. \tag{9}$$

The first term in the left-hand side of Equation(9) is the Brinkman term, second is the Darcy and third is the Forchheimer term $(n = 2)$, hence the momentum transfer in the porous region is governed by time dependent Brinkman-Forchheimer extended Darcy model.

The above equations have been rendered in dimensionless form by using the non-dimensional parameters defined in nomenclature.

The initial and boundary conditions in dimensionless form for the present problem are:

$$u = 0, \quad , \quad \text{for all } y \text{ when } t \le 0,\ u_f = B \text{ at } y = 0 \ ,$$

$$u_p = 0 \text{ at } y = 1\,, \text{ for } t > 0\,. \tag{10}$$

In modeling a composite fluid and porous system, the use of the two-domain approach for fluid and porous layers requires matching conditions at the interface.

In dimensionless form, they are obtained as follows:

$$\begin{cases} u_f = u_p \\ \gamma\dfrac{\partial u_p}{\partial y} = \dfrac{\partial u_f}{\partial y} \ , \end{cases} y = d \tag{11}$$

In describing the matching conditions at the fluid/porous interface in equation (11), continuity of velocity and shear stress are utilized as taken in [8] and [9].

Reference [10] and [11] used finite difference method in solving time dependent problems. The same method and computation procedure are adopted here for numerical solutions of equations (8) and (9) under the initial and boundary and matching condition which are discretized as follows:

$$\frac{u_f(i,j) - u_f(i,j-1)}{\Delta t} = \frac{u_f(i+1,j) - 2u_f(i,j) + u_f(i,j-1)}{(\Delta y)^2} \tag{12}$$

$$\frac{u_p(i,j) - u_p(i,j-1)}{\Delta t} = \gamma\frac{u_p(i+1,j) - 2u_p(i,j) + u_p(i,j-1)}{(\Delta y)^2} - \frac{u_p(i,j)}{Da} - \frac{C[u_p(i,j)]^n}{\sqrt[4]{Da^n}} \tag{13}$$

Table I: Comparison of Results; $d = 0.5, \gamma = 1.0, C = 5.0$

Da=0.1	y	**Anatical Solution**	**Numerical Solution (Implicit Finite-Difference Solution)**
	0.0	1.00000	1.00000
	0.1	0.85772	0.86796
	0.2	0.71914	0.73593
	0.3	0.58469	0.60390
	0.4	0.45480	0.47188
	0.5	0.32993	0.33985
	0.6	0.22865	0.22961
	0.7	0.15578	0.15133
	0.8	0.09844	0.09214
	0.9	0.04860	0.04368
	1.0	0.00000	0.00000
Da=0.01			
	0.0	1.00000	1.00000
	0.1	0.82879	0.83300
	0.2	0.65912	0.66600
	0.3	0.49118	0.49901
	0.4	0.32517	0.33202
	0.5	0.16134	0.16503
	0.6	0.05967	0.05978
	0.7	0.02220	0.02184
	0.8	0.00823	0.00790
	0.9	0.00304	0.00256
	1.0	0.00112	0.00000

5.0 Results and Discussion

Unsteady Couette flow between parallel plates partially filled with saturated porous material and partially with clear fluid was presented utilizing the Brinkman-Forchheimer extension of Darcy equation in the porous material region.

For the flow in the porous region $(n = 2)$, $d = 0.5,\ \gamma = 1.0, C = 5.0$ and $Da = 0.1$ and 0.01, the results of equations (1) & (2) have been compared with the implicit finite-difference solutions of equations (12) & (13) in Table I.

An approximate analytical solution was obtained and compared with the numerical finite difference solutions. The solution obtained is exactly same as the numerical finite difference to 1 decimal place as noticed in the Table. It can also be noticed that the approximate analytical solution presented in this work, though is simple, gives good and accurate results, and hence can be efficiently used to solve this class of nonlinear differential equation models.

References

[1]. S. Kakac, B Kilkis, F. Kulacki,; F. Arinc,: Convective heat and mass transfer in porous media, Kluwer Academic Publishers, Netherlands ,1991, 563-615

[2]. D.A. Nield, Bejan, A. "Convection in porous media" *Springer-Verlag, New York,* 1992.

[3]. M. Kaviany, "Principles of heat transfer in porous media", *Springer-Verlag, New York,*(1991)

[4]. K. Vafai, R. Thiyagaraja, "Analysis of flow and heat transfer at the interface region of a porous medium", *Int. jr. heat and mass transfer,* Vol. 30, 1987, 1391-1405

[5]. K. Vafai, S.J. Kim, "Fluid mechanics of the interface region between a porous medium and fluid layer-An exact solution" *Int. jr. heat and mass transfer,* Vol. 11, 1990, 254-256

[6]. A.V. Kuznetsov "Analytical investigation of coquette flow in a composite channel partially filled with a porous medium and partially with clear fluid" *Int. jr. of heat and mass transfer,* Vol.41, 1998, 2556-2560

[7]. Basant K. Jha and M.L.Kaurangini "Approximate analytical solutions for the nonlinear Brinkman-Forchheimer-extended Darcy flow model" *Applied Mathematics,* , Vol. 2, No 2, , 2011, 1432-1436

[8]. G. Neale, W. Nader, "Practical significance of Brinkman's extension of Darcy's law: Coupled parallel flows within a channel and a bounding porous medium, *Canadian jr. chem. Engng,* Vol. 52, 1974, 475-478

[9]. K. Vafai, S.J. Kim, "Analysis of surface enhancement by a porous substrate" *ASME jr. heat transfer,* Vol. 112, 1990, 700-706

[10]. Muhammad L. Kaurangini and Basant K. Jha "Steady and Transient Generalized Couette Flow in the Interface Region between Uniform Porous Medium and a Clear Fluid in Parallel-plate" *Journal of Porous Media, USA* Vol. 13, No.10, 2010, pp. 931-943, DOI: 10.1615/JPorMedia.v13.i10.60

[11]. M.L.Kaurangini and Basant K. Jha "Transient pressure driven Flow in the Interface Region between Uniform Porous Medium and a Clear Fluid in Parallel-plate with Suction/Injection" *International Journal of Management science and Engineering Management (IJMSEM), England UK,* Vol. 5, No 1, 2010, pp 15-23