Journal of the Nigerian Association of Mathematical Physics
***Volume* 22 (November, 2012), pp 399 – 406**
© J. of NAMP

Solution of a Mathematical Model of Pollutant Concentration in a Channel Flow using Adomian Decomposition Method

Ukoabasi Isip, *Terhemen Aboiyar and Stephen E. Onah

Department of Mathematics/Statistics/Computer Science,
University of Agriculture, Makurdi, Nigeria

Abstract

This paper focuses on the solution of a model for nonlinear dispersion of a pollutant ejected by an external source into a laminar flow of an incompressible fluid in a channel. The model equations are solved using the Adomian Decomposition Method, which is a semi-analytical method. The Adomian Decomposition Method (ADM) can be used to obtain exact or nearly exact solutions of nonlinear functional equations of various kinds without discretizing the equations or approximating the operators. The solution, when it exists, is found in a rapidly converging series form, and time and space are not discretized. Solutions of the mathematical model are presented in graphical form and given in terms of fluid velocity and pollutant concentration, for various parameter values. The results agree with results in literature obtained by high order finite difference methods.

Keywords: Kronecker product, braid group, Burau representation, irreducible.

1.0 Introduction

Water pollution, resulting from industrial waste discharge into water bodies such as rivers, lakes, streams among others, is a serious environmental concern, having large scale impact on both people and other living organisms in both small and large communities, especially in the riverine areas. An example of this is the accidental spillage of crude oil into water channels in the Niger Delta area of Nigeria, destroying both marine and land animals. In this paper, we are particularly concerned with the blockage of river channels as a result of excessive pollutant discharge. According to Taylor [1], spread of pollutants in a fluid flow depends largely on concentration coefficients. These can be determined empirically for each type of pollutant. Investigations such as the one by Shulka [2] can help identify the pollutant physical properties and the related mathematical parameters likely to cause the greatest harm in spreading the pollutant downstream. Chinyoka & Makinde [3], under the assumption that a pollutant is introduced nonlinearly into a channel flow via an external source presented the mathematical model described in this paper.

The Adomian Decomposition Method (ADM) allows exact solutions of nonlinear functional equations of various kinds without discretizing the equations or approximating the operators. The solution, when it exists, is found in a rapidly converging series form, and time and space are not discretized. The decomposition method yields rapidly convergent series solutions by using a few iterations for both linear and nonlinear deterministic and stochastic equations. The advantage of this method is that it provides a direct scheme for solving the problem, i.e., without the need for linearization, perturbation, massive computation and any transformation. This can be seen in various works in the literature such as [4-8].

In this paper, we will use the ADM to find a reliable and accurate solution to the model of pollutant concentration proposed Chinyoka & Makinde [3]. This yields an approximation to the exact solution in series form.

2.0 Methods

2.1 The Mathematical Model

We will first of all describe the model of Chinyoka & Makinde [3]. They considered a transient problem of fluid flow and nonlinear dispersion of pollutant in a rectangular channel, and provided the dimensionless model of the form:

*Corresponding author: ***Terhemen Aboiyar,*** E-mail: t_aboiyar@yahoo.co.uk, Tel. +234 7069121825

$$
\begin{aligned}
&\frac{\partial w}{\partial t} = K + \frac{\partial}{\partial y}\left(e^{\alpha\phi}\frac{\partial w}{\partial y}\right) + Gc\phi,\\
&\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial}{\partial y}\left(e^{\gamma\phi}\frac{\partial \phi}{\partial y}\right) + \lambda e^{n_0\phi},\\
&w(y,0) = m(1-y^2), \qquad \phi(y,0) = 2y,\\
&\frac{\partial w}{\partial y}(0,t) = \frac{\partial \phi}{\partial w}(0,t) = 0, \qquad \text{for } t>0\\
&w(1,t) = 0, \quad \phi(1,t) = 0, \qquad \text{for } t>0,
\end{aligned} \tag{1}
$$

where ϕ and y are the dimensionless equivalents for the pollutant concentration and flow velocity, respectively, λ is the pollutant external source parameter, Gc is the solutal Grashof number (property of the pollutant), K is the axial pressure gradient, Sc is the Schmidt number, α is the viscosity variation parameter, γ is the mass diffusivity variation parameter, and n_0 is the pollutant external source variation parameter. The dimensionless shear stress (C_f) and the mass transfer rate (Sh) at the channel wall are given by

$$
C_f = -\left.\frac{\partial w}{\partial y}\right|_{y=1}, \qquad \text{Sh} = -\left.\frac{\partial \phi}{\partial y}\right|_{y=1} \tag{2}
$$

Equation (1) is a system of first order nonlinear partial differential equations.

2.2 The Adomian Decomposition Method for Systems of Partial Differential Equations

We first consider a system of two first order partial differential equations

$$
\begin{aligned}
&\frac{\partial u}{\partial t} + \frac{\partial v}{\partial x} + N_1(u,v) = g_1\\
&\frac{\partial v}{\partial t} + \frac{\partial u}{\partial x} + N_2(u,v) = g_2
\end{aligned} \tag{3}
$$

with initial data

$$
\begin{aligned}
u(x,0) &= f_1(x)\\
v(x,0) &= f_2(x)
\end{aligned} \tag{4}
$$

Rewriting in operator form we have,

$$
\begin{aligned}
L_t u + L_x v + N_1(u,v) &= g_1\\
L_t v + L_x u + N_2(u,v) &= g_2
\end{aligned} \tag{5}
$$

with initial data

$$
\begin{aligned}
u(x,0) &= f_1(x)\\
v(x,0) &= f_2(x)
\end{aligned} \tag{6}
$$

where L_t and L_x are the first order partial differential operators, $\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial x}$, respectively, N_1 and N_2 are nonlinear operators, and g_1 and g_2 are non-homogenous terms. Applying the inverse operator L_t^{-1} to the system (5) and using the initial data (6) yields

$$
\begin{aligned}
u(x,t) &= f_1(x) + L_t^{-1}g_1 - L_t^{-1}L_x v - L_t^{-1}N_1(u,v),\\
v(x,t) &= f_2(x) + L_t^{-1}g_2 - L_t^{-1}L_x u - L_t^{-1}N_1(u,v).
\end{aligned} \tag{7}
$$

where $L_t^{-1} = \int_0^t (.)dt$

The linear terms $u(x,t)$ and $v(x,t)$ are thereafter decomposed by an infinite series of components

$$
u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \quad v(x,t) = \sum_{n=0}^{\infty} v_n(x,t), \tag{8}
$$

which are obtained systematically as the terms of the infinite series progress. The nonlinear operators $N_1(u,v)$ and $N_2(u,v)$ are defined by the infinite series of Adomian polynomials

$$
N_1(u,v) = \sum_{n=0}^{\infty} A_n, \tag{9}
$$

$N_2(u,v) = \sum_{n=0}^{\infty} B_n,$

which can be uniquely determined using the algorithm provided by Wazwaz [8] for each nonlinear operator.

Substituting (8) and (9) into (7) gives

$$\left.\begin{aligned}\sum_{n=0}^{\infty} u_n(x,t) &= f_1(x) + L_t^{-1}g_1 - L_t^{-1}\left(L_x\left(\sum_{n=0}^{\infty} v_n\right)\right) - L_t^{-1}\left(\sum_{n=0}^{\infty} A_n\right),\\ \sum_{n=0}^{\infty} v_n(x,t) &= f_2(x) + L_t^{-1}g_2 - L_t^{-1}\left(L_x\left(\sum_{n=0}^{\infty} u_n\right)\right) - L_t^{-1}\left(\sum_{n=0}^{\infty} B_n\right).\end{aligned}\right\} \tag{10}$$

Following Adomian analysis, the nonlinear system (7) is transformed into a set of recursive relations given by

$$\left.\begin{aligned}u_0(x,t) &= f_1(x) + L_t^{-1}g_1,\\ u_{k+1}(x,t) &= -L_t^{-1}(L_x v_k) - L_t^{-1}(A_k), \quad k \geq 0,\end{aligned}\right\} \tag{11}$$

and

$$\left.\begin{aligned}v_0(x,t) &= f_2(x) + L_t^{-1}g_2,\\ v_{k+1}(x,t) &= -L_t^{-1}(L_x u_k) - L_t^{-1}(B_k), \quad k \geq 0,\end{aligned}\right\} \tag{12}$$

The solution is thus obtained in series form as

$$\left.\begin{aligned}u(x,t) &= \sum_{n=0}^{\infty} u_n(x,t),\\ v(x,t) &= \sum_{n=0}^{\infty} v_n(x,t),\end{aligned}\right\} \tag{13}$$

3 Results

We solve the model equations above using the Adomian Decomposition Method by first simplifying them and then rewriting them in operator form. Simplifying using the product rule for differentiation we have the initial value problem

$$\left.\begin{aligned}\frac{\partial w}{\partial t} &= K + \left(e^{\alpha\phi}\right)_y \frac{\partial w}{\partial y} + e^{\alpha\phi}\frac{\partial^2 w}{\partial y^2} + Gc\phi,\\ \frac{\partial \phi}{\partial t} &= \frac{1}{Sc}\left(e^{\gamma\phi}\right)_y \frac{\partial \phi}{\partial y} + \frac{1}{Sc}e^{\gamma\phi}\frac{\partial^2 \phi}{\partial y^2} + \lambda e^{n_0\phi},\\ w(y,0) &= m(1-y^2), \quad \phi(y,0) = 2y\end{aligned}\right\} \tag{14}$$

We now rewrite the system of $PDEs$ (14) in operator form as follow

$$\left.\begin{aligned}L_t w(y,t) &= K + N_1(w,\phi) + N_2(w,\phi) + R\phi\\ L_t\phi(y,t) &= M_1(\phi) + M_2(\phi) + M_3(\phi)\end{aligned}\right\} \tag{15}$$

with initial data

$$w(y,0) = m(1-y^2), \qquad \phi(y,0) = 2y \tag{16}$$

where the operator

$$L_t(\cdot) = \tfrac{\partial}{\partial t}(\cdot)$$

and the inverse operator

$$L_t^{-1}(\cdot) = \int_0^t (\cdot)dt$$

The linear term is

$$R\phi = Gc\phi,$$

and nonlinear terms:

$$\left.\begin{aligned} N_1(w,\phi) &= \left(e^{\alpha\phi}\right)_y \frac{\partial w}{\partial y}, N_2(w,\phi) = e^{\alpha\phi}\frac{\partial^2 w}{\partial y^2} \\ M_1(\phi) &= \frac{1}{Sc}\left(e^{\gamma\phi}\right)_y \frac{\partial \phi}{\partial y}, M_2(\phi) = \frac{1}{Sc}e^{\gamma\phi}\frac{\partial^2 \phi}{\partial y^2}, M_3(\phi) = \lambda e^{n_0\phi} \end{aligned}\right\} \tag{17}$$

Applying the inverse operator L_t^{-1} to the system (15) and using the initial data (16) yields

$$\left.\begin{aligned} w(y,t) &= w(y,0) + Kt + L_t^{-1}\big(N_1(w,\phi)\big) + L_t^{-1}\big(N_2(w,\phi)\big) + L_t^{-1}R\phi \\ \phi(y,t) &= \phi(y,0) + L_t^{-1}\big(M_1(\phi)\big) + L_t^{-1}\big(M_2(\phi)\big) + L_t^{-1}\big(M_3(\phi)\big) \end{aligned}\right\} \tag{18}$$

which we simplify to obtain

$$\left.\begin{aligned} w(y,t) &= m(1-y^2) + Kt + L_t^{-1}\big(N_1(w,\phi)\big) + L_t^{-1}\big(N_2(w,\phi)\big) + L_t^{-1}R\phi \\ \phi(y,t) &= L_t^{-1}\big(M_1(\phi)\big) + L_t^{-1}\big(M_2(\phi)\big) + L_t^{-1}\big(M_3(\phi)\big) \end{aligned}\right\} \tag{19}$$

The Adomian decomposition method suggests that the linear terms $w(y,t)$ and $\phi(y,t)$ be decomposed by an infinite series of components:

$$\left.\begin{aligned} w(y,t) &= \sum_{n=0}^{\infty} w_n(y,t), \\ \phi(y,t) &= \sum_{n=0}^{\infty} \phi_n(y,t), \end{aligned}\right\} \tag{20}$$

and the nonlinear terms

$$\left.\begin{aligned} N_1(w,\phi) &= \sum_{n=0}^{\infty} A_n, \\ N_2(w,\phi) &= \sum_{n=0}^{\infty} B_n, \\ M_1(\phi) &= \sum_{n=0}^{\infty} C_n, \\ M_2(\phi) &= \sum_{n=0}^{\infty} D_n, \\ M_3(\phi) &= \sum_{n=0}^{\infty} E_n, \end{aligned}\right\} \tag{21}$$

where A_n, B_n, C_n, D_n and E_n, are the Adomian polynomials.
Substituting (20) and (21) into (19) gives

$$\left.\begin{aligned} \sum_{n=0}^{\infty} w_n(y,t) &= m(1-y^2) + Kt + L_t^{-1}\left(\sum_{n=0}^{\infty} A_n\right) + L_t^{-1}\left(\sum_{n=0}^{\infty} A_n\right) + L_t^{-1}\left\{Gc\sum_{n=0}^{\infty}\phi_n(y,t)\right\} \\ \sum_{n=0}^{\infty} \phi_n(y,t) &= 2y + L_t^{-1}\left(\sum_{n=0}^{\infty} A_n\right) + L_t^{-1}\left(\sum_{n=0}^{\infty} A_n\right) + L_t^{-1}\left(\sum_{n=0}^{\infty} A_n\right) \end{aligned}\right\} \tag{22}$$

Following Adomian analysis, the nonlinear system (21) is transformed into a set of recursive relations given by

$$\left.\begin{aligned} w_0(y,t) &= m(1-y^2) + Kt \\ &\vdots \\ w_{K+1}(y,t) &= L_t^{-1}A_k + L_t^{-1}B_k + L_t^{-1}\big(Gc\phi_k(y,t)\big), \end{aligned}\right\} \; k \geq 0 \tag{23}$$

and

$$\left.\begin{aligned} \phi_0(y,t) &= 2y \\ &\vdots \\ \phi_{K+1}(y,t) &= L_t^{-1}C_k + L_t^{-1}D_k + L_t^{-1}E_k, \qquad k \geq 0 \end{aligned}\right\} \tag{24}$$

It is an essential feature of the decomposition method that the zeroth components $w_0(y,t)$ and $\phi_0(y,t)$ are defined always by all terms that arise from initial data and from integrating the inhomogeneous terms. Having defined the zeroth pair (w_0,ϕ_0) the pair (w_1,ϕ_1) can be determined recurrently by using (23) and (24). The remaining pairs (w_k,ϕ_k); $k \geq 2$; can be determined in a parallel manner. It should be noted that additional pairs for the decomposition series normally account for higher accuracy. Having determined the components of $w(y,t)$ and $\phi(y,t)$, the solution $(w,\phi)$of the system follows immediately in the form of a power series expansion upon using (22). The series obtained can be summed up in many cases to give a closed form solution. For concrete problems, the nth term approximants can be used for numerical purposes.

Following Wazwaz [8], and using an algorithm designed in MAPLE, we obtain the Adomian polynomials for the nonlinear terms as follows:

$$A_0 = \left(e^{\alpha\phi_0}\right)_y \frac{\partial w_0}{\partial y}$$
$$A_1 = \left(e^{\alpha\phi_0}\right)_y \frac{\partial w_1}{\partial y} + \left(\alpha\phi_1 e^{\alpha\phi_0}\right)_y \frac{\partial w_0}{\partial y}$$
$$A_2 = \left(e^{\alpha\phi_0}\right)_y \frac{\partial w_2}{\partial y} + \left(\alpha\phi_1 e^{\alpha\phi_0}\right)_y \frac{\partial w_1}{\partial y} + \left(\alpha\phi_2 e^{\alpha\phi_0} + \frac{1}{2}\phi_1^2\alpha^2 e^{\alpha\phi_0}\right)_y \frac{\partial w_0}{\partial y}$$
$$A_3 = \left(e^{\alpha\phi_0}\right)_y \frac{\partial w_3}{\partial y} + \left(\alpha\phi_1 e^{\alpha\phi_0}\right)_y \frac{\partial w_2}{\partial y} + \left(\alpha\phi_2 e^{\alpha\phi_0} + \frac{1}{2}\phi_1^2\alpha^2 e^{\alpha\phi_0}\right)_y \frac{\partial w_1}{\partial y} +$$
$$\left(\alpha\phi_3 e^{\alpha\phi_0} + \alpha^2\phi_1\phi_2 e^{\alpha\phi_0} + \frac{1}{6}\alpha^3\phi_1^3 e^{\alpha\phi_0}\right)\frac{\partial w_0}{\partial y} \tag{25}$$

and,

$$B_0 = \left(e^{\alpha\phi_0}\right)_y \frac{\partial^2 w_0}{\partial y^2}$$
$$B_1 = \left(e^{\alpha\phi_0}\right)_y \frac{\partial^2 w_1}{\partial y^2} + \left(\alpha\phi_1 e^{\alpha\phi_0}\right)_y \frac{\partial w_0}{\partial y}$$
$$B_2 = \left(e^{\alpha\phi_0}\right)_y \frac{\partial^2 w_2}{\partial y^2} + \left(\alpha\phi_1 e^{\alpha\phi_0}\right)_y \frac{\partial^2 w_1}{\partial y^2} + \left(\alpha\phi_2 e^{\alpha\phi_0} + \frac{1}{2}\phi_1^2\alpha^2 e^{\alpha\phi_0}\right)_y \frac{\partial^2 w_0}{\partial y^2}$$
$$B_3 = \left(e^{\alpha\phi_0}\right)_y \frac{\partial^2 w_3}{\partial y^2} + \left(\alpha\phi_1 e^{\alpha\phi_0}\right)_y \frac{\partial^2 w_2}{\partial y^2} + \left(\alpha\phi_2 e^{\alpha\phi_0} + \frac{1}{2}\phi_1^2\alpha^2 e^{\alpha\phi_0}\right)_y \frac{\partial^2 w_1}{\partial y^2} +$$
$$\left(\alpha\phi_3 e^{\alpha\phi_0} + \alpha^2\phi_1\phi_2 e^{\alpha\phi_0} + \frac{1}{6}\alpha^3\phi_1^3 e^{\alpha\phi_0}\right)\frac{\partial^2 w_0}{\partial y^2} \tag{26}$$

and,

$$C_0 = \frac{1}{Sc}\left(e^{\gamma\phi_0}\right)_y \frac{\partial \phi_0}{\partial y}$$
$$C_1 = \left(e^{\gamma\phi_0}\right)_y \frac{\partial \phi_1}{\partial y} + \left(\gamma\phi_1 e^{\gamma\phi_0}\right)_y \frac{\partial \phi_0}{\partial y}$$
$$C_2 = \left(e^{\gamma\phi_0}\right)_y \frac{\partial \phi_2}{\partial y} + \left(\gamma\phi_1 e^{\gamma\phi_0}\right)_y \frac{\partial \phi_1}{\partial y} + \left(\gamma\phi_2 e^{\gamma\phi_0} + \frac{1}{2}\phi_1^2\gamma^2 e^{\gamma\phi_0}\right)_y \frac{\partial \phi_0}{\partial y}$$
$$C_3 = \left(e^{\gamma\phi_0}\right)_y \frac{\partial \phi_3}{\partial y} + \left(\gamma\phi_1 e^{\gamma\phi_0}\right)_y \frac{\partial \phi_2}{\partial y} + \left(\gamma\phi_2 e^{\gamma\phi_0} + \frac{1}{2}\phi_1^2\gamma^2 e^{\gamma\phi_0}\right)_y \frac{\partial \phi_1}{\partial y} +$$
$$\left(\gamma\phi_3 e^{\gamma\phi_0} + \gamma^2\phi_1\phi_2 e^{\gamma\phi_0} + \frac{1}{6}\gamma^3\phi_1^3 e^{\gamma\phi_0}\right)\frac{\partial \phi_0}{\partial y} \tag{27}$$

and,

$$D_0 = \frac{1}{Sc}\left(e^{\gamma\phi_0}\right)_y \frac{\partial^2 \phi_0}{\partial y^2}$$
$$D_1 = \left(e^{\gamma\phi_0}\right)_y \frac{\partial^2 \phi_1}{\partial y^2} + \left(\gamma\phi_1 e^{\gamma\phi_0}\right)_y \frac{\partial^2 \phi_0}{\partial y^2}$$
$$D_2 = \left(e^{\gamma\phi_0}\right)_y \frac{\partial^2 \phi_2}{\partial y^2} + \left(\gamma\phi_1 e^{\gamma\phi_0}\right)_y \frac{\partial \phi_1}{\partial y} + \left(\gamma\phi_2 e^{\gamma\phi_0} + \frac{1}{2}\phi_1^2\gamma^2 e^{\gamma\phi_0}\right)_y \frac{\partial \phi_0}{\partial y}$$
$$D_3 = \left(e^{\gamma\phi_0}\right)_y \frac{\partial^2 \phi_3}{\partial y^2} + \left(\gamma\phi_1 e^{\gamma\phi_0}\right)_y \frac{\partial^2 \phi_2}{\partial y^2} + \left(\gamma\phi_2 e^{\gamma\phi_0} + \frac{1}{2}\phi_1^2\gamma^2 e^{\gamma\phi_0}\right)_y \frac{\partial^2 \phi_1}{\partial y^2} +$$
$$\left(\gamma\phi_3 e^{\gamma\phi_0} + \gamma^2\phi_1\phi_2 e^{\gamma\phi_0} + \frac{1}{6}\gamma^3\phi_1^3 e^{\gamma\phi_0}\right)\frac{\partial^2 \phi_0}{\partial y^2}$$

and

$$E_0 = \lambda\left(e^{n_0\phi_0}\right)$$
$$E_1 = \lambda\left(n_0\phi_1 e^{n_0\phi_0}\right)$$
$$E_2 = \lambda\left(n_0\phi_2 e^{n_0\phi_0} + \frac{1}{2}\phi_1^2 n_0^2 e^{n_0\phi_0}\right)$$
$$E_3 = \lambda\left(n_0\phi_3 e^{n_0\phi_0} + \phi_1\phi_2 n_0^2 e^{n_0\phi_0} + \frac{1}{6}\phi_1^3 n_0^3 e^{n_0\phi_0}\right) \quad (29)$$

Computing the polynomials using MAPLE, we have the solutions:

$$w(y,t) = m(1-y^2) + (K - 4\alpha e^{2\alpha y}my - 4\alpha e^{2\alpha y}m + 2Gcy)t + \frac{1}{2}\Big\{-2\Big[\alpha\left(\frac{8\gamma^2 e^{2\gamma y}}{Sc} + 2\lambda n_0 e^{2n_0 y}\right)e^{2\alpha y} + 2\alpha^2\left(\frac{4\gamma e^{2\gamma y}}{Sc} + \lambda e^{2n_0 y}\right)e^{2\alpha y}\Big]my + 4\alpha e^{2\alpha y}(-8\alpha^2 e^{2\alpha y}my - 4\alpha e^{2\alpha y}m - 8\alpha^2 e^{2\alpha y}m + 2Gc) - 2\Big[\alpha\left(\frac{8\gamma^2 e^{2\gamma y}}{Sc} + 2\lambda n_0 e^{2n_0 y}\right)e^{2\alpha y} + 2\alpha^2\left(\frac{4\gamma e^{2\gamma y}}{Sc} + \lambda e^{2n_0 y}\right)e^{2\alpha y}\Big]m + Gc\left(\frac{4\gamma e^{2\gamma y}}{Sc} + \lambda e^{2n_0 y}\right)\Big\}t^2 + \cdots \quad (30)$$

and,

$$\phi(y,t) = 2y + \left(\frac{4\gamma e^{2\gamma y}}{Sc} + \lambda e^{2n_0 y}\right)t + \frac{1}{2Sc}\Big\{2\Big[\gamma\left(\frac{8\gamma^2 e^{2\gamma y}}{Sc} + 2\lambda n_0 e^{2n_0 y}\right)e^{2\gamma y} + 2\gamma^2\left(\frac{4\gamma e^{2\gamma y}}{Sc} + \lambda e^{2n_0 y}\right)e^{2\gamma y}\Big] + 2\gamma e^{2\gamma y}\left(\frac{8\gamma^2 e^{2\gamma y}}{Sc} + 2\lambda n_0 e^{2n_0 y}\right) + 2\gamma e^{2\gamma y}\left(\frac{16\gamma^3 e^{2\gamma y}}{Sc} + 4\lambda {n_0}^2 e^{2n_0 y}\right) + \lambda n_0 Sc\left(\frac{4\gamma e^{2\gamma y}}{Sc} + \lambda e^{2n_0 y}\right)e^{2n_0 y}\Big\}t^2 + \cdots \quad (31)$$

4.0 Numerical Experiments

Here we obtain the solutions of the model graphically, using various parameter values.

4.1 Transient Solutions

Figures 1 and 2 illustrate the changes in velocity (w) and concentration (ϕ) as time increases.

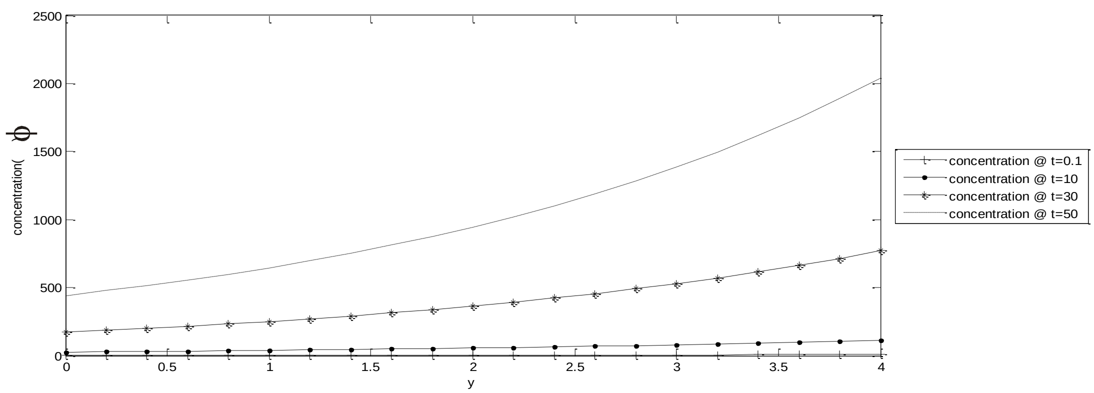

Figure 2: Graph of Concentration (ϕ) versus 'y' at t = 0.1, 10, 25, 50, using K=1, n_o=0.1, λ=0.5, Gc=0.1, Sc=0.6, $\gamma = 0.1$

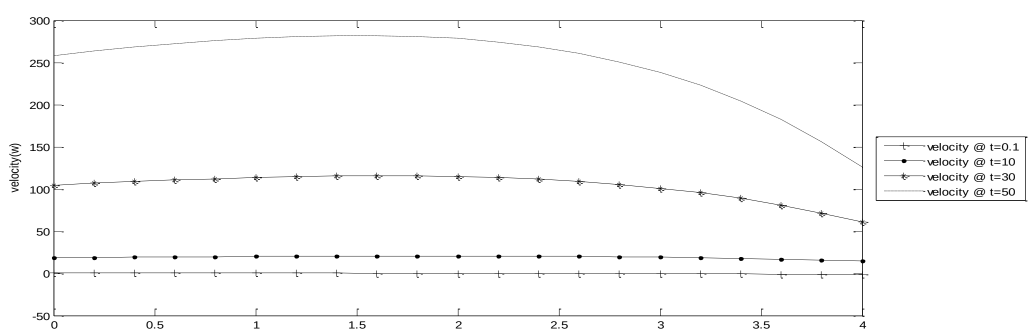

Figure 1: Graph of Velocity (w) versus 'y' at t = 0.1, 10, 30, 50

Figure 1 shows the time development of the velocity profile. As expected, the velocity increases, following a parabolic path. This increase is due to increased momentum, as the flow progresses. This is consistent with the results obtained by Chinyoka & Makinde [3]. Figure 2 shows increase in concentration profiles as time progresses. This increase is sustained till some future time when there is no further injection of pollutants.

4.2 Influence of Flow Parameters

Figures 3 to 6 show the dependence of velocity and concentration on the flow parameters. Two key parameters are modified in this analysis: λ (the pollutant external source parameter) and Sc the Schmidt number).

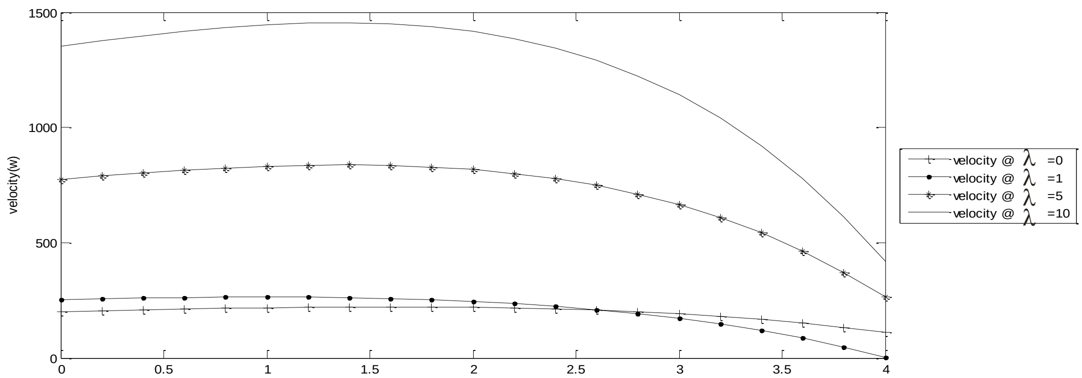

Figure 3: Graph of Velocity (w) versus 'y' at λ = 0, 1, 5, 10 using K=1, n_o=0.1, Gc=0.1, Sc=0.6, $\gamma = 0.1$

In the figure above, we observe the change in velocity using different values of λ, a property of the pollutant, which depends inversely on the pollutant viscosity. Figure 3 shows higher velocities for higher values of λ. This observation is consistent with those obtained by Chinyoka and Makinde [3].

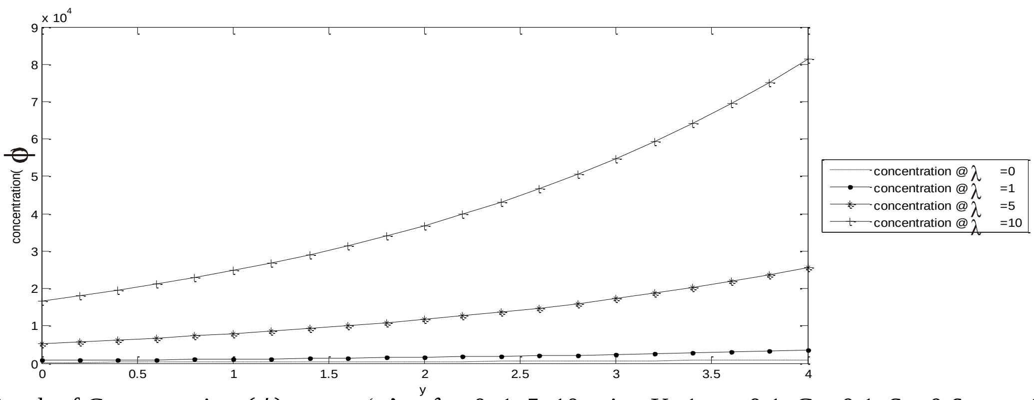

Figure 4: Graph of Concentration (ϕ) versus 'y' at λ = 0, 1, 5, 10 using K=1, n_o=0.1, Gc=0.1, Sc=0.6, $\gamma = 0.1$

Similar to the result obtained in Figure 3, Figure 4 shows that the concentration increases significantly as λ increases. Very high values of λ may however lead to a blow up in the concentration values.

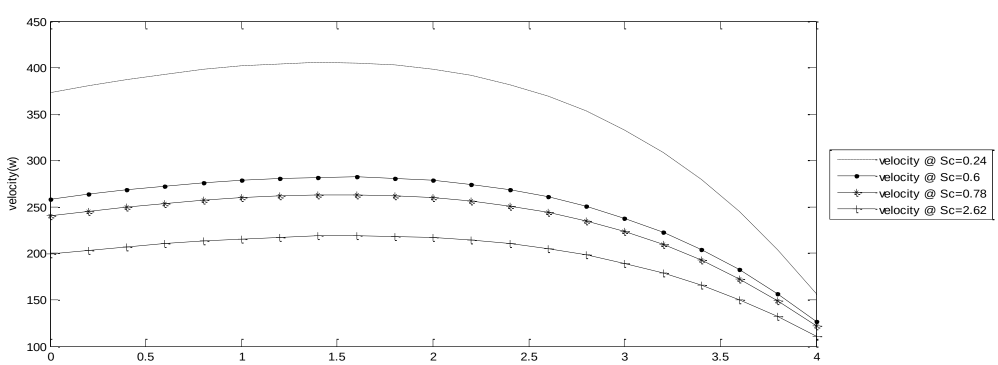

Figure 5: Graph of Velocity (w) versus 'y' at Sc = 0.24, 0.6, 0.78, 2.62 using K=1, n_o=0.1, λ=0.5, Gc=0.1, $\gamma = 0.1$

Sc is the Schmidt number, which depends on the chemical composition of the pollutant. Usually chemical compounds with higher Sc values are more viscous. As observed in Figure 5, lower values of Sc lead to higher velocity. This is expected, as a result of reduced viscous drag. Pollutants with lower Sc values are less likely to cause blockages in water channels.

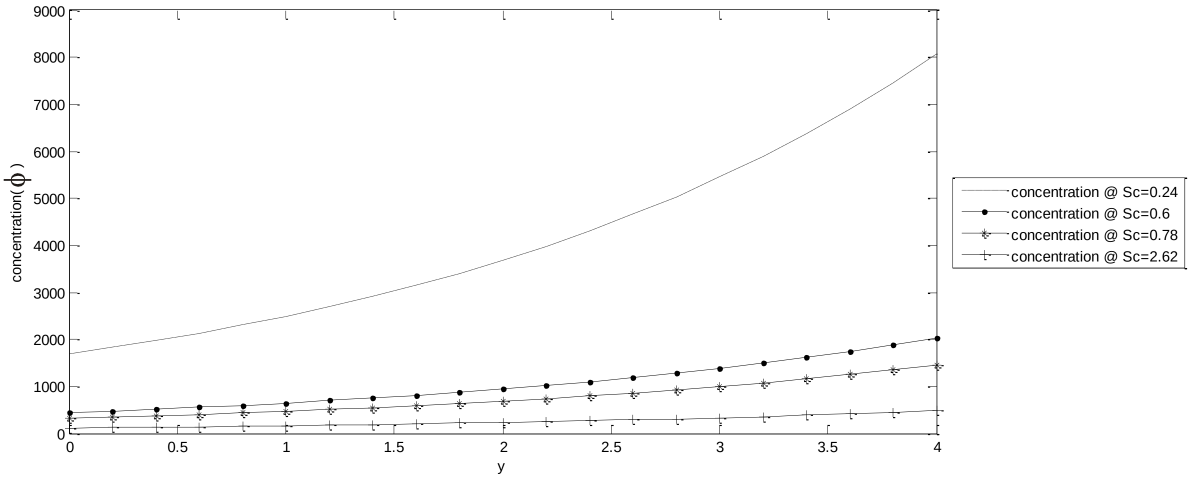

Figure 6: Graph of Concentration (ϕ) versus 'y' at $Sc = 0.24,\ 0.6, 0.78, 2.62$ using K=1, n_o=0.1, λ=0.5, Gc=0.1, $\gamma = 0.1$

In this case, we observe the change in concentration as we vary the Schmidt values. The reduced viscosity leads to a drastic increase in concentration with lower Sc values. This phenomenon is observed in Figure 6.

5.0 Conclusion

In this paper, we studied the model developed by Chinyoka and Makinde [3], and obtained solutions for the pollutant concentration in a channel flow, as well as the flow velocity, using the Adomian Decomposition Method. Graphical illustration of the solution shows a clear similarity with results from high order finite difference methods. The results obtained by the Adomian Decomposition Method shows clearer demarcations between the graphs for different parameter values and they can be more easily used for further analysis.

References

[1] Taylor, G. I. (1954). *The dispersion of matter in turbulent flow through pipe*, Proceedings of the Royal Society of London, Vol. 233, pp. 446–448.
[2] Shulka, P. (2002). *Analytical solutions for steady transport dispersion of nonconservative pollutant with time-dependent periodic waste discharge concentration*, Journal of Hydraulic Engineering, Vol. 129, No. 9, pp. 866–869.
[3] Chinyoka, T. and Makinde, O. D. (2010). *Analysis of Nonlinear Dispersion of a Pollutant Ejected by an External Source into a Channel Flow,* Mathematical Problems in Engineering, Volume 2010, pp.1-18.
[4] Abbaoui, K. and Cherrualt, Y. (1994a). *Convergence of Adomians method applied to differential equations*, Computers and Mathematics with Applications, Vol. 28, pp. 103-109.
[5] Abbaoui, K. and Cherrualt Y. (1994b). *Convergence of Adomians method applied to nonlinear equation*, Mathematical and Computer Modelling, Vol. 20, pp. 69-73.
[6] Lesnic, D. (2002). *The decomposition method for forward and backward time-dependent problems*, Journal of Computational and Applied Mathematics, Vol. 147**,** 27-38.
[7] Al-Khaled, K. and Allan, F. (2005). *Construction of solution for the shallow water equations by the decomposition method*, Mathematics and Computers in Simulation, Vol. 66**,** pp. 479-486.
[8] Wazwaz, A. (2000). *A new algorithm for calculating Adomian polynomials for nonlinear operators, Applied Mathematics and Computation,* Vol. 111, pp. 53-69.