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Modification of Richardson-Dushmaan Equation for the Figure of Merit of a Thermionic Energy Converter Using Pure Tungsten and Tantalum as the Emitters

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Abstract

In this paper, a comparative analysis of maximum conversion efficiencies of thermionic converters of heat to electricity made of pure Tungsten, W and that made of Tantalum, Ta metals as cathodes were done in terms of the potential difference between the top potential barrier in the inter electrode space and the Fermi level of the emitter, V_E , the potential drop across a load impedance connected in series to the converter, V_L and the potential drop to the necessary electrical connection to the collector, V_C. An expression for the maximum conversion efficiency has been developed, which yields optimum values of load impedance, collector lead geometry and emitter work function in terms of collector voltage, emitter temperature, effective emitter emmissivity using the theoretical Dushmann constant, A. The constant depends on the work functions of the emitting metal surfaces, which experimentally varies from one metal to another. The modification of the Richardson equation was done based on this variation. The results show that low value of collector voltage is required for a high efficiency. Low radiation heat loss is required for a high conversion efficiency and relatively low values of emitter work function are required for maximum conversion efficiency at ordinary emitter temperature. A constant deviation of 4% from the theoretical efficiency was observed. Also, the maximum conversion efficiency of Tantalum, Ta metal is higher than that of Pure Tungsten, W.

Keywords: Thermionic converters, emitter, potential drop, Richardson-Dushmann constant.

1.0 Introduction

Thermionic energy converter is a device that converts heat energy directly into electrical energy by utilizing thermionic emission. It consists of a thermionic cathode emitting electrons to an anode whose Fermi level is more negative than that of the cathode. Useful electrical power can be extracted by connecting a resistive electrical load between the cathode and the anode.

The process of converting thermal energy (heat) to a useful electrical work by the phenomenon of thermionic emission is the fundamental concept applied to a cylindrical version of the planner converter, considered as the building block for space nuclear power system (SNPS) at any power level. Space nuclear reactors based on this process can produce electrical power ranging 5kWh to 5MWh. This spectrum serves the need of current users such as National Aeronautic and Space Administration (NASA) [1]. Moreover, electrical power in this range is currently being considered for commercial telecommunications satellites, navigation, propulsion and planetary exploration mission to mention a few [2, 3].

Several attempts on the direct conversion of heat to electricity have been published [4 -8]. But all these employ the use of the theoretically assumed values of the Richardson-Dushmaan constant, A; in their analysis. However, it has been found experimentally that, A, vary from material to material as in Culp [9]. Using this fact the Richardson-Dushmaan equations were reviewed considering Molybdenum as the cathode (emitter) [10, 11]. The results obtained indicated serious deviation of the conversion efficiency from the theoretical results. The emission properties of some typical materials used are presented in Table 1.

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Table 1: Thermionic Emission properties of some materials [9]

Materials	**ϕ, (eV)**	**A (A/m^2K^2)**
Cs	1.89	$0.5x10^6$
Mo	4.20	$0.55x10^6$
Ni	4.61	$0.30x10^6$
Pt	5.32	$0.32x10^6$
Ta	4.19	$0.55x10^6$
W	4.52	$0.6x10^6$
W+Cs	1.50	$0.03x10^6$
W+Ba	1.60	$0.015x10^6$
W+Th	2.70	$0.04x10^6$
BaO	1.50	$0.001x10^6$
SrO	2.20	$1.00x10^6$

The analyses in the existing work use both the practically obtained A value as presented by Culp [9] and the theoretically obtained value, for realistic results and hence the expected efficiency of the thermionic converters.

In the operation of the thermionic converter, electrons "*boil-off*" from the emitter material surface in a refractory metal such as tungsten, when heated to high temperatures (1600K-2000K). The electrons then traverse the small inter electrode gap, to a colder (800K-1000K) collector where they condense, producing an output voltage that drives the current through the electrical load and back to the emitter [12], (See Fig. 1). The flow of electrons through the electrical load is sustained by the temperature difference and the difference in surface work functions ϕ of the electrodes [3].

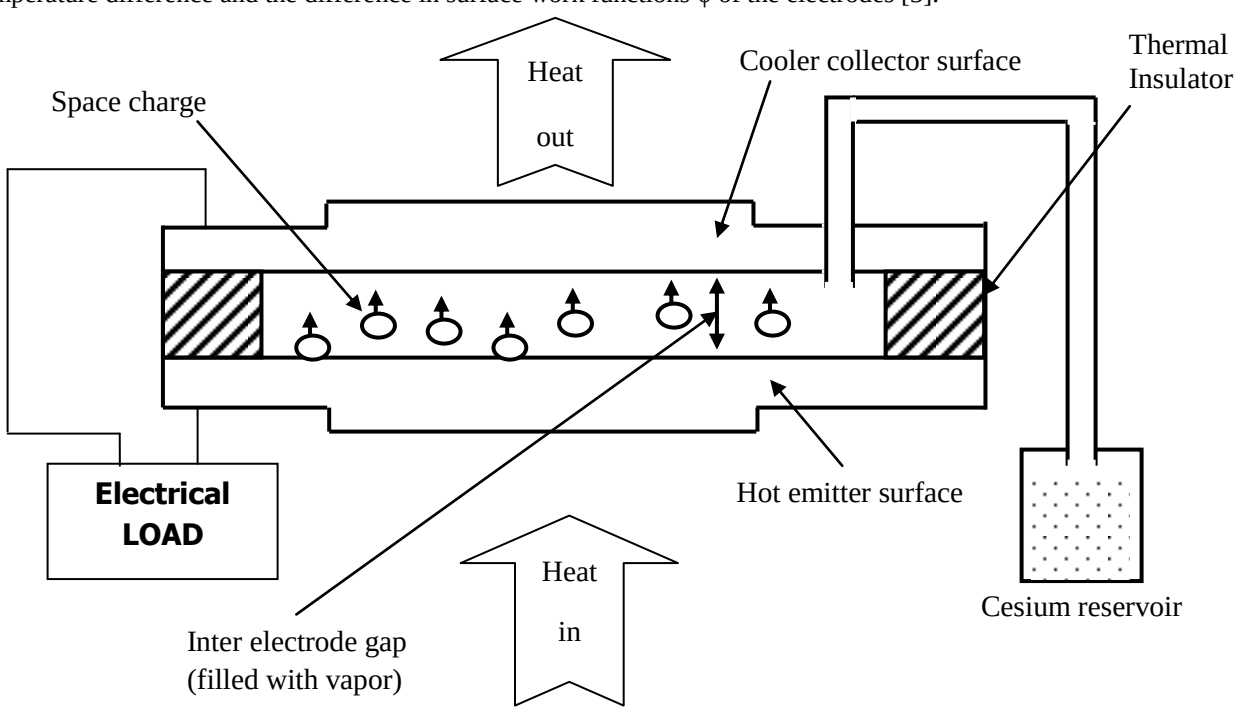

Fig. 1: Schematic diagram of an elementary Thermionic converter

Operating regime

Emitter temperature: 1600K – 2000K Emitter material: W or Ta metal
Collector temperature: 800K – 1000K Collector material: also W or Ta metal
Insulator: Al_2O_3, Al_2O_3/Nb Electrode atmosphere: Cs at 1.0 Torr

2.0 Methods and Theoretical development

The converter output voltage

If we designate the work function of the emitter (cathode) as ϕ_E and that for the collector (anode) as ϕ_C, then the total output voltage is, V_{out}, is

$$V_{\text{out}} = \phi_E - \phi_C \tag{1}$$

where V_{out} signifies the voltage across the load and the leads applied between the emitter and the collector.

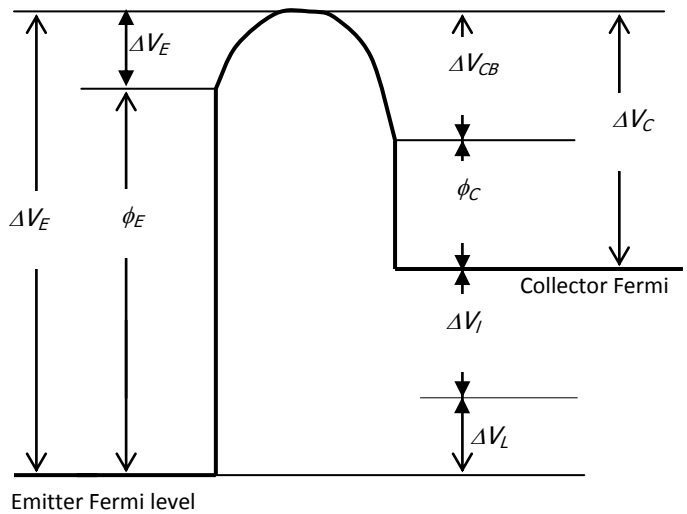

Fig. 2: Potential diagram of a thermionic vacuum diode

Note that as long as $V_{out} + \phi_C < \phi_E$, the barrier to electron flow is ϕ_E and the current is independent of the thermionic device voltage which is called saturation current, j given by [12] & [13].

$$j = AT_E^2 \exp\left(-\frac{\phi_E}{k_B T_E}\right) \tag{2}$$

Where, T_E, is the emitter temperature, ϕ_E is the emitter work function, k_B is the Boltzmann constant and A is the Richardson-Dushmann constant. However, when $V_{out} + \phi_C > \phi_E$, then the barrier is $V_{out}+\phi_C$ and any increase in V_{out} will reduce j.

Figure 2 shows the potential diagram used in this work, where subscripts E and C denote emitter and collector respectively. And ϕ denotes work function, V_E the potential difference. But the top of the potential barrier and the Fermi level of the emitter is seen to be equal to $\Delta V_c + \Delta V_L + \Delta V_l$ which is the voltage across the collector, load and the leads. The net current density in the system is equal to $j_E - j_C$, which gets over the potential barrier, j_E and j_C are given by the Richardson-Dushmann equation as

$$j_E = AT_E^2 \exp\left[-\left(\frac{e\Delta V_E}{k_B T_E}\right)\right] \tag{3}$$

$$j_C = AT_C^2 \exp\left[-\left(\frac{e\Delta V_C}{k_B T_C}\right)\right] \tag{4}$$

The effect of space charge

Once the electron cloud builds up between the electrodes, the flow of the electrons from the emitter is retarded by an additional potential, ΔV_{EB} (symbolising emitter barrier voltage). Adding in the voltage loss across the leads ΔV_l and the voltage loss across the load, ΔV_L as in Fig. 2 gives

$$j = AT_E^2 \exp\left[-\left(\frac{\phi_C + \Delta V_{CB} + \Delta V_l + \Delta V_L}{k_B T_E}\right)\right] \tag{5}$$

where V_{CB} is the collector barrier voltage, V_{EB} is the emitter barrier voltage, V_l is the lead voltage and V_L is the load voltage.

Note that in Thermionics, large current requires small work function, and large ΔV_{EB} ($V_{out} \equiv \phi_E - \phi_C$) requires large work function.

Efficiency computation

Efficiency is defined as the useful electrical power output per unit area of the emitter divided by the power input per unit area of the emitter.

$$\eta = \frac{power\ output / unit\ area\ of\ emitter}{power\ input / unit\ area\ of\ emitter} \times 100\% \tag{6}$$

The case of practical interest, of course, is that for $j_C << j_E$, otherwise there would be negligible power output from the device. Therefore, the useful electrical power output is given by $(j_E - j_C)\ V_L \approx jV_L$, where $j \equiv j_E - j_C$ is the current density. This work would be restricted to the case for which the above condition applies.

Consider equations (3) and (4), when $j_C << j_E$ then

$$\left(\frac{\theta_C}{\theta_E}\right)^2 \exp\left[\left(\frac{\Delta V_E}{\theta_E}\right) - \left(\frac{\Delta V_C}{\theta_C}\right)\right] << 1 \tag{7}$$

where $\theta_i \equiv k_B T_i / e$, and the subscript i could be emitter, E, or collector, C. For practical purposes therefore, the neglect of j_C in comparison with j_E in the following analysis is justified.

In the steady state, the heat input to the emitter is expected to be equal to the heat loss from the emitter.

The heat loss from the emitter consists of mainly three terms, which are as follows: - (i) Electron emission cooling term, P_e (W/cm^2) which is the sum of the potential energy, P.E imparted to the electrons and the kinetic energy, K.E at the emitter temperature, (ii) radiation heat losses, P_r (W/cm^2) radiated from the hot emitter, and (iii) heat conduction and I^2R losses, P_l (W/cm^2) conducted away from the emitter through the electrical connections. In the case of the gas-filled converter there is an additional loss P_g due to the conduction of heat in the gas. However, this term is probably very small and it has been neglected in this analysis.

(a) Electron emission cooling term, P_e

Only those electrons emitted from the emitter with an x- component of velocity greater than $\left[2\left(\frac{e}{m}\right)\left(\Delta V_E - \phi_E\right)\right]^{1/2}$ can get over the potential barrier ($\Delta V_E - \phi_E$) to the anode, and each such electron takes away from the cathode (emitter) an energy equal to $e\phi + \frac{m}{2}\left(u^2 + v^2 + w^2\right)$ where m is the electronic mass; u, v and w are the x, y and z components of velocity, respectively. Then if, n, is the total number of electrons per unit volume just outside the emitter, the total energy taken away from the emitter per unit area is given as

$$P_e = \int_a^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} nu\left[e\phi + \frac{m}{2}\cup^2\right]\left(\frac{m}{2\pi k_B T_E}\right) \times \exp\left(-\frac{m}{2k_B T_E}\cup^2\right) du\, dv\, dw \tag{8}$$

where

$$a^2 \equiv \sqrt{2(e/m)(\Delta V_E - \phi_E)}\ and\ \cup^2 \equiv u^2 + v^2 + w^2. \tag{9}$$

Thus, the electron emission cooling term is

$$P_e = j_n\left(\Delta V_E + \frac{2k_B T_E}{e}\right) \tag{10}$$

But $\Delta V_E = \Delta V_L + \Delta V_l + \Delta V_C$ *and* $\Delta V_l = j_n A_E R_l$ (Fig. 2) Therefore, we get

$$P_e = j_n\left(\Delta V_L + j_n A_E R_l + \Delta V_C + \frac{2k_B T_E}{e}\right) \tag{11}$$

There is another term in (11) which accounts for the energy received by the cathode from the electrons emitted from the anode which gets over the potential barrier. But for $j_C << j_E$ this term is negligible.

(b) Radiation loss term, P_r

This term is given by

$$P_r = \sigma\left[T_E^4 - T_C^4\right]\left\{\left(\frac{1}{\varepsilon_E}\right) + \left(\frac{1}{\varepsilon_C}\right) - 1\right\}^{-1} \tag{12}$$

where ε_E is the emissivity of the emitter, ε_C is the emissivity of the collector and σ is the Stefan-Boltzmann constant. It should be noticed that the above equation shows that using materials with low emissivities can reduce heat loss.

(c) Heat conduction and thermal losses, P_l

i) Conduction loss, P_k

Heat loss due to conduction is given by

$$P_k = \frac{K_l A_l}{A_E}\left(\frac{T_E - T_L}{l}\right) \tag{13}$$

where A_E is the surface area of the emitter, A_l is the crossectional area of the lead, K_l is the conductivity of the lead and l is the length of the lead.

From the definition of resistivity, ρ the length of the lead, l is given by

$$l = \frac{R_l A_l}{\rho_l} \tag{14}$$

Therefore, a useful expression for P_k is obtained as

$$P_k = \frac{K_l \rho_l}{A_E}\left(\frac{T_E - T_L}{R_l}\right) \tag{15}$$

However, from the Wideman – Franz law, one gets

$$\rho_l K_l = \left(\frac{\pi^2}{6}\right)\left(\frac{k_B}{e}\right)^2 \left(T_E + T_L\right) \tag{16}$$

where $T_l = \left(\frac{T_E + T_L}{2}\right)$, which leads to

$$P_K = \left[\frac{1}{A_E}\right]\left(\frac{\pi^2}{6}\right)\left(\frac{k_B}{e}\right)^2 \left(T_E^2 - T_L^2\right) \tag{17}$$

ii) Thermal Loss, P_j (Joule heating):

This is given by [13] as:

$$P_j = \left[\frac{1}{A_E}\right]\left(j_n A_E\right)^2 R_l \tag{18}$$

Assuming that half of the loss flows towards the cathode, then

$$P_j = \frac{1}{2}\left[\frac{1}{A_E}\right]\left(j_n A_E\right)^2 R_l \tag{19}$$

The combined loss ($P_k + P_j$)

The combined loss for the (i) and (ii) above is written thus

$$P_l = \left\{\frac{1}{A_E}\right\}\sum\left[\frac{\pi^2}{6A_E R_l}\left(\frac{k_B}{e}\right)^2\left(T_E^2 - T_L^2\right) - \frac{1}{2}\left(j_n A_E\right)^2 R_l\right] \tag{20}$$

The efficiency of the converter (diode), η, is

$$\eta = \frac{P_L}{P_e + P_r + P_l} \times 100\% \tag{21}$$

where $P_L = j_n \Delta V_L$ (useful load/unit area of emitter).

Substituting the results for P_e, P_r and P_l into (21) gives

$$\eta = \frac{j_n \Delta V_L}{j_n\left(\Delta V_C + \Delta V_L + \Delta V_l\right) + 2 j_n \theta_E + P_r + P_l} \tag{22}$$

where $\theta_E \equiv k_B T_E/e$ has been used. Dividing the numerator and the denominator of the right hand side of the above equation by $j_n\theta_E$ and noting that $V_i = j_n A_E R_l$ we can write the efficiency as

$$\eta = \frac{\psi_L}{\psi_L + \psi_C + 2 + \left(P_r / j_n \theta_E\right) + \tfrac{1}{2}\left(\pi^2/3\psi\right)\psi_l} \tag{23}$$

where $\psi_i = V_i/\theta_E$, θ_C^2 has been neglected compared with θ_E^2 and j_n is given by

$$j_n = j_o \exp\left(-\psi_C - \psi_L - \psi_l\right) \tag{24}$$

where $j_o \equiv A(e/k_B)^2\theta_E^2$. According to (23) the efficiency can be interpreted as the ratio of power delivered to the load to the sum of powers delivered to the load and the anode (collector).

In optimizing ψ_L and ψ_l (i.e. V_L and V_l) , it is convenient to work with the reciprocal of the efficiency, which from (23) is

$$\frac{1}{\eta} = 1 + \frac{1}{\psi_L}\left[\psi_C + 2 + \frac{P_r}{j_n\theta_E} + \frac{\pi^2}{3\psi_l} + \frac{1}{2}\psi_l\right] \tag{25}$$

where ψ_C, θ_E and P_r are constant parameters. For η to be maximum (i.e. 1/η to be minimum) it is required that:

$$\frac{\partial\left(\frac{1}{\eta}\right)}{\partial\psi_l} = -\frac{P_r}{j_n^2\theta_E}\frac{\partial j_n}{\partial\psi_l} - \frac{\pi^2}{3\psi_l^2} + \frac{1}{2} = 0 \tag{26}$$

$$\frac{\partial\left(\frac{1}{\eta}\right)}{\partial\psi_L} = -\frac{1}{\psi_L}\frac{P_r}{j_n^2\theta_E}\frac{\partial j_n}{\partial\psi_L} - \frac{1}{\psi_l^2}\left(\psi_C + 2 + \frac{P_r}{j_n\theta_E} + \frac{\pi^2}{3\psi_l} + \frac{1}{2}\psi_l\right) = 0 \tag{27}$$

and from (24) one gets

$$\frac{\partial j_n}{\partial\psi_l} = \frac{\partial j_n}{\partial\psi_L} = -j_n \tag{28}$$

Therefore, from (26) and (27) one gets

$$\psi_l = \pi\left(\frac{2}{3}\right)^{1/2}\left[1 + 2\left(\frac{P_r}{j_n\theta_E}\right)\right]^{-1/2} \tag{29}$$

$$\psi_L = \psi_C + 2 + \left(\frac{P_r}{j_n\theta_E}\right) + \psi_L\left[1 + \left(\frac{P_r}{j_n\theta_E}\right)\right]\left(\frac{P_r}{j_n\theta_E}\right)^{-1} \tag{30}$$

Equation (29) and (30) are not explicit solutions for the optimum values of ψ_l and ψ_L because j_n depends exponentially on these two parameters. Instead one has two equations, which must be solved simultaneously for the optimum values of ψ_l and ψ_L. It turns out however, that first working with j_n alone can do this indirectly. Substituting equations (29) and (30) into (24) taking the logarithm of each side, and then simplifying gives

$$\frac{P_r}{j_n\theta_E} = \frac{\psi_C + 2 + \pi\left(2/3\right)^{1/2}\left[1 + 2\left(P_r/j_n\theta_E\right)\right]^{1/2}}{\ln\left(j_0\theta_E/P_r\right) + \ln\left(P_r/j_n\theta_E\right) - \left(\psi_C + 1\right)}$$, with $P_r/j_n\theta_E \equiv \beta$, we can write

$$\beta = \frac{\left(\frac{e\Delta V_C}{K_B T_E}\right) + 2 + \pi\left[\frac{2(1+2\beta)}{3}\right]^{1/2}}{\ln\left(\frac{AK_B T_E^3}{eP_r}\right) + \ln\beta - \left(\frac{e\Delta V_C}{K_B T_E}\right) - 1} \tag{31}$$

where $\beta = P_r/j_n\theta_E = ep_r/j_n k_B T_E$ .Equation (31) is the condition on j_n and hence on ψ_l and ψ_L for which η is a maximum.

Substituting (29) and (30) into (31) and simplifying the results gives maximum efficiency in terms of the optimum value of $P_r/j_n\theta_E$ obtained from (31) as

$$\eta_{\max} = \frac{1}{1 + \left(P_r/j_n\theta_E\right)_{opt}} \quad \text{or} \quad \eta_{\max} = \frac{1}{1+\beta} \tag{32}$$

Thus the maximum efficiency for particular values of V_C and T_E depends on the ratio of the radiation loss, P_r to the optimum value of $2j_n\ \theta_E$, which is the kinetic energy, K.E. of the electrons that reach the anode (collector) from the cathode (emitter).

The optimum values of cathode lead resistance R_l and load impedance R_L can be obtained in terms of β from (29) and (30) by using the relation $R_i = (\theta_E/j_n A_E)\psi_i$ as

$$\left(R_l\right)_{opt} = \pi\left(\frac{2}{3}\right)^{1/2}\frac{\left(k_B T_E\right)^2}{e^2 P_r A_E}\frac{\beta}{\left(1+2\beta\right)^{1/2}} \tag{33}$$

and

$$\left(R_L\right)_{opt} = \frac{\left(\frac{k_B T_E}{e}\right)^2}{P_r A_E}\left[\frac{e\Delta V_C}{k_B T_E} + \left(2+\beta\right) + \pi\left(\frac{2}{3}\right)^{1/2}\frac{1+\beta}{\left(1+2\beta\right)^{1/2}}\right] \tag{34}$$

For the maximum efficiency, the following interrelated conditions must be satisfied.

(a) The current in the circuit must satisfy equation (31)
(b) The cathode or emitter lead resistance and the load impedance must satisfy equations (33) and (34) respectively.
(c) The optimum cathode lead geometry l/A_E can be obtained directly from equation (20)

3.0 Data Generation

The data were generated by first solving equation (31) iteratively for different values of T_E and V_C. The results were used in connection with equation (32) to obtain the maximum conversion efficiency. Since to produce useful quantities of electricity the temperature of the collector has to be maintained in the same range as that of electron tube i.e. 800 K to 1000 K, while the emitter is to be heated to about twice that temperature i.e. 1500 to 2000 K, therefore, the emitter temperature, T_E was varied from 1500 K to 5000 K in steps of 500 K and the collector voltage, V_C was varied from 1.0V to 3.0 V in steps of 0.5 V. This was done for the metals considered (Tungsten, W and Tantalum, Ta) with experimental Richardson-Dushmann constant, A as in table 1 as well as with the theoretical A value i.e. (A = 120 A/cm^2K^2). Tables of values were then computed based on both the theoretical and experimental values of A (see Tables 2 to 5).

4.0 Results and Discussions

The graphs in figures 3 and 4 were plotted using the values in Tables 2 to 5. From the tables it was observed that:- (i) The values for the efficiencies increase as the β (as earlier defined) decreases. (ii) The values of the efficiencies decrease along the row as the V_C increases. (iii) The values of the efficiencies increase along the column as the temperature increases. (iv) There were no values for the efficiencies at V_C = 2.5 V and 3.0 V at T_E = 1500 K. This suggests that at this temperature and for these voltages the electrons do not have enough energy to cross the potential barrier for these metal surfaces. This imply that no voltage is obtained if the emitter temperature does not exceed 1500 K both for **W** and **Ta**.

It was observed from Fig. 3 that:- The curves for the theoretical Richardson-Dushmaan constant, ***A*** are higher than that for the experimental ***A*** for both metals. Figure 4 shows that the conversion efficiencies decreases linearly with the out put collector voltages,both theoretically and experimentally. ***A*** constant difference between the theoretically obtain efficiency and the experimentally available efficiency for the metals considered of approximately 4% was obtained for both metals at all collector voltages V_C.

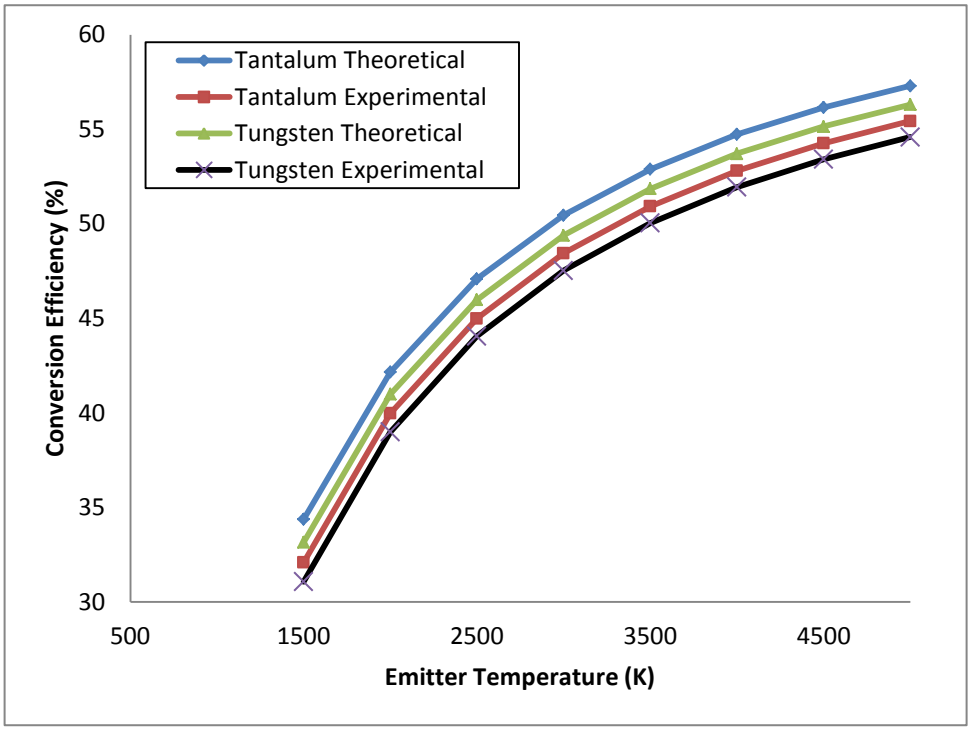

Fig. 3: Conversion efficiency versus emitter temperature at V_C =1.0V for both metals using the Theoretical Richardson-Dushmann Constant, (A =120 A/cm^{2}K^2) and Experimental Richardson-Dushman constant, (A_W = 60 A/cm^{2}K^2 and A_{Ta} =55 A/cm^{2}K^2)

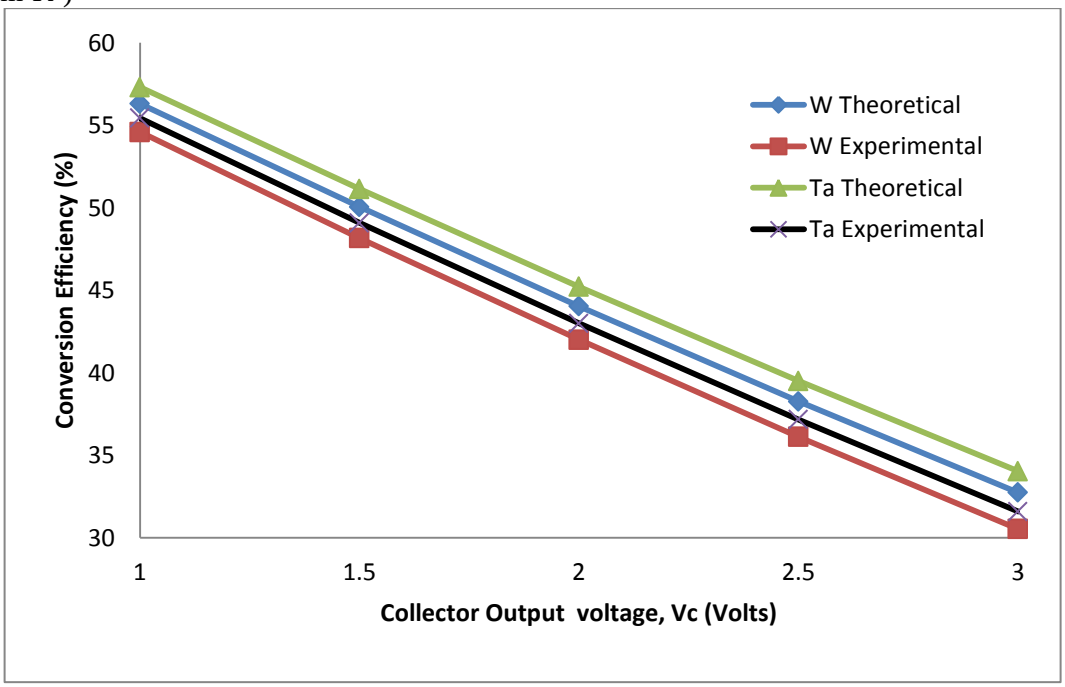

Fig. 4: Comparison of Maximum conversion efficiencies of the two metals with their collector out put voltage for both theoretical and Experimental values of Richardson-Dushmann, A at T_E = 5000K

5.0 Conclusion

In summary, it is clear that variation in the Richardson-Dushmann constant ***A*** due to variations in the wok functions of different metals affects their conversion efficiencies. In essence all the results of the thermionic conversion of heat to electricity obtained by assuming ***A*** to be 120 A/cm^{2}K^2 has this much deviation from the observed A value on the converters under review. To resolve this discrepancy, the following has to be considered (i) the effect of the reflection coefficient (ii) the effect of the emitter work function (iii) the surface ruggedness and (iv) the effect of the external electric field all of which bring about the deviation of the Richardson-Dushman constant from its theoretical value. In addition it is clear that Tantalum, Ta has higher conversion efficiency than Tungsten, W.

Table 2: Computed maximum conversion efficiency for **Tungsten, W** converter using Theoretical A (120 *A*/cm^{2}K^2) value

T_E (K)	Pr (W/m^2)	V_C = 1.0(V)		V_C = 1.5(V)		V_C = 2.0(V)		V_C = 2.5(V)		V_C = 3.0(V)	
		β	η(%)	β	η(%)	B	η(%)	B	η(%)	B	η(%)
1500	5.2764	2.0168	33.15	4.6930	17.57	15.9560	5.89	-	-	-	-
2000	17.6800	1.4393	40.99	2.5806	27.93	5.0494	16.53	22.4543	4.26	50.7098	1.93
2500	43.8324	1.1750	45.98	1.8664	34.89	3.0383	24.76	5.3225	15.82	10.8534	8.44
3000	91.3900	1.0248	49.39	1.5139	39.78	2.2445	30.82	3.4254	22.60	5.5553	15.25
3500	169.7064	0.9286	51.85	1.3051	43.38	1.8259	35.39	2.5838	27.90	3.7620	20.99
4000	289.8400	0.8619	53.71	1.1675	46.14	1.5689	38.93	2.1152	32.10	2.8921	25.69
4500	464.5464	0.8131	55.15	1.0702	48.30	1.3956	41.74	1.8182	35.48	2.3848	29.54
5000	708.2855	0.7760	56.31	0.9980	50.05	1.2710	44.03	1.6138	38.26	2.0544	32.74

Table 3: Computed maximum conversion efficiency for **Tungsten, W** converter using Experimental A (60 *A*/cm^{2}K^2) value

T_E (K)	Pr (W/m^2)	V_C = 1.0(V)		V_C = 1.5(V)		V_C = 2.0(V)		V_C = 2.5(V)		V_C = 3.0(V)	
		β	η(%)	β	η(%)	β	η(%)	β	η(%)	β	η(%)
1500	5.2764	2.2190	31.07	5.4551	15.49	21.3253	4.48	-	-	-	-
2000	17.6800	1.5642	38.99	2.8804	25.77	5.9177	14.46	24.2867	3.95	79.5650	1.24
2500	43.8324	1.2696	44.06	2.0516	32.77	3.4292	22.58	6.2765	13.74	13.8399	6.74
3000	91.3900	1.1039	47.53	1.6511	37.72	2.4913	28.64	3.9015	20.40	6.5849	13.18
3500	169.7064	0.9983	50.04	1.4169	41.38	2.0082	33.24	2.8926	25.69	4.3181	18.80
4000	289.8400	0.9254	51.94	1.2637	44.18	1.7157	36.82	2.3441	29.90	3.2622	23.46
4500	464.5464	0.8723	53.41	1.1561	46.38	1.5203	39.68	2.0017	33.31	2.6611	27.31
5000	708.2855	0.8320	54.59	1.0764	48.16	1.3808	42.00	1.7685	36.12	2.2759	30.53

Table 4: Computed maximum conversion efficiency for **Tantalum, Ta** converter using Theoretical A (120 A/cm^{2}K^2) value.

T_E (K)	Pr (W/m^2)	V_C = 1.0(V)		V_C = 1.5(V)		V_C = 2.0(V)		V_C = 2.5(V)		V_C = 3.0(V)	
		β	η(%)	β	η(%)	β	η(%)	β	η(%)	β	η(%)
1500	3.4676	1.9099	34.37	4.3167	18.81	13.6432	6.83	-	-	-	-
2000	11.6189	1.3723	42.15	2.4249	29.20	4.6234	17.78	10.7660	8.49	-	-
2500	28.8066	1.1238	47.09	1.7684	36.12	2.8379	26.06	4.8577	17.07	9.5103	9.51
3000	60.0610	0.9819	50.46	1.4405	40.98	2.1153	32.10	3.1840	23.90	5.0565	16.51
3500	111.5307	0.8906	52.89	1.2450	44.54	1.7295	36.64	2.4241	29.20	3.4827	22.31
4000	190.4819	0.8272	54.73	1.1151	47.28	1.4907	40.15	1.9953	33.39	2.7024	27.01
4500	305.2989	0.7808	56.15	1.0238	49.41	1.3288	42.94	1.7213	36.75	2.2412	30.85
5000	465.4837	0.7453	57.30	0.9555	51.14	1.2120	45.21	1.5315	39.50	1.9383	34.03

Table 5: Computed maximum conversion efficiency for **Tantalum, Ta** converter using Experimental A (55 A/cm^{2}K^2) value

T_E (K)	Pr (W/m^2)	V_C = 1.0(V)		V_C = 1.5(V)		V_C = 2.0(V)		V_C = 2.5(V)		V_C = 3.0(V)	
		β	η(%)	β	η(%)	β	η(%)	β	η(%)	β	η(%)
1500	3.4676	2.1176	32.08	5.0652	16.49	18.4564	5.14	-	-	-	-
2000	11.6189	1.5019	39.97	2.7292	26.82	5.4718	15.45	14.1015	6.62	-	-
2500	28.8066	1.2225	44.99	1.9588	33.79	3.2312	23.63	5.7856	14.74	12.2646	7.54
3000	60.0610	1.0646	48.44	1.5826	38.72	2.3671	29.69	3.6596	21.46	6.0541	14.18
3500	111.5307	0.9637	50.92	1.3611	42.35	1.9168	34.28	2.7367	26.76	4.0347	19.86
4000	190.4819	0.8939	52.80	1.2153	45.14	1.6423	37.85	2.2290	30.97	3.0748	24.54
4500	305.2989	0.8430	54.26	1.1134	47.32	1.4580	40.68	1.9097	34.37	2.5218	28.39
5000	465.4837	0.8042	55.43	1.0374	49.08	1.3261	42.99	1.6911	37.16	2.1646	31.59

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