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# Simple Algorithm for the Generation of Adomian Polynomial

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## *Abstract*

***In this paper, we shall introduce a simple Class I Accelerated algorithm for the generation of Adomian polynomial in Adomian decomposition method .The class I Accelerated algorithm which is more precise and accurate in the generation of Adomian polynomial is also compared to the classical method and formula for the generation of Adomian polynomial.***

**Keywords:** Adomian decomposition method, Adomian polynomial, Linear and non-linear Functions ODE.

## 1.0 Introduction

Several reseachers [1-5] have presented powerful decomposition methodology based on the Adomian approach for practical solution of linear or nonlinear and deterministic or stochastic operator equations, including ordinary differential equations, partial differential equations, integral equations e.t.c.

Adomian decomposition method(ADM) is a significant, powerful method, which provides an efficient means for the analytics and numerical solution of differential equation which model real-life physical applications.

Adomian decomposition methods allow us to solve non-linear differential equation having to appeal to the decidedly questionable practice of perturbation or linearization.

The solution algorithm yields a rapidly convergent sequence of analytics approximant, which is readily computable, without recourse to linearization.

ADM consist of splitting the given equation into linear and non-linear parts, leaving the highest – order derivative operation contained in the linear operation in both sides, identifying the initial and / or the boundary condition and the terms involving the independent variable alone as initial approximation, decomposing the unknown function into series whose component are to be determined, decomposing the non linear function in terms of special polynomial called Adomian's polynomials and finding the successive terms of the series solution by recurrent relation using Adomian's polynomial .ADM is quantitative rather than qualitative, analytic, requiring neither linearization nor perturbation and continuous with no resort to discretization and consequent computer-intensive calculations.

In this study we shall be finding other ways mathematically, on how to generate Adomian Polynomial outside the conventional approach.

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## 2.0 Discuss methodology

Given

$FU = LU + RU + NU = g$ (1.1)

a nonlinear ordinary differential equation

such that

g = Special analytic function

F = General nonlinear stochastic operator

L= the linear operator

R= Random operator

N= Nonlinear operator per Adomian's notation.

Then for simplicity,
we consider an Engineering model from the Adomian's differential equation
such as,

$$LU + NU = g \quad (1.2)$$

where generally, the operator F has linear and nonlinear components. Adomian [1] partitioned L into components for the sake of easy integrations, such that,

$$L = \mathcal{L} + R \quad (1.3)$$

Where

$$L = \frac{d^n}{dx^n}\, t \quad (1.4)$$

And generally

$$R_1 = \sum_{v-0}^{n-1} \propto_v \ (x)\frac{d^v}{dx_v} \quad (1.5)$$

(1.5) the remainder ( R) operator linear terms.

For the power of illustration, we specialize the nonlinear operator N to be a simple non linearity such that

$$Nu = R_2\, f(u) \quad (1.6)$$

where $f(u)$ is an analytic function of the solution u.
In this case, the operator $R_2$ may have a similar form as the remainder $R_1$.
For simplicity of exposition, we have

$$Lu - R_1 u - \ R_2\, f(u) = g \quad (1.7)$$

Now if L is an inverted operator

$$L^{-1} = \emptyset + A^{-1} \quad (1.8)$$

Where $A^{-1}\ is\ n - fold\ indefinite\ integral\ operator$
$\int \ldots \ldots \int (.)\, dx \ldots \ldots \ldots \ldots \ldots dx\ and\ \emptyset\ compose\ of\ constants\ of\ integration$
and

$$\phi = \sum_{v=0}^{p-1} B_v \frac{x^v}{v!} \qquad [5] \quad (1.9)$$

it means that

$$L^{-1} = \int_{\circ}^{\prime} \circ \circ \circ \int_{\circ}^{\prime} (^{\circ})\ dt \ldots .\, dt \quad (2.0)$$

for IVP order differential equation.

$$L^{-1} = \beta_{\circ} + \beta_{\prime} r + \int r^{-2} \int r^2\, (0) d_r d_r \quad (2.1)$$

By Banach theorem [4] on (1.6) , we have

$$\|L^{-1}R_1\| <| \quad (2.2)$$

and

$$\|L^{-1}R_2\| <| \quad (2.3)$$

With the operator L we have

$$L^{-1}\, Lu = L^{-1}g + L^{-1}R_1 u + L^{-1}R_2\, f(u) \quad (2.4)$$

which yield Adomian integrator

$$U = \emptyset +\ L^{-1}g + L^{-1}R_1 u + L^{-1}R_2\, f(u). \quad (2.5)$$

Equation (2.4) is the decomposition IVP First order and Boundary value problem respectively.
This can be [4]

$$U = \sum_{m=0}^{\infty} U_m \lambda^m \ f(u) = \sum_{m=0}^{\infty} A_m \lambda^m \quad (2.6)$$

Where $\lambda =$ parametrized variable.
By the Paramerized Taylor expansion series (2.6)

$$u_m = \frac{1}{m!} \frac{\partial^m f(u(x:\lambda)}{\partial \lambda^m}\bigg|_{\lambda=0} \quad (2.7)$$

$$A_m = \frac{1}{m!} \frac{\partial^m f(u(x:\lambda)}{\partial \lambda^m}\bigg|_{\lambda=0} \quad (2.8)$$

Equation (2.6) by implicit differentiation is call Adomian polynomial formula

**The generation of Adomian polynomial by (2.6)**

$$A_m = \frac{1}{m!} \frac{\partial^m f(u(x:\lambda)}{\partial \lambda^m}\bigg|_{\lambda=0}$$

When n=0

$$A_0 = \frac{1}{1}\left[\frac{1}{1} f(u_0)\right] = f(u_0)$$

When n=1

$$A_1 = \frac{1}{1!}\left[\frac{d}{d\lambda} N(\sum_{k=0}^{1} \lambda u_k\bigg|_{\lambda=0})\right]$$

$$A_1 = \frac{d}{d\lambda} N(\lambda^0 u_0 + \lambda^1 u_1)$$

$$A_1 = f(u_0) + f(\lambda^1 u_1)$$

$$A_1 = u_1 f^I(u_0) + u_1 f(\lambda^1 u_1)$$

$$A_1 = u_1 f^I(u_0)$$

When n=2

$$A_2 = \frac{1}{2!}\left[\frac{d^2}{d\lambda^2} N(\lambda^0 u_0 + \lambda^1 u_1 + \lambda^2 u_2)\right]$$

$$= \frac{1}{2!}\left[\frac{d^2}{d\lambda^2} f(\lambda^0 u_0 + \lambda^1 u_1 + \lambda^2 u_2)\right]$$

$$= \frac{1}{2!}\left[\frac{d^2}{d\lambda^2} f(\lambda^0 u_0 + f(\lambda^1 u_1) + f(\lambda^2 u_2)\right]$$

$$= \frac{1}{2!}\frac{d}{d\lambda}[0 + u_1 f^I(\lambda^I u_1) + 2\lambda u_2 f^I(\lambda^2 u_2)]$$

$$= \frac{1}{2!}[u_1^2 f^{II}(\lambda^I u_1) + 2\lambda u_2 2\lambda u_2 f^{II}(\lambda^2 u_2) + 2\lambda u_2 f^I(\lambda^2 u_2)]$$

$$= \frac{1}{2!}[u_1^2 f^{II}(\lambda^I u_1) + 4\lambda^2 u_2^2 f^{II}(\lambda^2 u_2) + 2\lambda u_2 f^I(\lambda^2 u_2)]$$

$$as\ \lambda \Rightarrow 0$$

$$= \frac{1}{2!}[u_1^2 f^{II}(\lambda^I u_1) + 2\lambda u_2 f^I(\lambda^2 u_2)]$$

$$= u_2 f^I(u_2) + \frac{u_1^2}{2!} f^{II}(u_0)$$

therefore $A_2 = u_2 f^I(u_2) + \frac{u_1^2}{2!} f^{II}(u_0)$

When n=3

$$A_3 = \frac{1}{3!}\left[\frac{d^3}{d\lambda^3} f(\textstyle\sum_{n=0}^{\infty} \lambda^k u_k)\right]$$

$$A_3 = \frac{1}{3!}\left[\frac{d^3}{d\lambda^3} f(\lambda^0 u_0 + \lambda^1 u_1 + \lambda^2 u_2 + \lambda^2 u_2)\right]$$

$$= \frac{1}{3!}\left[\frac{d^3}{d\lambda^3}(f(\lambda^0 u_0) + f(\lambda^1 u_1) + f(\lambda^2 u_2) + f(\lambda^3 u_3))\right]$$

$$= \frac{1}{3!}\left[\frac{d^2}{d\lambda^2}(u_1 f^I(\lambda u_1) + 2\lambda u_2 f^I(\lambda^2 u_2) + 3\lambda^2 u_3 f^I(\lambda^3 u_3))\right]$$

$$= \frac{1}{3!}\left[\frac{d^1}{d\lambda^1}(u^2 f^{II}(\lambda u_1) + 2u_2 f^I(\lambda^2 u_2) + 2\lambda u_2 2\lambda u_2 f^{II}(\lambda^2 u_2) + 6\lambda \ \ u_3 f^I(\lambda^3 u_3) + 3\lambda^2 u_3 \ 3\lambda^2 u_3 \ f^{II}(\lambda^3 u_3))\right]$$
$$= \frac{1}{3!}\left[\frac{d^1}{d\lambda^1}(u^2 f^{II}(\lambda u_1) + 2u_2 f^I(\lambda^2 u_2) + 4\lambda^2 u_2^2 f^{II}(\lambda^2 u_2) + 6\lambda \ \ u_3 f^I(\lambda^3 u_3) + 9\lambda^4 u_3^2 f^{II}(\lambda^3 u_3))\right]$$
$$= \frac{1}{3!}\left[(u^3 f^{III}(\lambda u_1) + 2u_2 f^I(\lambda^2 u_2) + 2\lambda u_2 \ 2\lambda u_2 f^{II}(\lambda^2 u_2) + 4u_2^3 u_3 f^{III}(\lambda^2 u_2) + 6\lambda u_3 \ 3\lambda^2 u_3 f^{II}(\lambda^3 u_3) \right.$$
$$\left. + 36\lambda^3 u_3^2 f^{II}(\lambda^3 u_3) + 9\lambda^4 u_3^2 3\lambda^2 u_3 f^{III}(\lambda^3 u_3)\right]$$

$$= \frac{1}{3!}\left[(u^3 f^{III}(\lambda u_1) + 2u_2 f^I(\lambda^2 u_2) + 4\lambda^2 u_2^2 f^{II}(\lambda^2 u_2) + 4u_2^3 u_3 f^{III}(\lambda^2 u_2) + 6u_3 \ f^I(\lambda^3 u_2) + 18\lambda^3 u_3^2 f^{II}(\lambda^3 u_3) \right.$$
$$\left. + 36\lambda^3 u_3^2 f^{II}(\lambda^3 u_3) + 27\lambda^6 u_3^3 f^{III}(\lambda^3 u_3)\right]$$

Appling the principle of $\lambda = 0$ after differentiating and the other condition of u

$$A_3 = u_1 f^I(u_0) + u_1 u_2 \frac{u_1^2}{2!} f^{II}(u_0) + \frac{u_1^3}{3!} f^{III}(u_0)$$

**The algorithm of Adomian Polynomial by Class I accelerated$(\mathbf{A_n^{(1)}})$**

The class 1 value for the decomposition parameters $P_1^{(1)}(m) = m, P_2^{(1)}(m) = \infty \ and \ P_3^{(1)}(m) = \infty$, where the parameterized partial sum $\mathrm{S_m^{(1)}}$ $(\sum A_n)$ define the accelerated Adomian polynomials $\mathrm{A_n^{(1)}}$

$$S_m^{(1)}(\sum A_n) = \sum_{k=1}^{n} \left\| \frac{1}{n!} \left( \sum_{m-0}^{\infty} U_v - U_\circ \right)^n \frac{\partial^n}{\partial U_\circ} f(U_\circ) \right\| \ v, m \ [1] \tag{2.9}$$

$$S_m^{(1)}(\sum A_n) = \sum_{n=0}^{\infty} \frac{S_m(\sum u_v) - U_\circ)^n}{n!} \ \frac{\partial^n}{\partial U_o^n} f(U_o) \tag{3.0}$$

We note that

$$(\sum u_v) = \sum_{v=0}^{\infty} (U_o) \ also \tag{3.1}$$

$$\frac{\partial^n}{\partial U_o^n} f(U_o) = f^n(U_o) \tag{3.2}$$

However the degenerate sums U and $f(u)$ are unique therefore

$$S_m^{(1)}\left(\sum^{\infty} A_n^{(1)}\right) = \phi_m^{(1)}[f(u)] = \phi_m^{(1)} = \sum_{n=o}^{\infty} A_n^{(1)} \tag{3.3}$$

$$\sum_{n=o}^{\infty} \frac{\sum_{v=1}^{\infty} U_v^{(1)}}{n!} f^n(U_o) = \sum_{n=o}^{\infty} \frac{\left(\phi_m^{(1)} - U_o\right)^n}{n!} f^n(U_o) = f\left(\phi_m^{(1)}\right) \tag{3.4}$$

We now emphasize the equality for $m \geq 1$

$a$nd

$$\phi_m^{(1)} = \sum_{n=o}^{\infty} \frac{\left(\phi_m^{(1)} - U_o\right)^n}{n!} f^n(U_o) = f\left(\phi_m^{(1)}\right) \tag{3.5}$$

this can now sequel to

$A_m^{(1)} = \phi_m^{(1)}$ . For $m \geq 1$ (3.6)

implies $A_m^{(1)} = \phi_{m+1}^{(1)} - \phi_m^{(1)}$ (3.7)

we have

$$\phi_m^{(1)} = f^n(U_o) = f\left(\phi_m^{(1)}\right) \tag{4.8}$$

$$\phi_2^{(1)} = \sum_{n=o}^{\infty} \frac{\left(U_m^{(1)}\right)^n}{n!} f^n(U_o) \tag{4.9}$$

$$\phi_3^{(1)} = \sum_{n=o}^{\infty} \frac{\left(U_m^{(1)} + U_m^{(2)}\right)^n}{n!} f^n(U_o) \tag{5.0}$$

$$\phi_4^{(1)} = \sum_{n=o}^{\infty} \frac{\left(U_m^{(1)} + U_m^{(2)} + U_m^{(3)}\right)^n}{n!} f^n(U_o) \tag{5.1}$$

$$\phi_5^{(1)} = \sum_{n=o}^{\infty} \frac{\left(U_m^{(1)} + U_m^{(2)} + U_m^{(3)} + U_m^{(4)}\right)^n}{n!} f^n(U_o) \tag{5.2}$$

Since

$$A_m^{(1)} = \phi_1^{(1)}\ for\ m \geq 1 \tag{5.3}$$

Therefore the General equation of Class 1 Adomian Polynomial Generation

$$A_m^{(1)} = \phi_1^{(1)} = \sum_{n=0}^{\infty} \frac{(\sum_{v-1}^{n-1} U_v)^n}{n!}\ f(U_o) \tag{5.4}$$

**THE GENERATION OF ADOMIAN POLYNOMIAL BY CLASS 1**

$At\ n = 1$

$$A_o = \phi_1^{(1)} = \sum_{n=0}^{\infty} \frac{(\sum_{v-1}^{0} U_v)^n}{n!}\ f(U_o) = f(U_o) \tag{5.5}$$

$At\ n = 2$

$$\emptyset_2 = \sum_{n=0}^{\infty} \frac{\left(U_m^{(1)}\right)^n}{n!} f^n(U_o) = \frac{U_1^o}{0!}\ f^{(o)}(U_o) = \ U_1\, f^{(2)}(U_o) + \frac{U_1^{(2)}}{2!} = U_1 f^{(o)}(U_o) \tag{5.6}$$

$At\ n = 3$

$$\emptyset_3^{(1)} = \sum_{n=0}^{\infty} \frac{(\sum_{v=2}^{2} U_v)^n}{n!} = \sum_{n=0}^{\infty} \frac{(U_1 + U_2)^n}{n!} f^n(U_o) \tag{5.7}$$

$$= \frac{(U_1 + U_2)^0}{0!} f^{(0)}(U_o) + \frac{(U_1 + U_2)^1}{1!} f^1(U_o) + \frac{(U_1 + U_2)^2}{2!} f^2(U_o) + \frac{(U_1 + U_2)^3}{3!}\ f^{(3)}\ (U_0) \tag{5.8}$$

$$\emptyset_3 = U_2\, f^{(1)}(U_o) + \frac{U_1^2}{2!} f^2(U_o) \tag{5.9}$$

$At\ n = 4$

$$\emptyset_4^{(1)} = \sum_{n=0}^{\infty} \frac{(\sum_{v=1}^{3} U_v)^n}{n!} = \sum_{n=0}^{\infty} \frac{(U_1 + U_2 + U_3)^n}{n!} f^n(U_o) \tag{6.0}$$

$$= \frac{(U_1 + U_2 + U_3)^0}{0!} f^n(U_o) + \frac{(U_1 + U_2 + U_3)^1}{1!}\ f^n(U_o) + \ \frac{(U_1 + U_2 + U_3)^2}{2!}\ f^n(U_o) + \frac{(U_1 + U_2 + U_3)^3}{3!}\ f^n(U_o) + \frac{(U_1 + U_2 + U_3)^4}{4!}\ f^n(U_o) \tag{6.1}$$

$$\emptyset_4 = U_3\, f^{(1)}(U_o) + U_1 U_2 f^2(U_o) + \frac{U_1^3}{3!} f^3(U_o) \tag{6.2}$$

$At\ n = 5$

$$\emptyset_5^{(1)} = \sum_{n=0}^{\infty} \frac{(\sum_{v=1}^{4} U_v)^n}{n!} = \sum_{n=0}^{\infty} \frac{(U_1 + U_2 + U_3 + U_4)^n}{n!} f^n(U_o) \tag{6.3}$$

$$\emptyset_5^{(1)} = U_4\, f^{(1)}(U_o) + \frac{U_2^2}{2!}\ f^2(U_o) + U_1 U_3 f^3(U_o) + \frac{U_1^4}{4!} f^4(U_o) \tag{6.4}$$

$$A_4 = U_4\, f^I(U_o) + \frac{U_2^2}{2!}\ f^{II}(U_o) + U_1 U_3 f^{III}(U_o) + \frac{U_1^4}{4!} f^{Iv}(U_o)$$

$$A_{n-1} = \emptyset_n \sum_{n=0}^{\infty} \frac{(\sum_{v=1}^{n-1} U_v)^n}{n!} \tag{6.5}$$

## Conclusion

The higher the order of differential in (2.8) makes the Generation of Adomian polynomial very complex, tedious, cumbersome to generate.

But, class I Acceleration algorithm creates a convenient, precise and fast method of generating the polynomial.

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