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An Economic Production Quantity (EPQ) Model for Delayed Deteriorating Items with Stock-Dependent Demand Rate and Time Dependent Deterioration Rate

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Abstract

In this paper, an economic production quantity (EPQ) model is presented for single product with delayed deterioration in which the production rate is constant, demand rate is inventory level dependent before and after production and the rate of deterioration is time dependent. The main objective is to determine the optimal replenishment cycle time such that the total variable cost is minimised. We present some numerical examples to illustrate the application of the model developed and use the examples to study the effect of various changes in some possible combinations of model parameters.

Keywords: Inventory level - dependent demand, delayed deterioration, variable deterioration rate.

1.0 Introduction

The basic assumptions of classical Economic Order Quantity (EOQ) model proposed by Harris in 1915 has over time been modified by considering more realistic factors in order to make the EOQ models correspond with reality. In many inventory models, it is often assumed that the demand rate is uniform. However, many marketing researchers and practitioners have long recognised that the demand of many retail items is proportional to the amount of inventory displayed. Whitin [1] observed, without empirical evidence that, 'An increase in inventories may bring about increased sales of some items'. Wolfe [2] presented empirical evidence of this relationship, noting that the sales of style merchandise, such as women's dresses or sport clothes, are proportional to the amount of inventory displayed. Levin *et al* [3] and Silver and Peterson [4] also observed that sales at the retail level tend to be proportional to inventory displayed and that a large pile of goods displayed in a supermarket will lead customers to buy more. These observations imply that in real life, the demand rate may be influenced by the stock levels. Gupta and Vrat [5] were the first to incorporate these observations into inventory models, developing an inventory model with stock-dependent consumption rate, which is a function of the initial stock level. Baker and Urban [6] developed an inventory model assuming the demand rate to be in a polynomial functional form and depending on the on-hand inventory. Mandal and Phaudjar [7] assumed the demand rate to be dependent linearly on the on-hand inventory at any instant time. Datta and Pal [8] discussed a model of inventory- level- depended rate in which the demand rate depends on inventory level down to a certain stock level and then it becomes constant for the rest of the cycle. They assumed that at the end of each cycle, the inventory level is zero. Urban [9] relaxed the terminal condition of zero-ending inventory and suggested that, in an inventory system with inventory-level-dependent demand rate, it would be more profitable to utilize higher inventory levels resulting in greater demand. Later on, Goh [10] discussed the model of Baker and Urban [6] relaxing the assumption of a constant holding cost. In the model, the author employed the holding cost as (i) a nonlinear function of the length of time the item is held in stock, and (ii) a nonlinear function of the amount of the on-hand inventory.

In many inventory models a general assumption is that products have indefinitely long lives. However, in many cases, items deteriorate over time. Often the rate of deterioration is so low that there is little need to consider the deterioration in the development of the model. Nevertheless, there are many products in the real world that are subject to a significant rate of deterioration. The loss due to deteriorating items affects the total average cost of a business and hence should not be neglected in the decision process of inventory modeling. Padmanabhan and Vrat [11] developed an inventory model for

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initial stock dependent consumption rate and exponential decay. Datta and Pal [8] extended work in [7]for deteriorating items with the assumption that the demand rate is a linear function of the on-hand inventory and allowing shortages, which are completely backlogged for both finite and infinite time horizons. Giri and Chaudhari [12] extended the work of Goh [10] to cover instantaneous deteriorating items where the rate of deterioration is a constant fraction of the on-hand inventory. Later on, inventory models of instantaneous deteriorating items with inventory-level-dependent demand were developed [11, 13 - 19].

In the literature of inventory systems, inventory models for deteriorating items mostly assume that deterioration starts as soon as the retailer receives the commodities. However, in real life, many goods would have a span of maintaining quality or the original condition for some period. That is during that period there is no deterioration occurring, and that phenomenon is termed as “delayed deterioration” in Musa and Sani [20] or “non - instantaneous deterioration” in Wu *et al* [21].

Wu *et al* [21] modified the model of Padmanabhan and Vrat [14] by considering a problem of determining the optimal replenishment policy for non-instantaneous deteriorating items with linearly stock dependent demand and constant unit holding cost. In the model, shortages are allowed and the backlogging rate is a variable and dependent on the waiting time for the next replenishment. Other inventory models using non-instantaneous deteriorating rate with linearly stock-dependent demand rate include those in [22, 23] who used linear functional form to represent stock-dependent demand rate. Mahata and Goswani [24] modified the model of Giri and Chaudhuri [12] by considering non-instantaneous deteriorating items with fussy deteriorating rate for power-form stock-dependent demand rate. Baraya and Sani [25] presented a deterministic inventory model for delayed deteriorating items with inventory level dependent demand rate.

Many authors studied in the area of production-inventory for deteriorating items.These authors include Misra [26] who first studied the EPQ model for deteriorating items with varying and constant rate of deterioration and others [27 - 31]. With the assumption that the demand rate, production rate, and deteriorating rate are all constant, Liao [32, 33] established a production-inventory model for deteriorating items under the condition of the supplier providing the retailer with trade credit. In all the models stated above, shortages are not allowed. However, Lin *et al*. [34] established a production-inventory model with constant production rate, demand rate and deteriorating rate that allowed for shortages. Zhou *et al*. [35] also considered production-inventory problem in which each cycle of a production-inventory schedule starts with replenishment and ends with a shortage. In the research of Sana *et al*. [36] and Zhou and Gu [37], shortages are allowed and occur at the end of a cycle.

All the deteriorating rates in the models stated above are constant but some authors studied situations with time-dependent deteriorating rates. Such authors include Skouri and Papachristos [38] and Chen *et al*. [29] who developed a production-inventory model in which storages are allowed at the beginning of the cycle. In contrast, Manna and Chaudhuri [39] and Balkhi's research [40], also studied production models in which shortages are also allowed but occur at the end of each cycle. Abad [41] studied the pricing and lot-sizing problem for deteriorating items under the conditions of finite production, partial backlogging, and lost sale. Teng *et al.* [42] extended Abad's model by adding the backlogging cost and the cost of lost goodwill. Tripathy *et al.* [43] obtained an EPQ model for linear deteriorating item with variable holding cost.

Mandal and Phaujdar [7] presented an EPQ model with a variable rate of deterioration and linearly dependent demand rate. Gupta and Agarwal [44] studied EPQ model in which production rate and demand rate during production remain constant and just as the production stops the demand rate becomes stock dependent. This continued up to a certain level of inventory and then the demand rate becomes constant for the rest of the cycle. Sarker *et al*. [45] developed an inventory model in which the demand is considered as a composite function consisting of a constant component, and a variable component which is proportional to the inventory level in the periods when there is a positive inventory buildup. The rate of production is considered finite and the decay rate as exponential. Teng and Chang [18] established an EPQ model for deteriorating items when the demand rate is dependent on both the stock level and the selling price per unit, and in which the deteriorating rate is a constant. Gary *et al.* [46] obtained production cycle time for an Economic Lot-Size Production (ELSP) with deteriorating items. Sugapriya and Jeyaraman [47] proposed an EPQ model for single product subject to non-instantaneous deterioration and in which the holding cost varies with time. Sugapriya and Jeyaraman [48] also developed an ELSP model for non- instantaneous deteriorating items using price discount and permissible delay in payments. Ruxian *et al.* [49] made a detailed review of deteriorating inventory study including production - inventory study. Baraya and Sani [50] proposed an economic production quantity (EPQ) model for delayed deteriorating items with stock-dependent demand rate and linear time dependent holding cost.

In this paper, we present an EPQ model for delayed deteriorating items with linear inventory level dependent demand rate while the deteriorating rate is a linear increasing function of time. The main difference between the model of Baraya and Sani [50] and this paper is that the former considers constant rate of deterioration while the later considers variable rate of deterioration. While the constant rate simplifies the determination of the control parameters, it cannot reflect the real situation of some deterioration phenomena. We develop an EPQ model for the inventory system and then present some numerical examples to illustrate the application of the model developed. We later use the examples to study the effect of various changes in some possible combinations of model parameters on the decision variables of the system.

2.0 Notation and Modelling Assumptions

The inventory system is developed on the basis of the following model assumptions and notation:

Notation:

λ uniform production rate per unit time

C_s set up cost per period

$\theta(t)$ $\theta(t) = \omega\, t,\ 0 < \theta(t) < 1$, variable time dependent deterioration rate

δ unit inventory holding cost per unit time

μ production unit cost per cycle

T optimal production cycle time

T_1 production run period

T_2 time during which there is no production of the product and deterioration immediately sets in

$q_1(t)$ inventory level for the product during the production period

$q_2(t)$ inventory level for the product during the period when there is no production

Q maximum inventory level

P production lot-size

TVC (T) total inventory cost per unit time.

Assumptions:

The following are the assumptions applied in the development of the model:

1 The demand rate for the product is linearly dependent on the inventory level
2 Production rate is known and constant and is greater than demand rate
3 Shortages are not allowed and replenishment is finite
4 Rate of inflation is constant
5 The time horizon of the inventory system is infinite. Only a typical planning schedule of cycle length is considered, all remaining cycles are identical
6 Once a unit of the product is produced, it is available to meet demand
7 Once the production is terminated the product starts deteriorating, thus delay in deterioration is during production period
8 There is no replacement or repair for a deteriorated item.

3.0 Model Development

The objective of the inventory problem here is to determine the optimal production quantity so as to keep the total relevant cost as low as possible.

At the start, t = 0, the inventory level is zero. The production and supply start simultaneously and the production stops at t = T_1 during which the maximum inventory Q is reached. The demand rate during production is assumed to be $\alpha + \beta q_1(t)$, where, $0 < \beta < 1$ and so the inventory built up is at the rate $\lambda - (\alpha + \beta q_1(t))$ in the interval $[0, T_1]$, and in that interval there is no deterioration. After the time T_1, the produced units start deteriorating. There is no fall in demand, when the inventory reduces to zero level and production run begins. The inventory level of the product at time t over period [0, *T*] can be represented by the following differential equations:

$$\frac{d}{dt}\big(q_1(t)\big) = \lambda - (\alpha + \beta q_1(t)),\ \ for\ 0 \le t \le T_1 \tag{1}$$

$$\frac{d}{dt}\big(q_2(t)\big) + \theta(t) q_2(t) = -(\alpha + \beta q_2(t)) \tag{2}$$

where $\theta(t) = \omega t$ with $0 < \omega < 1$ for $T_1 \le t \le T \Leftrightarrow 0 \le t \le T_2$

That is, the demand rate after production is $\alpha + \beta q_2(t)$ and the product has then started deteriorating at the rate of θ(t), i.e. the rate of deterioration increases linearly.

The pictorial representation of the model is given in Figure 1.

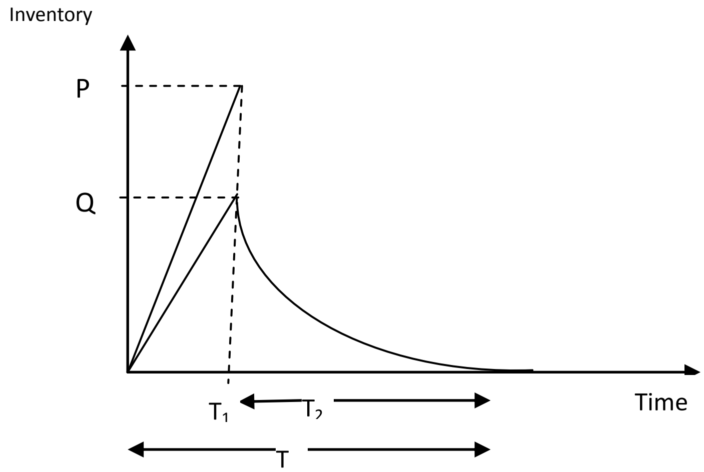

Figure1: The inventory system of the situation above

λ = constant production rate, $\alpha + \beta q_1(t)$ = demand rate during production period,

$\lambda - \left(\alpha + \beta q_1(t) \right)$ = rate of inventory built up, where $\lambda - \left(\alpha + \beta q_1(t) \right) > 0$

On integrating (1) with respect to t, we have

$$q_1 e^{\beta t} = \frac{(\lambda - \alpha) e^{\beta t}}{\beta} + k_1 \tag{3}$$

where k_1 is a constant of integration.

Using the boundary condition $q_1(0) = 0$ in equation (3), we have

$$q_1 e^{\beta t} = \frac{(\lambda - \alpha) e^{\beta t}}{\beta} - \frac{(\lambda - \alpha)}{\beta} \quad , \; 0 \le t \le T_1$$

and from which

$$q_1(t) = \frac{(\lambda - \alpha)}{\beta} \left(1 - e^{-\beta t} \right)$$

Using the Taylor series expansion for exponential function, we have

$$q_1(t) = \left(\frac{\lambda - \alpha}{\beta} \right) \left\{ 1 - \left(1 - \beta t + \frac{(\beta t)^2}{2} - \cdots \right) \right\}$$

Approximating the value by neglecting those terms in β t $\left(\text{since } 0 < \beta < 1 \right)$ of degree greater than 2, we have

$$q_1(t) = (\lambda - \alpha) \left(t - \frac{\beta t^2}{2} \right), \qquad 0 \le t \le T_1 \tag{4}$$

From (2) we have

$$\frac{d}{dt}\left(q_2(t) \right) + (\omega t + \beta) q_2(t) = -\alpha$$

Also integrating it with respect to t gives

$$q_2(t) \exp\left(\frac{\omega t^2}{2} + \beta t \right) = -\alpha \int \exp\left(\frac{\omega t^2}{2} + \beta t \right)$$

$$= -\alpha \int \left\{ 1 + \left(\frac{\omega t^2}{2} + \beta t \right) + \frac{1}{2!} \left(\frac{\omega t^2}{2} + \beta t \right)^2 + \ldots \right\} dt$$

Since $0 < \beta < 1$ and $0 < \omega < 1$, then neglecting all terms of $\beta\omega$ of degree greater than or equal to 1 in the Taylor`s series expansion, we have

$$q_2(t)\exp\left(\frac{\omega t^2}{2} + \beta t\right) = -\alpha \int\left\{ 1 + \beta t + \frac{\omega t^2}{2} \right\}dt$$

$$= -\alpha\left\{ t + \frac{\beta t^2}{2} + \frac{\omega t^3}{6} \right\} + k_2 \qquad (5)$$

where k_2 is a constant of integration.

And using the condition $q_2(T_2) = 0$ in (5) gives

$$k_2 = \alpha\left(T_2 + \frac{\beta T_2^{\,2}}{2} + \frac{\omega T_2^{\,3}}{6}\right)$$

and

$$q_2(t)\exp\left(\frac{\omega t^2}{2} + \beta t\right) = \alpha\left(T_2 + \frac{\beta T_2^{\,2}}{2} + \frac{\omega T_2^{\,3}}{6} - t - \frac{\beta t^2}{2} - \frac{\omega t^3}{6} \right)$$

Thus

$$q_2(t) = \alpha\left(T_2 + \frac{\beta T_2^{\,2}}{2} + \frac{\omega T_2^{\,3}}{6} - t - \frac{\beta t^2}{2} - \frac{\omega t^3}{6} \right)\exp\left(-\left(\frac{\omega t^2}{2} + \beta t\right)\right) \qquad (6)$$

The production cost per period is given by

$$\mathrm{PC} = \mu\lambda T_1 \qquad (7)$$

The holding cost per period is given by

$$\mathrm{HC} = \delta\int_0^{T_1} q_1(t)\,dt + \delta\int_0^{T_2} q_2(t)\,dt$$

$$= \delta\int_0^{T_1}(\lambda - \alpha)\left(t - \frac{\beta t^2}{2}\right)dt + \alpha\delta\int_0^{T_2}\left(T_2 + \frac{\beta T_2^{\,2}}{2} + \frac{\omega T_2^{\,3}}{6} - t - \frac{\beta t^2}{2} - \frac{\omega t^3}{6} \right)\exp\left(-\left(\frac{\omega t^2}{2} + \beta t\right)\right)dt$$

We take the first two terms from the Taylor`s series expansion of the exponential function, since $0 < \beta, \omega < 1$, to have

$$HC = \delta\int_0^{T_1}(\lambda - \alpha)\left(t - \frac{\beta t^2}{2}\right)dt + \alpha\delta\int_0^{T_2}\left(T_2 + \frac{\beta T_2^{\,2}}{2} + \frac{\omega T_2^{\,3}}{6} - t - \frac{\beta t^2}{2} - \frac{\omega t^3}{6} \right)\left(1 - \beta t - \frac{\omega t^2}{2}\right)dt$$

$$= \delta\int_0^{T_1}(\lambda - \alpha)\left(t - \frac{\beta t^2}{2}\right)dt + \alpha\delta\int_0^{T_2}\left(\xi_1(t) - \beta\xi_2(t) - \frac{\omega}{2}\xi_3(t)\right)dt \qquad (8)$$

where

$$\xi_1(t) = \left(T_2 + \frac{\beta T_2^{\,2}}{2} + \frac{\omega T_2^{\,3}}{6} - t - \frac{\beta t^2}{2} - \frac{\omega t^3}{6} \right)$$

$$\xi_2(t) = \left(T_2 t + \frac{\beta T_2^{\,2} t}{2} + \frac{\omega T_2^{\,3} t}{6} - t^2 - \frac{\beta t^3}{2} - \frac{\omega t^4}{6}\right)$$

$$\xi_3(t) = \left(T_2 t^2 + \frac{\beta T_2^{\,2} t^2}{2} + \frac{\omega T_2^{\,3}}{6}t^2 - t^3 - \frac{\beta t^4}{2} - \frac{\omega t^5}{6}\right)$$

Evaluating the integral (8) with respect to t, we have

$$HC = \delta(\lambda - \alpha)\left(\frac{T_1^{\,2}}{2} - \frac{\beta T_1^{\,3}}{6}\right) + \alpha\delta\left\{\left(\frac{T_2^{\,2}}{2} + \frac{\beta T_2^{\,3}}{3} + \frac{\omega T_2^{\,4}}{8}\right) - \beta\left(\frac{T_2^{\,3}}{6} + \frac{\beta T_2^{\,4}}{8} + \frac{\omega T_2^{\,5}}{20}\right)\right\}$$

$$- \frac{\alpha\delta\omega}{2}\left(\frac{T_2^{\,4}}{12} + \frac{\beta T_2^{\,5}}{15} + \frac{\omega T_2^{\,6}}{36}\right) \qquad (9)$$

The demand in the time period T_2 is given by

$$MD = \int_0^{T_2} (\alpha + \beta t)\, dt = \alpha T_2 + \frac{\beta T_2^2}{2} \tag{10}$$

The number of deteriorated items per cycle is given by

$$DP = q_2(0) - MD$$

$$= \alpha\left(T_2 + \frac{\beta T_2^2}{2} + \frac{\omega T_2^3}{6} \right) - \left(\alpha T_2 + \frac{\beta T_2^2}{2} \right)$$

$$= \frac{\alpha\omega T_2^3}{6} + \frac{\alpha\beta T_2^2}{2} - \frac{\beta T_2^2}{2} \tag{11}$$

The average total variable inventory cost per unit time is given by $TVC(T) = \frac{1}{T}\left(C_s + PC + \mu DP + HC\right)$

$$= \frac{C_s}{T} + \frac{1}{T}(\mu\lambda T_1) + \frac{\mu}{T}\left(\frac{\alpha\omega T_2^3}{6} + \frac{\alpha\beta T_2^2}{2} - \frac{\beta T_2^2}{2}\right) + \frac{1}{T}\left(\delta(\lambda - \alpha)\left(\frac{T_1^2}{2} - \frac{\beta T_1^3}{6}\right)\right)$$
$$+ \frac{\alpha\delta}{T}\left\{ \left(\frac{T_2^2}{2} + \frac{\beta T_2^3}{3} + \frac{\omega T_2^4}{8}\right) - \beta\left(\frac{T_2^3}{6} + \frac{\beta T_2^4}{8} + \frac{\omega T_2^5}{20}\right) - \frac{\omega}{2}\left(\frac{T_2^4}{12} + \frac{\beta T_2^5}{15} + \frac{\omega T_2^6}{36}\right)\right\} \tag{12}$$

To minimize the total variable cost per unit time TVC (T), we differentiate TVC (T) with respect to T and set the result to zero with $T_2 = T - T_1$ as follows:

$$\frac{d\left(TVC(T)\right)}{dT} = -\frac{C_s}{T^2} - \frac{\mu\lambda T_1}{T^2} + \frac{\mu}{T^2}\left\{ \frac{\alpha\omega}{6}\left(2T^3 - 3T^2T_1 + T_1^3\right) + \frac{\alpha\beta}{2}\left(T^2 - T_1^2\right) - \frac{\beta}{2}\left(T^2 - T_1^2\right)\right\}$$
$$- \frac{\delta(\lambda - \alpha)}{T^2}\left(\frac{T_1^2}{2} - \frac{\beta T_1^3}{6}\right)$$
$$+ \frac{\alpha\delta}{T^2}\left\{ \frac{T^2 - T_1^2}{2} + \frac{\beta}{3}\left(2T^3 - 3T^2T_1 + T_1^3\right) + \frac{\omega}{8}\left(3T^4 - 8T^3T_1 + 6T^2T_1^2 - T_1^4\right)\right\}$$
$$- \frac{\alpha\delta\beta}{T^2}\left\{ \frac{1}{6}\left(2T^3 - 3T^2T_1 + T_1^3\right) + \frac{\beta}{8}\left(3T^4 - 8T^3T_1 + 6T^2T_1^2 - T_1^4\right)\right\}$$
$$- \frac{\alpha\delta\beta\omega}{12T^2}\left(4T^5 - 15T^4T_1 + 20T^3T_1^2 - 10T^2T_1^3 + T_1^5\right)$$
$$- \frac{\alpha\delta\omega}{24T^2}\left(3T^4 - 8T^3T_1 + 6T^2T_1^2 - T_1^4\right)$$
$$- \frac{\alpha\delta\omega^2}{72T^2}\left(5T^6 - 24T^5T_1 + 45T^4T_1^2 - 40T^3T_1^3 + 15T^2T_1^4 - T_1^6\right) \tag{13}$$

Setting equation (13) to zero and multiplying both sides by $360T^2$, simplifying and rearranging the terms, yields

$$-360C_s - 360\mu\lambda T_1 + \mu\left\{60\alpha\omega\left(2T^3 - 3T^2T_1 + T_1^3\right) + 180\alpha\beta\left(T^2 - T_1^2\right) - 180\beta\left(T^2 - T_1^2\right)\right\}$$
$$- \delta(\lambda - \alpha)\left(180T_1^2 - 60\beta T_1^3\right).$$
$$+ \alpha\delta\left\{180\left(T^2 - T_1^2\right) + 120\beta\left(2T^3 - 3T^2T_1 + T_1^3\right) + 45\omega\left(3T^4 - 8T^3T_1 + 6T^2T_1^2 - T_1^4\right)\right\}$$
$$- \alpha\delta\beta\left\{60\left(2T^3 - 3T^2T_1 + T_1^3\right) + 45\beta\left(3T^4 - 8T^3T_1 + 6T^2T_1^2 - T_1^4\right)\right\}$$
$$- 30\alpha\delta\beta\omega\left(4T^5 - 15T^4T_1 + 20T^3T_1^2 - 10T^2T_1^3 + T_1^5\right)$$

$$-15\alpha\delta\omega\left(3T^4-8T^3T_1+6T^2T_1^2-T_1^4\right)$$

$$-5\alpha\delta\omega^2\left(5T^6-24T^5T_1+45T^4T_1^2-40T^3T_1^3+15T^2T_1^4-T_1^6\right) \quad = \quad 0$$

This further simplifies to

$$\psi_1T^6+\psi_2T^5+\psi_3T^4+\psi_4T^3+\psi_5T^2+\psi_6=0 \tag{14}$$

$\psi_1=-25\alpha\delta\omega^2$ where $\psi_2=120\alpha\delta\omega^2T_1-120\alpha\delta\beta\omega$

$$\psi_3=-225\alpha\delta\omega^2T_1^2+450\alpha\delta\beta\omega T_1-135\alpha\delta\beta^2+90\alpha\delta\omega$$

$$\psi_4=200\alpha\delta\omega^2T_1^3-600\alpha\delta\beta\omega T_1^2-240\alpha\delta\omega T_1+120\alpha\delta\beta+360\alpha\delta\beta^2T_1+120\mu\alpha\omega$$

$$\psi_5=-75\alpha\delta\omega^2T_1^4+300\alpha\delta\omega\beta T_1^3+180\alpha\delta\omega T_1^2{}_1-270\alpha\delta\beta^2T_1^2-180\alpha\delta\beta T_1$$
$$+180\alpha\delta-180\mu\alpha\omega T_1+180\mu\alpha\beta-180\mu\beta$$

$$\psi_6=5\alpha\delta\omega^2T_1^6-30\alpha\delta\beta\omega T_1^5-30\alpha\delta\omega T_1^4+45\alpha\delta\beta^2T_1^4-180\alpha\delta T_1^2+60\alpha\delta\beta T_1^3-360C_s$$
$$-360\mu\lambda T_1+60\mu\alpha\omega T_1^3-180\mu\alpha\beta T_1^2+180\mu\beta T_1^2-\delta(\lambda-\alpha)\left(180T_1^2-60\beta T_1^3\right)$$

The roots of equation (14) can be solved by any suitable numerical approximation method such as Newton-Raphson method when the values of the different parameters are prescribed. Such a positive root T* of equation (14) for which $\frac{d^2\left(TVC(T)\right)}{dT^2}>0$ gives a minimum for the total system cost. This is found to be so for all our examples below.

4.0 Numerical Examples

Four tables (Table 1, Table 2, Table 3 and Table 4) of examples are presented to illustrate the model developed and the effects of changes of some model parameters on the decision variables. From Table 1, an inventory system in the first example has the following values of parameters: $\alpha=90$, $\beta=0.7$, $\omega=0.05$, $\delta=2$, $\lambda=110$, C_s = ₦ 120 per order, μ = ₦ 10 per unit, $T_1=0.1$ time units, the optimum cycle length of time T* is determined by Newton-Raphson method using expression (13) as $T^*=0.74$ units and corresponding minimum total system cost per unit time is determined using the expression (12) as TVC* = ₦548.70. For other values of the parameters as reflected in the table, the optimum cycle length of time T* and the corresponding minimum total system cost per unit time TVC* are similarly determined. The results are tabulated in Table 1. Also, Table 2, Table 3 and Table 4 are similarly drawn up, with changes as indicated in the heading of the table. Note that in all examples given, the sufficient condition $\frac{d^2}{dT^2}\left(TVC(T)\right)>0$ is satisfied implying that T* so determined is indeed the minimum in the considered region.

Table 1: Effects of changing the stock-dependent demand rate β on the decision variables

S/No	α	β	ω	δ	λ	C_s	T_1	μ	T*	TVC*
1	90	0.7	0.05	2	110	120	0.1	10	0.74	548.70
2	90	0.5	0.05	2	110	120	0.1	10	0.83	492.69
3	90	0.3	0.05	2	110	120	0.1	10	0.97	425.65

Table 2: Effects of changing the deterioration rate ω on the decision variables

S/No	α	β	ω	δ	λ	C_s	T_1	μ	T*	TVC*
1	100	0.5	0.1	5	130	150	0.05	11	0.62	655.35
2	100	0.5	0.3	5	130	150	0.05	11	0.60	666.51
3	100	0.5	0.5	5	130	150	0.05	11	0.58	676.95

Table 3: Effects of changing the set up cost C_s on the decision variables

S/No	α	β	ω	δ	λ	C_s	T_1	μ	T*	TVC*
1	80	0.7	0.1	4	90	120	0.01	8	0.55	453.75
2	80	0.7	0.1	4	90	100	0.01	8	0.51	413.92
3	80	0.7	0.1	4	90	80	0.01	8	0.46	370.65

Table 4: Effects of changing the constant production rate λ on the decision variables

S/No	α	β	ω	δ	λ	C_s	T_1	μ	T*	TVC*
1	110	0.5	0.2	3	160	130	0.08	7	0.73	533.86
2	110	0.5	0.2	3	200	130	0.08	7	0.77	565.53
3	110	0.5	0.2	3	500	130	0.08	7	0.99	771.89

5.0 Results and Discussion

A careful study of the computational results as shown in Tables 1 - 4, and within the range of values of the chosen parameters, reveal the following observations:

(1) From Table 1, a higher value of β results in lower values of T*, but higher values of TVC*. This implies that increase in the stock-dependent demand rate will result in the decrease of optimal cycle, but higher values optimal total system cost per unit time. This is expected since if stock dependent demand rate is higher, stock will finish earlier and so T* will decrease. On the other hand, increasing the stock- dependent demand parameter β will effectively increase the total demand which, in turn, will increase the overall system cost per unit time.

(2) From Table 2, a higher value of ω results in lower values of T*, but higher values of TVC*. This implies that increase in deterioration rate will result in decrease in the optimal length. The total variable cost per unit time, however, increases in this case, which is also expected since when deterioration cost increases, the total variable cost per unit time will also increase. With deterioration rate higher however, stock will finish earlier resulting in lower T*.

(3) From Table 3, higher values of C_s result in higher values of T*and TVC*. In this case, it implies that increase in setup cost will result in the increase of optimal cycle length and the total variable cost per unit time. This is clearly expected since excess stocking is encouraged as a result of higher set up cost. The total variable cost per unit time is therefore expected to increase due to increase in stocking cost, and to avoid more frequency of ordering, T* increases.

(4) From Table 4, higher values of λ result in higher values of T*and TVC*. In this case, it implies that increase in production rate will result in the increase of optimal cycle length of time and the total variable cost per unit time. The total variable cost per unit time is also expected to increase due to increase in stocking cost and higher production cost. Clearly also T* should increase since much has been produced and so takes longer time to finish.

6.0 Conclusion and Suggestions

In this article, an economic production quantity (EPQ) model is presented for single product with delayed deterioration in which the production rate is constant, demand rate is inventory level dependent in a linear functional form before and after production and deteriorating rate is a linear increasing function of time. The rate of deterioration is a linear increasing function of time. An analytic formulation of the problem on the framework described above has been provided and also an optimal solution procedure. We present some numerical examples to illustrate the application of the model developed and use the examples to study the effect of various changes in some possible combinations of model parameters on the decision variables of the system. We find from the above results that the effects of changing the model parameters $\beta, \omega, C_{s.}$ and λ on the optimal replenishment policy reveal the following:

(1) When the stock-dependent consumption rate β is increasing, the optimal cost is increasing.

(2) When the deterioration rate ω is increasing, the optimal cost is increasing.

(3) When the set up cost C_s is increasing, the optimal cost is increasing.

(4) When the production rate λ is increasing, the optimal cost is increasing.

The model we have presented in this study provides a basis for several possible extensions. For future research, the model can be enriched by incorporating planned shortages, inflation and time value of money, variable holding cost, other forms of stock dependent demand rate function, etc.

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