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Multi-objective optimal design of a three phase induction motor using geometric programming

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Abstract

This paper presents the use of Geometric Programming optimisation in the optimal design of a three phase induction motor. In order to get the best result we used multi-objective functions with the same constraints. Comparison is made between the independent variables and performance index obtained from these objective functions. The efficiency objective function has the best performance index with efficiency been 95%, power factor 0.87 and torque 47.8 N.m.

Keywords: Geometric Programming, Optimisation, Efficiency, Losses, Power factor Induction Motor, Variables, Constraints, Minimisation, Multi- objective functions, Design.

1.0 Introduction

Three phase induction motors are the most frequently used machines in various electrical drives. About 70% of all industrial loads on a utility are represented by induction motors [1].The wide applications of induction motors have led to the quest of improving its energy consumption, efficiency and power factor. In the design of induction motors, minimization of certain parameters leads to the production of better and efficient energy saving motors. The optimal design of an induction motor is mainly a proper sizing technique which is subject to a set of constraints. These constraints could be thermal, mechanical or users specifications [1]. Optimization is a science of determining the best solution to certain mathematically defined problems, which are often models of physical reality. Optimization involves the setting up of an, or a set of objective function(s) and making it or these to reach a maximum or minimum value(s) while keeping all variables within an acceptable limit or range. The objective function of an induction motor could be the efficiency [2-3], stator, rotor, iron losses or a combination of two or all the losses [4], cost of material [5] or the torque [6]. Several optimization techniques can be used in solving multi-objective functions derived from an induction motor subject to its stated constraints. Most of the expressions used in the formulation of the objective functions for induction motors are nonlinear and this has prompted some authors [4,7] to solve these objective functions using nonlinear optimization techniques. Some optimization techniques that have been applied in the design of induction motor are, the Genetic Algorithm [4,7-10], the Finite Element Method [11-12], the Particle Swarm optimization[13], and Simulated annealing[14]. All these aforementioned non linear optimization technique has been found to yield good results, and most of them can detect if the solution to the problem is feasible or infeasible at the early stage of the simulation process.

The aim of this paper is to use the Geometric Programming technique in the optimal design of a three phase induction motor using five objective functions namely the maximization of the Efficiency, minimization of the Stator Copper loss, Rotor Copper Loss, Stator Iron loss and the Cost of Material, with the improvement in efficiency, and power factor in mind. The Geometric Programming technique has been proved successful in the cost minimization of transformers [15] and

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synchronous motors [16]. A comparison is performed on these objective functions and the best of the five is sorted out. The condition for selecting the best is the improvement in efficiency, torque, power factor and reduction in cost of production. These are termed the performance index. The performance indexes are derived from the independent variables which are the sizing parameters.

The paper is organised as follows; section 2 sets out the optimization problem which includes the design variables and objective functions. Section 3 outlines a basic review on Geometric programming, section 4 deals with the design of an induction motor using GP, section 5 indicates the optimisation results and discussion, and we conclude in section 6.

2.0 Optimization Problem

2.1 Design Variables

The following quantities are chosen as the independent design variables for the multi-objective optimization: Stator bore diameter D , Stator tooth width W_{st}, Rotor tooth width W_{rt}, Depth of stator slot d_{ss} , Depth of stator core d_{sc} , Depth of rotor slot d_{rs} , air gap length l_g, Stator outer diameter D_o , Stator axial length L

2.2 Limits of Variables

The design carried out is for a 7.5 kW, three phase induction motor operating under a 380 Volts, 50 Hz power supply. The minimum and maximum value limits of the independent variables are as shown in table I.

Table I: Minimum and maximum limits of independent variables

	Values (m)	
	min	max
D	0.1200	0.1500
W_{st}	0.0045	0.0065
d_{ss}	0.0160	0.0190
W_{rt}	0.0035	0.0050
d_{rs}	0.0160	0.0185
d_{sc}	0.0280	0.0320
L	0.1400	0.1900
l_g	0.0035	0.0050
D_o	0.1800	0.2200

Table II: Nomenclature

Nomenclature	Meaning
ρ_{fe}	mass density of iron
K_{fe}	cost of iron per kg
ρ_{cu}	electrical resistivity of copper
K_{cu}	cost of copper conductor per kg
N	number of turns per phase
I	stator phase current
m	number of phase
J	stator current density
d_b	depth of rotor bar
w_b	width of rotor bar
S_r	number of rotor slots
a_e	cross section of end ring
D_e	mean diameter of end ring
S_s	number of stator slots
p	number of poles
L_b	length of rotor bar

2.3 Objective Function

In this work the following objective functions to be maximized or minimized are considered; 1) Material Cost C_M , 2) Stator Copper Loss W_{scu} , 3) Rotor Copper Loss W_{rcu} , 4) Stator Iron Loss W_{sfe} and 5) Efficiency of motor η .

2.3.1 Material Cost

The material cost C_M considered here are those that have direct bearing with the independent variables. These are the cost of iron C_{fe} and copper C_{cu}. The total cost of material is,

$$C_M = C_{fe} + C_{cu} \tag{1}$$

The cost of iron C_{fe} is given as,

$$C_{fe} = \rho_{fe} K_{fe} (L d_{cs} \pi (D + 2d_{ss} + d_{cs}) + S_s W_t d_{ss} L) \tag{2}$$

and the cost of copper C_{cu} (stator winding and rotor bars including end rings) is given as,

$$C_{cu} = \rho_{cu} K_{cu} \left(\frac{NIm}{J} \left(2L + \frac{2.3\pi D}{p} + 0.24 \right) + d_b w_b L_b S_r + 2a_e \pi D_e \right) \tag{3}$$

Table II contains all the symbols used in 1,2 and 3 and their corresponding meaning

2.3.2 Stator Copper Loss

The stator copper loss W_{scu} is given as,

$$W_{scu} = \rho_{cu} mNJI \left(2L + \frac{2.3\pi D}{p} + 0.24 \right) \tag{4}$$

2.3.3 Rotor Copper Loss

The rotor copper loss Rotor Copper Loss W_{rcu} is given as,

$$W_{rcu} = \rho_{cu} (2mk_w NI)^2 \left(\frac{L_b}{S_r w_b d_b} + \frac{2D_e}{\pi p^2 a_e} \right) \tag{5}$$

2.3.4 Stator Iron Loss

The stator iron loss W_{sfe} is given as,

$$W_{sfe} = \rho_{fe} \left(s_s W_{st} L d_{ss} k_1 f^{k_2} B_{st}^{k_3} + \pi (D + 2d_{ss} + d_{sc}) d_{sc} L k_1 B_{sc}^{k_3} f^{k_2} \right) \tag{6}$$

where, W_t is the width of stator teeth , k_1 core iron parameter , k_2 core iron constant , k_3 core iron constant, f is power utility frequency, B_{st} is the tooth flux density, B_{sc} is the stator core flux density.

2.3.5 Efficiency

The efficiency η of a machine is given as,

$$\eta = \frac{P_{out}}{P_{out} + W_{cu} + W_{sfe}} \tag{7}$$

The overall copper losses W_{cu} occurring in the stator and rotor slots of a three phase induction motor is given as

$$W_{cu} = \rho_{cu} \left(mNJ \left(2L + 2.3\pi \frac{D}{p} + 0.024 \right) I + (2mNk_w I)^2 \left(\frac{L_b}{s_s d_b w_b} + \frac{2D_e}{\pi p^2 a_e} \right) \right) \tag{8}$$

where, k_w is the winding factor.

$$\eta = \frac{P_{out}}{P_{out} + \rho_{cu} \left(mNJ \left(2L + 2.3\pi \frac{D}{p} + 0.024 \right) I + (2mNk_w I)^2 \left(\frac{L_b}{s_s d_b w_b} + \frac{2D_e}{\pi p^2 a_e} \right) \right) \cdots + \rho_{fe} \left(s_s W_{st} L d_{ss} k_1 f^{k_2} B_{st}^{k_3} + \pi (D + 2d_{ss} + d_{sc}) d_{sc} L k_1 B_{sc}^{k_3} f^{k_2} \right)} \tag{9}$$

2.4 Constraints

The following equality and inequality constraints shown in table III are imposed on the optimization problem. These constraints are so chosen so as to bring the best in the design. All the objective functions were subjected to these same constraints.

Table III: Constraints

constraints	
equality	inequality
$D/2p = dss$	$D + 2d_{ss} + 2d_{cs} \leq D_o$
$(0.95Wt\,Ss)/Sr = Wrt$	$1.15Ss\ Wt/\pi p \leq dcs$
$k\pi D/p = L$	$D_r + 2l_g \leq D$
$(\pi DBg)/(1.7Ss) = Wt$	$0.0002 + 0.002\sqrt{DL} \leq l_g$
$Pp/(2.22f\pi\text{^}2\ Bg\ D\text{^}2\ kw\ Q) = L$	$Q\pi D/(2mI) \leq N$

3.0 Overview of Geometric Programming

A geometric programme (GP) is an optimisation problem of the form [17-20],
minimise $f_0(x)$

subject to $f_i(x) \leq 1 \quad i = 1, \dots., m,$

$$g_i(x) = 1, \quad i = 1, \dots \dots, p. \tag{10}$$

where f_i are posynomial functions, g_i are monomials, and x_i are the optimisation variables.
A monomial function is defined as,

$$g_i(x) = c_i x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n} \tag{11}$$

where c is a positive real constant called the monomial coefficient, and $a_{1,\cdots,}a_n$ are real and may be negative or fractional constants that are referred to as the exponents of the monomial.
The sum of monomial functions is named a posynomial function; that is,

$$f_i(x) = \sum_{k=1}^{K} c_k x_1^{a_{1k}} x_2^{a_{2k}} \cdots x_n^{a_{nk}} \tag{12}$$

If a posynomial is multiply by a monomial the result is a posynomial, similarly a posynomial can be divided by a monomial with the result being a posynomial.
The geometric programming technique has many similarity to linear programming but has advantage over it in that, a non-linear objective function can be used, the constraints can be non-linear and the optimal value can be determined with a dual without first determining the specific value of the primal variables [22]

4.0 Design of Induction Motor using GP

In this design the geometric programming (GP) is used in the design of a three phase induction motor whose parameters are as presented in table IV. It is used in finding a set of design variables which ensure that the objective function $f_0(x)$ has a minimum or maximum value and all the constraint are satisfied.

Table IV: Motor specification and design constants

Motor specification		Design Constants	
Power rating (kW)	7.5	k_1	1.9×10^{-3}
Voltage (V)	380	k_2	1.24
frequency(f)	50	k_3	2
No. of poles	4	ρ_{cu}	$1.72 \times 10^{-3}\Omega - cm$
		ρ_{fe}	$4800kg/m^3$
		K_{fe}	1000/kg
		K_{cu}	500/kg

5.0 Results and Discussions

The results of the design using the multi-objective functions are shown in table V. From the table the following are noted. The independent variables obtained from the simulations are within the minimum and maximum limits. The size of the induction motor is almost the same as shown from the value of the stator outer diameter. The air gap length is within the limit apart from that obtained from W_{sfe}. The higher efficiencies and hence the low losses seen in C_m ,W_{rcu}, and η is due to a low depth of the rotor slots. In all the results of the independent variables obtained from the efficiency and rotor copper loss objective functions gives improved performance indexes. From the results obtained the efficiency objective function is the best of the five though the rotor loss objective function shows improve power factor and low cost of materials its computed torque is lower than the calculated torque (47.7 N.m)

Table V: Optimal design results with various objective functions

Variables/indices	C_M	W_{scu}	W_{rcu}	W_{sfe}	η
Independent Variables					
Stator bore diameter D (m)	0.1321	0.1306	0.1467	0.1321	0.1343
Stator outer diameter D_o (m)	0.2097	0.2152	0.2200	0.2106	0.2179
Stator tooth width W_{st} (m)	0.0021	0.0034	0.0029	0.0024	0.0034
Rotor tooth width W_{rt} (m)	0.0024	0.0030	0.0026	0.0021	0.0042
Depth of stator core d_{sc} (m)	0.0151	0.0233	0.0183	0.0165	0.0240
Depth of stator slot d_{ss} (m)	0.0165	0.0163	0.0183	0.0165	0.0168
Depth of rotor slot d_{rs} (m)	0.0081	0.0047	0.0183	0.0082	0.0108
Stator axial length L (m)	0.1557	0.1538	0.1728	0.1557	0.1582
Air gap length $l_g(m)$	0.0056	0.0051	0.0050	0.0056	0.0050
Dependent variables					
Stator current density $J_s(A/m^2)$	$2.7\text{x}10^6$	$1.2\text{x}10^6$	$2.68\text{x}10^6$	$2.68\text{x}10^6$	$1.2\text{x}10^6$
Rotor current density $J_r(A/m^2)$	$5.6\text{x}10^6$	$5.6\text{x}10^6$	$4.5\text{x}10^6$	$5.6\text{x}10^6$	$4.5\text{x}10^6$
Gap flux density $B_g(Wb/m^2)$	0.4716	0.5000	0.3796	0.3500	0.3857
Performance Index					
Full load efficiency	0.7898	0.8578	0.9373	0.8080	0.9476
Full load power factor	0.9990	0.9988	0.9371	0.9977	0.8694
Full load torque $(N.m)$	41.55	66.2	45.00	31.83	47.80
Rotor Losses (W)	1709	980.59	184.28	1451	123.5
Stator losses (W)	247.29	208.24	234.67	292.65	232.75
Iron losses (W)	41.09	54.39	57.97	37.98	58.300
Cost $(Naira)$	7458.5	11551	11198	8100.7	11279

6.0 Conclusion

The geometric programming optimisation process has been applied on five objective functions namely, Material Cost C_M , Stator Copper Loss W_{scu} , Rotor Copper Loss W_{rcu} , Stator Iron Loss W_{sfe} and Efficiency of motor η in the design of a three phase induction motor with the task of maximizing the efficiency and power factor of the motor. The optimization leads to the determination of the stator and rotor geometry under certain constraining conditions. The results of the stator and rotor geometry variables obtained from the optimization are within the specified limits. As shown in Table IV, the independent variables and performance indexes obtained from the efficiency objective function are the best.

References

[1] Giampaolo, L ., Stefano, L., Francesco, P and Marco, W (2003) "Multiobjective Optimization Techniques for the Design of Induction Motors" IEEE Transaction on Magnetics, Vol.39, No.3, pp.1261-1264

[2]Millie Pant, Radha Thangaraj and Singh, V.P (2008) "Efficiency optimization of electric motors: a comparative study of stochastic algorithms" ISSN 1 746-7233, England, UK World Journal of Modelling and Simulation, Vol.4, No.2, pp. 140-148.

[3] Khalil B, Mohammad, B.B and Jafar Mohammadi (2005) "Induction Motor Efficiency Estimation using Genetic Algorithm" WA of Science, Engineering and Technology 3
[4]] Mehmet Cunkas and Ramazan Akkaya (2006) "Design Optimization of Induction Motor By Genetic Algorithm and Comparison with Existing Motor" Mathematical and Computation Applications, Vol.11, No.3, pp. 193-203.
[5] Mendes, E, Baba, A and Razek, A (1995) "Losses minimization of a field oriented controlled induction machine" Proc. 7th Int. Conf. on Electrical machines and Drives, Durham, UK, 310-314.
[6] Pillay, P, Nolan, R and Hague T (1997) "Application of Genetic Algorithms to Motor Parameter Determination for Transient Torque Calculations" IEEE Trans. Industry Application, 33 (5), 1273-1282.
[7] Sivaraju, S.S and Devarajan, N (2011) "Novel Design of Three Phase Induction Motor Enhancing Efficiency, Maximizing Power Factor and Minimizing Losses" European Journal of Scientific Research, Vol. 58 No.3 pp.423-432.
[8] Subramanian, R, Sivanandam, S.N, and Vimalarani C (2010) "An Optimization of Design for S4-Duty Induction Motor Using Constraints Normalization Based Violation Technique" Journal of Computer Science 6(2): 107-111.
[9] Hassan Ibrahim and Mostafa Metwaly (2011) "Genetic Algorithm Based Performance Analysis of Self Excited Induction Generator" Engineering, 3, 859-864 www.SciRP.org/journal/eng
[10]Sujitjorn, S and Areerak, K.L (2002) "Loss Minimization in an Induction Motor Driven by a Voltage-Source-Inverter" Asian J. Energy., Vol. 3, Issues 1-2, pp. 53-78.
[11]Feyzi, M.R and Kalankesh, H.V (2001) "Optimization of induction Motor Design by using the Finite Element Method" IEEE Canadian Conference on Electrical and Computer Engineering, Toronto, May 2001.
[12] Arkkio, A (1990) "Finite Element Analysis of Cage Induction Motors fed by Static Frequency Converters" IEEE Transaction on Magnetics, Vol.26, No.2, pp. 551-554.
[13] Sakthivel, V.P, Bhuvaneswari, R, and Subramanian, S (2010) "Economic Design of Three-phase Induction Motor by Particle Swarm Optimization" J.Electromagnetic Analysis and Applications 2:301-310, www.sciRP.org/journal/jemaa
[14]Kannan, R, Subramanian, S and Bhuvaneswari, R (2010) "Multiobjective Optimal Design of Three - Phase Induction Generator Using Simulated Annealing Technique" IJEST, Vol. 2(5), pp. 1359-1369
[15] Jabr, R.A (2005) "Application of geometric programming to transformer design" Magnetics, IEEE Transactions on, vol.41, No.11, pp.4261-4269,
[16]Candela, C, Morin, M, Blazquez, F and Platero, C.A (2008) "Optimal design of a salient poles permanent magnet synchonous motor using geometric programming and finite element method" Electrical Machines, ICEM 2008. 18th International Conference on Electric Machines, pp.1-9
[17] Dembo, R.S. and Avriel, M. (1978) "Optimal design of membrane separation process using geometric programming" Math. Prog., Vol. 15, pp. 12-25
[18] Boyd, S and Vandenberghe, L (2004) "Convex Optimization" Cambridge University Press.
[19]Boyd, S and etal (2007) "A tutorial on geometric programming" Optim Eng (2007) 8:67-127
[20] Islam, S and Roy, T.K (2005) "Modified Geometric Programming Problem and its Applications" J.Appl. Math. & Computing Vol.17 No.1-2,pp.121-144
[21] Fletcher R. (2006) "Practical Method of Optimization" second edition, John Wiley & Sons Ltd, England,pp.339-344.