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Temperature distribution over a stretching surface with viscous dissipation

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Abstract

We investigate the effect of viscous dissipation on the temperature profile of the flow of an incompressible, second order fluid over a stretching sheet in the boundary layer theory and found that the effect of viscous dissipation depends largely on the Brinkmann's number of the material. When the Brinkmann's number is large, there is an initial rapid increase in the temperature near the wall before a gradual decrease away from it. Also increase in the elasticity of the material does not affect appreciably the temperature profile of the material when the viscous dissipation is prominent and the Prandtl number is held constant.

Keywords: second order fluid, stretching sheet, viscous dissipation, Prandtl number, Brinkmann's number, temperature distribution.
2010 AMS Classification: 80A20

1.0 Introduction

Non Newtonian fluids have become increasingly important in industries most especially in polymer industries where we encounter the flow of a viscous fluid over a stretching sheet. For example in the extrusion of a polymer sheet from a die the sheet is sometimes stretched and the properties of the final product depend to a great extent on the rate of cooling. If one is able to control the rate of cooling, a final product of desired characteristics can be achieved. Beard and Walters [1] used the boundary theory to study the flow of an idealised elastico-viscous fluid known as second-order fluid. They were able to formulate the equations governing the flow. Rajagopal et al [2] examined the non-linear equation of an incompressible fluid in the boundary layer over a stretching sheet for the case of a small elastic parameter. They showed that the skin friction decreases with increase in elastic parameter. Dandapat and Gupta [3] considering the same flow investigated the heat transfer. Their analysis of heat transfer revealed that when the wall and the ambient temperature are held constant, temperature at a point increases with increase in elastic parameter for fixed Prandtl number. Hassanien et. al. [4] considered flow and heat transfer in a power law fluid over a porous surface with a variable temperature and found out that the friction factor and heat transfer results exhibit similar strong dependence on fluid parameters. Basant and Haruna [5] investigated temperature field in a flow over a stretching sheet with internal generation and uniform heat flux. The velocity of the sheet was taken to be proportional to the distance from the slit. They found that increase in Prandtl number leads to decrease in heat conductivity.

When we are interested in controlling the rate of cooling, the effect of the viscous dissipation term may be significant within certain region in analysing the flow distribution [6] and heat transfer [7]. In this work we consider the effect of the dissipation factor on the temperature distribution. Recently some authors [8-10] investigated the effect of dissipation on various type of flows. Olanrewaju et al, [9] established that the rate of cooling can also be controlled by the use of convective boundary condition. In this work we also present numerical values to determine the effect of the variation in elastic parameter on the temperature distribution.

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2.0 Governing Equations

The usual two-dimension boundary layer equations for a flow past a semi-infinite flat plate where there is no pressure gradient are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} \tag{2}$$

where u, v are the velocity components along and perpendicular to the surface and υ the kinematic viscosity. We consider a mode of the flow of an incompressible viscous fluid past a flat sheet coinciding with the plane y = 0, the flow being confined to y > 0. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. This model displays normal stress difference in shear flow and is an approximation to a simple fluid in the sense of retardation. The model is applicable to some dilute polymer solutions and is valid at low rates of shear. It is assumed that the speed of a point on the plate is proportional to its distance from the slit and the temperature difference between the sheet and its immediate surrounding is zero. We should however note that due to the entrainment of the ambient fluid, this boundary layer over a stretching sheet is quite different from that in a Blasius flow past a flat plate. Beards and Walter [1] obtained equation (2) for this type of flow as

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - k\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial u}{\partial y}\left(\frac{\partial^2 v}{\partial y^2}\right) + v\left(\frac{\partial^3 u}{\partial y^3}\right)\right] \tag{3}$$

$$k = \alpha_1 / \rho \tag{4}$$

where α_1 is the material constant and ρ is the density.

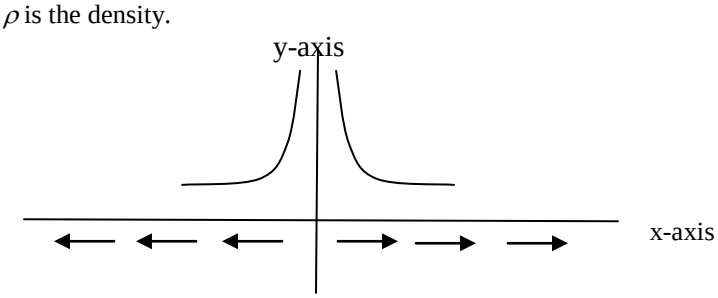

Fig. 1: Geometry of the problem.

The energy equation in the boundary layer approximation including the viscous dissipation term is given as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \lambda\frac{\partial^2 T}{\partial y^2} + \frac{u}{\rho c_1}\left(\frac{\partial u}{\partial y}\right)^2 \tag{5}$$

where T is the temperature and λ the thermal diffusivity and c_1 is the heat capacity. The last term in equation (5) is the viscous dissipation term. The boundary conditions are

$$T = T_w \ at \ y = 0, \ T \to T_\infty \ as \ y \to \infty \tag{6}$$

where T_w *and* T_∞ are constant temperature at the wall and very far away from the wall. We introduce the non-dimensionless temperature $\theta(\eta)$ by

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{7}$$

and self-similar solution of the form [3]

$$\left.\begin{aligned} & u = cx\frac{df(\eta)}{d\eta}\ ,\ v = -(\upsilon c)^{\frac{1}{2}} f(\eta) \\ & \text{where} \qquad \eta = \left(\frac{c}{\upsilon}x\right)^{\frac{1}{2}} y \end{aligned}\right\} \quad \text{c is a constant} \tag{8}$$

Equation (5) becomes

$$\frac{d^2\theta}{d\eta^2} + \frac{\upsilon}{\lambda} f \frac{d\theta}{d\eta} + \frac{\mu c^2 x^2}{\rho c_1 \lambda (T_w - T_\infty)}\left(\frac{d^2 f}{d\eta^2}\right)^2 = 0 \tag{8}$$

which can be written as

$$\frac{d^2\theta}{d\eta^2} + \sigma f \frac{d\theta}{d\eta} + Br_x\left(\frac{d^2 f}{d\eta^2}\right)^2 = 0 \tag{9}$$

where $\sigma = \dfrac{\upsilon}{\lambda}$

is the Prandtl number and

$$Br_x = \frac{\mu c^2 x^2}{\rho c_1 \lambda (T_w - T_\infty)} \tag{10}$$

is the Brinkmann's number. This shows that the Brinkmann's number is proportional to the square of the sheet velocity. This equation is similar to the equation derived by [3] except for the last term, which is a measure of the viscous dissipation. Equation (8) is solved subject to the boundary conditions.

$$\theta(0) = 1,\ \theta(\infty) = 0 \tag{11}$$

We adopt Dandapat and Gupta [3] method of solution by assuming that

$$f = (1-k)^{\frac{1}{2}}\left(1 - e^{-\eta(1-k)}\right)^{-\frac{1}{2}} \tag{12}$$

for the case $k << 1$ and transforming the ensuing equation we get

$$z\frac{d^2\vartheta}{dz^2} + \frac{d\vartheta}{dz} = \sigma(1-k)\left(\frac{d\vartheta}{dz} - z\frac{d\vartheta}{dz}\right) + Br_x z = 0 \tag{13}$$

where $z = e^{\eta(1-k)}$, $\theta(\eta) = \vartheta(z)$ (14)

$$k = \frac{a_1 c}{\rho\upsilon}\ldots \tag{15}$$

where a_1 is the material constant and ρ is the density.

The transformed boundary conditions are

$$\vartheta(0) = 0,\ \ \vartheta(1) = 1 \tag{16}$$

Without loss of generality we assume the Prandtl number $\sigma = 10$, equation (13) becomes

$$z\frac{d^2\vartheta}{dz^2} + \frac{d\vartheta}{dz} - 10(1-k)\left(\frac{d\vartheta}{dz} - z\frac{d\vartheta}{dz}\right) + Br_x z = 0 \tag{17}$$

Equation (17) is solved by the canonical series solution using the boundary conditions by letting

$$\left.\begin{aligned} & \vartheta = \vartheta_0 + \sum_{i=1}^{\infty} A_i \vartheta_i \ \ (\text{A is constant})\text{such} \\ & \vartheta_0(0) = 0,\ \vartheta_0(1) = 1\ and\ \vartheta_i(0) = 0,\ \vartheta_i(1) = 0 \end{aligned}\right\} \tag{18}$$

$$\vartheta_0 = a_1 + a_2 z \Rightarrow a_1 = 0,\ a_2 = 1, \Rightarrow \vartheta_0 = z$$

$$\vartheta_1 = b_1 z + b_2 z^2 \Rightarrow b_1 + b_2 = 0 \Rightarrow \vartheta_1 = b_1(z - z^2)$$

$$\vartheta_2 = c_2 z + c_3 z^2 \Rightarrow c_2 + c_3 = 0 \Rightarrow \vartheta_2 = c_2(z^2 - z^3)$$

$$\vartheta_3 = d_1\left(z^3 - z^4\right)$$

Thus we determine ϑ in the form

$$\vartheta = z + b_1\left(z - z^2\right) + c_1\left(z^2 - z^3\right) + d_1\left(z^3 - z^4\right) + \cdots$$

which we write in the form

$$\vartheta = Az + Bz^2 + Cz^3 + Dz^4 + \cdots$$

$$\theta = Ae^{-\eta\left(\sqrt{1-k}\right)^{\frac{1}{2}}} + Be^{-2\eta\left(\sqrt{1-k}\right)^{\frac{1}{2}}} + Ce^{-3\eta\left(\sqrt{1-k}\right)^{\frac{1}{2}}} + De^{-4\eta\left(\sqrt{1-k}\right)^{\frac{1}{2}}} + \cdots \qquad (19)$$

To determine the constants A, B, C, and D we substitute equations (15), (16), and (19) into (17) and equate the residue to zero within the interval [0, 1] for values of Br_x and k. Tables (1-6) show the values of the constants A, B, C, and D for various values of Br_x and different values of k.

Table 1: Values of the constants of equation (19) for various values of Br_x; k = 0.005

Br_x	0.1	0.5	0	2	5	10	20	50	100
A	1.6466	1.6161	1.6543	1.5018	1.2730	0.8917	0.1292	-2.1583	-5.9709
B	-6.8793	-6.7761	-6.9052	-6.3893	-5.6154	-4.3257	-1.7462	5.9920	18.8892
C	11.0010	11.0510	10.9885	11.2386	11.6138	12.2393	13.4899	17.2422	23.4959
D	-4.7683	-4.8910	-4.7376	-5.3511	-6.2714	-7.8053	-10.8729	-20.0759	-35.4142

Table 2: Values of the constants of equation (19) for various values of Br_x; k=0.01

Br_x	0.1	0.5	0	2	5	10	20	50	100
A	1.6396	1.6081	1.6474	1.4903	1.2545	0.8615	0.0755	-2.2824	-6.2122
B	-6.8610	-6.7537	-6.8877	-6.3516	-5.5472	-4.2066	-1.5254	6.5181	19.9239
C	10.9958	11.0402	10.9847	12.2065	11.5393	12.0939	13.2031	16.5307	22.0767
D	-4.7744	-4.8946	-4.7444	-5.3452	-6.2466	-7.7488	-10.7532	-19.7664	-34.7884

Table 3: Values of the constants of equation (19) for various values of Br_x; k = 0.05

Br_x	0.1	0.5	0	2	5	10	20	50	100
A	1.5760	1.5369	1.5857	1.3905	1.0977	0.6097	-0.3663	-3.2944	-8.1746
B	-6.6862	-5.4717	-6.7213	-6.0182	-4.9637	-3.9866	0.3092	10.8549	28.4312
C	10.9198	10.9157	10.9187	10.9070	10.8896	10.8603	10.8021	10.8549	10.3362
D	-4.8079	-4.9071	-4.7831	-5.2793	-6.0236	-7.2640	-9.7450	-17.1879	-29.5928

Table 4: Values of the constants of equation (19) for various values of Br_x; k = 0.1

Br_x	0.1	0.5	0	2	5	10	20	50	100
A	1.4773	1.4286	1.4895	1.2459	0.8803	0.2712	-0.9472	-4.6022	-10.6939
B	-6.3928	-6.2085	-6.4388	-5.5175	-4.1353	-1.8319	2.7751	16.9939	39.6310
C	10.7228	10.6585	10.7388	10.4175	9.9353	9.1319	7.5259	2.7039	5.3310
D	-4.8073	-4.8786	-4.7895	-5.1459	-5.6803	-6.5712	-8.3528	-13.6978	-22.6061

Table 5: Values of the constants of equation (19) for various values of Br_x; k = 0.2

Br_x	0.1	0.5	0	2	5	10	20	50	100
A	1.2102	1.1420	1.2273	0.8864	0.3750	-0.4773	-2.1818	-7.2955	-15.8182
B	-5.5216	-5.2443	-5.5909	-4.2046	-2.1250	1.3409	8.2727	29.0682	63.7273
C	9.9500	9.7500	10.0000	9.0000	7.0000	5.0000	0.0000	-15.0000	-40.0000
D	-4.6386	-4.6477	-4.6364	-4.6818	-4.7500	-4.7500	-5.0909	-5.7727	-6.9091

Table 6: Values of the constants of equation (19) for various values of Br_x; k = 0.5

Br_x	0.1	0.5	0	2	5	10	20	50	100
A	-0.1067	-0.2039	-0.0824	-0.5685	-1.2976	-2.5129	-4.9434	-12.2349	-24.3875
B	-0.2641	0.2615	-0.3956	2.2329	6.1756	12.7468	25.8892	65.3163	131.0282
C	3.6539	3.0183	3.8129	0.6346	-4.1329	-12.0786	-27.9702	-75.6448	-155.1024
D	-2.2831	-2.0759	-2.3349	-1.2990	0.2549	0.8447	8.0244	23.5634	49.4617

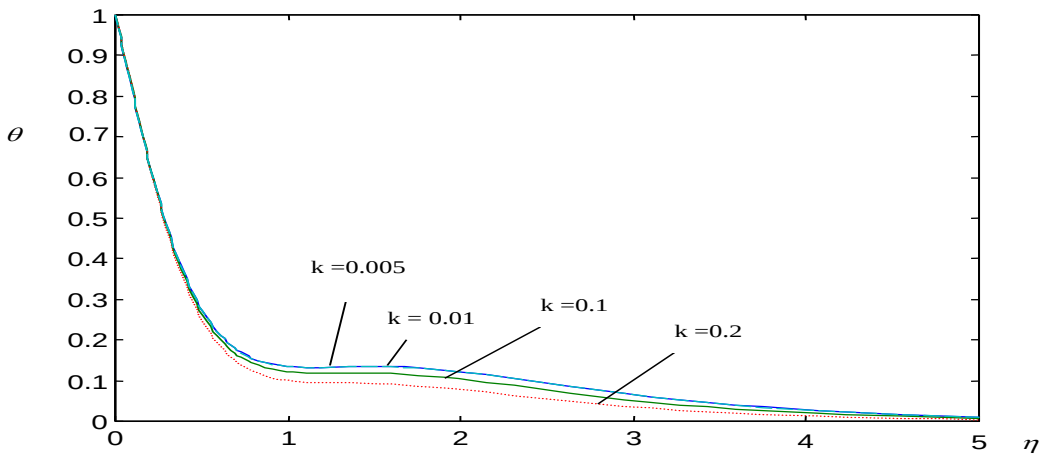

Fig 2: Variation of $\theta(\eta)$ with η when k_1 varies for small values of $Br_x = 0.1$

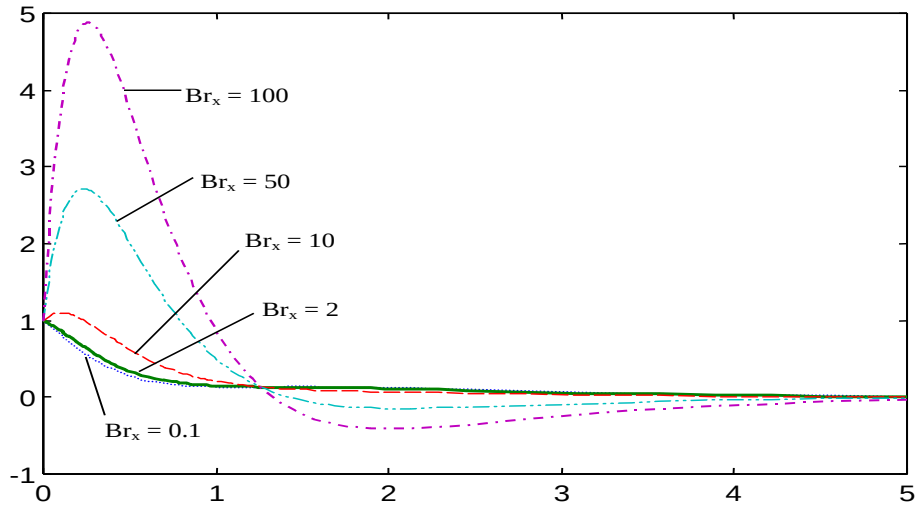

Fig 3. Variation of $\theta(\eta)$ with η when k_1 is fixed and Br_x, varies. $Br_x = 0.1, 2, 10, 50, 100$.

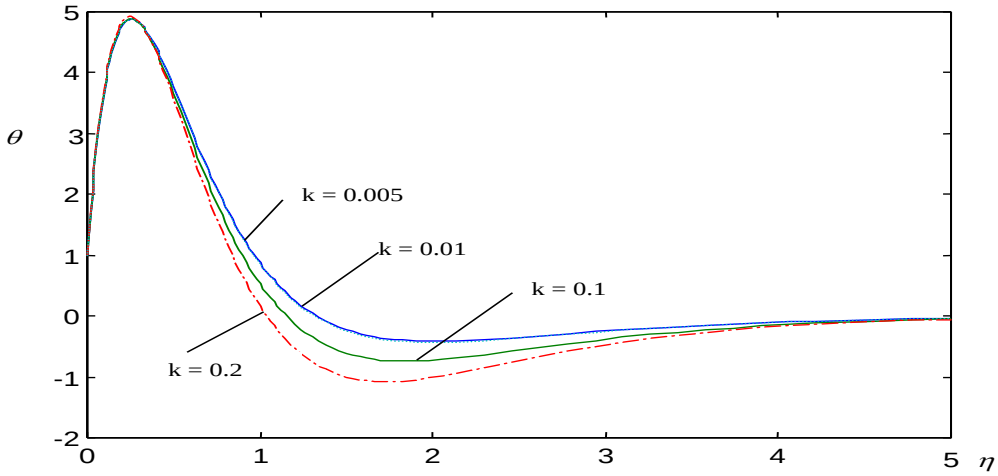

Fig 4. Variation of $\theta(\eta)$ with η when k_1 varies for high value of Br_x, $Br_x = 100$

4.0 Discussion and Conclusion

Fig. 2 is a plot of the temperature distribution for small values of the Brinkmann's number ($Br_x = 0.1$) which is a measure of the viscous dissipation when k_1 varies. For as long as $k << 1$, there is no significant change in the temperature distribution. Intuitively one can infer that a slightly elastic fluid ($k = 0.005$), will produce a boundary layer only slightly altered in its dimensions from a viscous one. However as the elasticity increases ($k = 0.5$) there seems to be a slight change in the temperature profile. When the viscous dissipation term becomes prominent by increasing Br_x (e.g. 50, 100) (see Fig.3) the temperature profile changes considerably. There was an initial significant rise in the temperature near the wall before it begins to fall away from it. As the viscous dissipation becomes more pronounced, the increase in temperature also becomes very significant. This is generally seen for all values of the elastic parameter $k<1$.

In Fig. 4 when k varies and $Br_x = 100$, (viscous dissipation very prominent), the increase in the elasticity of the material does not affect the profile of the temperature appreciably when the Prandtl number is constant. Comparing Fig 2 and Figure 4, we are able to appreciate the effect of the Brinkmann number when the skin friction varies. The implication of this is that the effect of the viscous dissipation of a material may be significant depending on the Brinkmann's number. Therefore viscous dissipation can only be ignored if the Brinkmann's constant of the material is very small especially less than 2 as shown in Figure 3.

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