

**Application of the theory of revealed preference to a special  
Problem in the maritime transport market**

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***Abstract***

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*The economic concept of “revealed preferences” suggests that individual behavior can be used to infer individual and collective preferences that would otherwise be known to be chaotic. This idea is used in this study to analyze specific problems in the maritime transport market, and using market forms to describe the structure of maritime transport.*

*The study therefore, presents an interpretation of economic modeling that can assist in judging future trends in the shipping market and in supply and demand models of the shipping market.*

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**Keywords:** Revealed Preference, Economic theory, Axiom, Ordinal utility, Neo classical model of choice, Utility Maximization, Reflexivity, Transitivity, Behavioral preference.

## **1.0 Introduction**

The theory of utility maximization implies a set of restrictions on the resulting Marshallian and Hicksian demand functions. However, parametric specification of functional forms is prone to error. An alternative is to look only at observed quantities without any reference to demand functions. The theory of revealed preference allows such a non-parametric approach to the study of choice.

The starting point for revealed preference analysis in modern economics was comparison of aggregate economics patterns. Classic work by Samuelson [1] and others [2,3] have argued that disagreements about the direction and magnitude of macroeconomic changes could be resolved by drawing inferences from the relative amounts of money allocated for goods and services.

Revealed preference theory was born with Samuelson’s [4] seminal paper. Unlike the traditional approach to consumer theory taking as primitive, an ordinal utility function, he directly started from a demand function. Thereby, he based the theory “upon those elements which must be taken as data by economic science” i.e. upon choice behavior, as opposed to utility judgments. Samuelson not only disposed of utility as a primitive of consumer theory, but completely removed the concept of utility from the theory. As he noted himself [4], such a purely behaviorist theory is silent about any notion of tastes or welfare.

The core meaning of the classical concept of revealed preference is retained. Individuals are assumed to act as if they were voting by allocating scarce resources over a finite set of choices [5,6]

The revealed preference hypothesis is considered as a major breakthrough in the theory of demand, because it has made possible the establishment of the law of demand directly (on the basis of revealed preference axiom) without the use of indifference curves and all of their restrictive assumptions. Regarding the ordering of consumer preference, the revealed preference has the advantage of not establishing the existence and convexity of the indifference curves (it does not accept them axiomatically).

## **2.0 The Revealed Preference Axiom**

The consumer by choosing a collection of goods in any one budget situation, reveals his preference for that particular collection. The chosen bundle is revealed to be preferred among all other alternative bundles available under the budget constraint. The chosen ‘basket of goods’ maximizes the utility of the consumer. The revealed preference for a particular collection of goods implies (axiomatically) the maximization of the utility of the consumer.

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### Types Of Axioms:

Definitions by Danan [7] of binary preference ordering that obeys the axioms of completeness, reflexivity, and transitivity;

$a \succ a'$  shall henceforth be interpreted as  $a$  is strictly preferred to  $a'$   
 $a \sim a'$  shall be henceforth be interpreted as  $a$  and  $a'$  are indifferent  
 $a \bowtie a'$  shall henceforth be interpreted as  $a$  and  $a'$  are incomparable  
 $a \succeq a'$  shall henceforth be interpreted as  $a$  is weakly preferred to  $a'$   
 we say that the binary relation  $\succeq$  is

(i). reflexive if  $\forall a \in A, a \sim a$

(ii). complete if  $\exists a, a' \in A$  such that  $a \bowtie a'$

(iii). transitive if  $\forall a, a', a'' \in A, a \succeq a' \succeq a'' \Rightarrow a \succeq a''$

Clearly, completeness implies reflexivity. Define the binary relations

$\succeq^+$  and  $\succeq^-$  on  $A$  by  $\succeq^+ = A \times A$ ,  $\succeq^- = \{(a, a) : a \in A\}$  i.e  $\succeq^+$  is the maximal (with respect to set inclusion) binary relation on  $A$  and  $\succeq^-$  is the minimal reflexive binary relation on  $A$ .

Conversly, a behavioral preference relation is a complete binary relation. A cognitive preference relation is a reflexive binary relation.

### Consumer Theory Is Based On Three Axiom of Choice

According to Andreoni & Miller [8] a binary preference ordering that obeys the axioms of completeness, reflexivity, and transitivity can be characterized by a utility function. If preferences are well behaved, that is they are convex and monotonic, then the utility function will generate smooth downward sloping demand curve.

## 3.0 Definition

### Directly Revealed Preferred

An allocation  $X$  is directly revealed preferred to a different allocation  $Y$  if  $Y$  was in the budget set when  $X$  was chosen. Then if a well-behaved utility function could have generated the data, the data will satisfy WARP.

### Weak axiom of revealed preference (WARP)

If allocation  $X$  is directly revealed preferred to  $Y$ , then  $Y$  cannot be directly revealed preferred to  $X$ .

Suppose  $X^0$  is chosen when prices are  $P^0$ , and  $X^1$  is chosen when prices are  $P^1$ . If

$P^1 X^1 \succeq P^1 X^0$ , we say that  $X^1$  is revealed preferred to  $X^0$ , and write  $X^1 RP X^0$ .

This definition is motivated by the fact that if  $P^1 X^1 \succeq P^1 X^0$ , the consumer can buy  $X^0$  at  $P^1$  and has  $X^1$  instead. Note that if  $P^1 X^1 \preceq P^1 X^0$ , then  $X^1$  is not revealed preferred to  $X^0$ , but that does not mean that  $X^0$  is revealed preferred to  $X^1$ . The condition for  $X^1 RP X^0$  is that.  $P^0 X^0 \succeq P^0 X^1$ .

In weak axiom, if  $X^1 RP X^0$ , then it is not the case that  $X^1 RP X^0$  (assuming they are distinct bundles of goods).

The weak axiom is equivalent to a compensated law of demand (i.e., it be equivalently stated in terms of the demand response to compensated price changes). Suppose that the function  $x(p, w)$  is homogeneous of degree one and satisfies Walras' law. Then  $x(p, w)$  satisfies the weak axiom of revealed preference if and only if the following property holds:

Consider any "compensated" price change from an initial price-wealth pair  $(p^1, w^1)$  to a new price-wealth pair  $(p^2, w^2)$ , where  $w^2 = p^2 \cdot x(p^1, w^1)$ . Then:

$(p^2 - p^1) \cdot [x(p^2, w^2) - x(p^1, w^1)] \leq 0$

With strict inequality whenever  $x(p^2, w^2) \neq x(p^1, w^1)$ .

WARP is a necessary condition on choices to be consistent with utility theory.

### Definition of Revealed Preferred:

If an allocation  $A$  is directly revealed preferred to  $B$ ,  $B$  is directly revealed preferred to  $C$ ,  $C$  is directly revealed preferred to  $Z$  and  $A$  and  $Z$  are not the same bundle, then  $A$  is revealed preferred to  $Z$ . That is, the revealed preferred relation is the transitive closure of the directly revealed preferred relation.

### Strong Axiom of Revealed Preferred (SARP):

If allocation  $X$  is revealed preferred to  $Y$ , then  $Y$  will never be revealed preferred to  $X$ . If preferences are strictly convex, then choice will conform to SARP. Moreover, if choice conforms to SARP then there exists a well-behaved preference ordering that could have generated the data. That is, utility theory is valid for the data observed.

SARP is a strong tool for the economist to use to verify that an individual's behavior is "rational". It is consistent with neoclassical choice theory. Mathematically, the strong axiom of revealed preferred states that if  $X^1 PR X^2$ ,  $X^2 RP X^3$ , .....  $X^n PR X$ , then it is not the case that  $X PR X^1$ . It is clear that utility maximization implies the strong axiom, but it can also be shown that if the observed data satisfy the strong axiom, then there exists a utility function such that maximization of this utility function will generate a choice behavior identical to the observed pattern. In other words, the strong axiom is a necessary and sufficient condition for utility maximization. See Afriat [9] and Varian [10]

Suppose the price vector changes from  $p$  to  $p + dp$  and the original demand bundle is  $x(p, m)$ . Under the new prices, the original demand bundle may no longer be affordable. But suppose money income is also changed from  $m$  to  $m + dm$ , such that

$$(p + dp) x(p, m) = m + dm = (p + dp) x(p + dp, m + dm) \quad (1)$$

Then at prices  $(p + dp)$  both  $x(p + dp, m + dm)$  and  $x(p, m)$  are affordable.

Therefore  $x(p + dp, m + dm)$  RP  $x(p, m)$  by the weak axiom, and this implies that  $x(p, m)$  is not revealed preferred to  $x(p + dp, m + dm)$ . Therefore we write

$$px(p, m) < px(x + dp, m + dm) \quad (2)$$

Subtracting the first condition from the second gives

$$-dp x(p, m) < -dp x(p + dp, m + dm). \quad (3)$$

Or equivalently  $dp(x(p + dp, m + dm) - x(p, m)) < 0$ .

### **Generalized Axiom Of Revealed Preference (garp):**

If an allocation  $X$  is revealed preferred to  $Y$ , then  $Y$  is never strictly directly revealed preferred to  $X$ . That is,  $X$  is never strictly within the budget set when  $Y$  is chosen.

Mathematically, let  $(q_1^0, q_2^0)$  be the quantity of goods 1 and 2 demanded at prices  $P_1^0, P_2^0$ . At prices  $P_1^1, P_2^1$  the quantity demanded is  $(q_1^1, q_2^1)$ .

Further let

$$P_0^0 q_0 = P_1^0 q_1^0 + P_2^0 q_2^0 \quad (4)$$

and

$$P_1^1 q_1 = P_1^1 q_1^0 + P_2^1 q_2^0 \quad (5)$$

be the expenditures on bundles  $(q_1^0, q_2^0)$  and  $(q_1^1, q_2^1)$  respectively, with  $P_0^0 q_0 \geq P_0^1 q_1$ , such that the  $q_0$  is revealed preferred to  $q_1$ , then  $P_1^1 q_1 \geq P_0^1 q_1$ , i.e.  $q_1$  is never revealed preferred to  $q_0$ , such that  $P_1^1 q_1 \geq P_0^1 q_1$ . Similarly, with  $P_1^1 q_0 \geq P_0^1 q_1$ ,  $q_1$  is revealed preferred to  $q_0$ , hence according to the revealed axiom,

$$P_1^1 q_0 < P_0^1 q_1, \quad (6)$$

### **In What Way Or Ways Can The Theory Of Revealed Preference Be Applied To A Specified Problem In The Maritime Transport Market?**

Consider the specific situation where a consumer is faced with a price-income vector  $(P^0, M^0)$  and chooses the consumption bundle  $X^0$ , where  $P^0 X^0 = M^0$ , that is, the budget equation is satisfied. In doing so, we shall say that the consumer reveals a preference for bundle  $X^0$  over some other bundle, say  $X^1$ , which was not chosen. We say  $X^0$  is revealed preferred to  $X^1$ . The phrase " $X^0$  revealed preferred to  $X^1$ " means that where the consumer was confronted with two affordable consumption bundles  $X^0$  and  $X^1$  and  $X^1$  was not chosen, it shows that  $X^1$  was more expensive than  $X^0$ .

It is not likely that we would be able to formulate a hypothesis about choices if the chosen bundle were less expensive than the non-chosen one. People choose to charter larger tonnages than smaller tonnages as a result of economics of scale. Size plays an important role in reducing shipping costs because in most cases, operating costs, voyage and capital costs do not increase in proportion to the deadweight of the vessel.

For example, a Very Large Crude Carrier (VLCC) of 280,000 dirty product tanker requires only the same number of crew as a 29,000 dirty product tanker, and fuel consumption per deadweight ton is substantially lower.

People choose larger tonnage instead of smaller tonnages to freight their limited quantity of cargoes, not necessarily because they prefer larger tonnages to smaller tonnages but because the later costs more. This statement that  $X^1$  is no more expensive than  $X^0$  is written.

$$P_1^1 q_0 < P_0^1 q_1$$

## **4.0 Discussion**

What value do consumers place on very large crude carrier vessel (VLCC) with 29 crew members when there is 29,000 gross ton dirty product tanker and fuel consumption per deadweight ton is substantially lower?

Increasingly, we have tackled this problem by asking consumers to state their preferences given a hypothetical set of alternatives.

The basic starting point for revealed preference analysis must take account of the logic of the choices that is implicit in a particular social situation and transaction data. It was not possible to test the revealed preference patterns described in the study, but the simplicity of our deductions provide opportunities for testing predictions about maritime transport market popularity as well as validating inferences about investing resources on chartering vessels to maximize expenditures.

According to Dowling [10], in maritime applications, the concept of level of service translated numerical volume / capacity ratios into letter grades that the general public and decision makers could more readily understand. As trends of time changes, level of services was switched from a measure of available capacity to an assessment of the facility user's degree of satisfaction with the service provided by the facilities.

*Journal of the Nigerian Association of Mathematical Physics Volume 22 (November, 2012), 341 – 344*

Consequently, maritime application is centered on

1. Identifying measures of performance regarding Throughput, Costs, Delay, and Safety.
2. Assessing user perception of performance with regard to delay and conflicts.
3. Developing methodologies to forecast performance and user satisfaction.
4. Conveying results to decision-makers.

The game plan for the development of a maritime capacity is the following four steps:

1. First identify those measures of maritime system performance that are important to system designers, system operators, decision makers and the maritime community.
2. Identify the subset of those system performance measures that best correlate with maritime community perceptions of quality of service.
3. Develop methodologies useful to maritime planners and designers for predicting the degree of maritime community satisfaction with the facility design and performance.
4. Establish a letter grade system for conveying the results of the analysis to decision makers and the general public.

Also, the revealed preference theory can be used to explain shipping lines patronages of ports or shippers choice of shipping lines.

### **Conclusion**

This work illustrates that not all “non-economic” behavior is beyond economic analysis. Our maintained assumption as economists is that individual behavior is consistent with self-interest. At its weakest, self-interest only means that choice conforms to some underlying preference ordering that is complete, reflexive and transitive, and hence, some utility function can be used to describe behavior.

However, the assumption of self-interest does not tell us what variables are in that utility function. What does? Our methodology dictates that people themselves, through their actions, will do so. What we have shown here is that unselfish behavior in experiments can indeed be captured by a model of self-interested agents, but that self-interested agents are not always money-maximizing. When we define the choice set appropriately, unselfish acts can be described and predicted with the standard neo-classical model of choice.

Consequently, the Revealed Preference has enabled us to deduce from the study, that, choice of very large crude carrier (VLCC) is synonymous with economics of scale, minimization of expenditure, and safety. Hence, the use of equation (6) of revealed axiom satisfies the account of the choice that is implicit in a particular social situation.

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