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Theoretical Verification of General Relativistic Theory of Radar Sounding To The Order Of C^{-3}

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Abstract

In a paper titled: Differences between General Relativistic and Dynamical Theory of Gravitation in the Resolution of Radar sounding Phenomenon to the Order of C^{-5}, the expression for the time delay according to General Relativistic Theory was derived. In this paper a theoretical verification was undertaken to confirm this equation for GPS measurements.

Keywords: Radar Sounding, Time delay, GPS time error.

1.0 Introduction

In our earlier paper [1], it was shown that the total time needed for the trip of the radar signals could be obtain from the world line element as follows:
The Schwardschild's centro – symmetric metric has the form [2]

$$ds^2 = c^2(1+f)dt^2 - (1+f)^{-1}dr^2 - r^2(d\theta^2 + sin^2 d\emptyset^2) \tag{1}$$

or

$$c^2d\tau^2 = (1+f)c^2dr^2 - (1+f)^{-1}dr^2 - r^2(d\theta^2 + sin^2\theta d\emptyset^2) \tag{2}$$

Where

$$ds^2 = c^2d\tau^2 \tag{3}$$

and τ is proper time. Then for radar signal traveling in a radial direction,

$$d\tau = 0 \tag{4}$$

and

$$d\theta = 0 = d\emptyset \tag{5}$$

Thus equation (2) becomes

$$0 = (1+f)c^2dt^2 - (1+f)^{-1}dr^2 \tag{6}$$

Substituting for f(r) in equation (6) we have:

$$\left(1 - \frac{2k}{r}\right)^{-1} dr^2 = \left(1 - \frac{2k}{r}\right) c^2dt^2 \tag{7}$$

Where

$$K = \frac{GM}{c^2} \tag{8}$$

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It follows from equation (7) that the velocity of the radar signals in the radial direction within the vicinity of the spherical massive sun is:

$$\frac{dr}{dt} = \pm\left(1 - \frac{2k}{r}\right)c \tag{9}$$

and the coordinate time is given by:

$$dt = \pm\frac{1}{c}\left(1 - \frac{2k}{r}\right)^{-1} dr \tag{10}$$

The total coordinate time for the whole round trip of the radar signal is given as:

$$dt = \frac{1}{c}\int \frac{dr}{\left(1-\frac{2k}{r}\right)} \tag{11}$$

Application of Radar Sounding In the Computation of Time Delay Error in GPS Measurements.

The expression obtained in equation (11) will be used to compute the time delay of the radar signals to the order of C^{-3}, also it shall be used to compute the time error in GPS measurements. The relationship between the proper time $d\tau$ and the coordinate time dt is generally given as [3].

$$\mathrm{d}\tau = (1 - \frac{2\mathrm{k}}{\mathrm{r}})^{\frac{1}{2}}\, dt \tag{12}$$

where

$$k = \frac{MG}{c^2} \tag{13}$$

Methods of Data Acquisition

We used a real time GPS data for month of February, 2012 of the monitoring GPS station of National Space Research and Development Agency Centre for Geodesy and Geodynamics, Toro, Bauchi State, Nigeria. Also we used a MATLAB software as a technical computing tool for the coordinate time (dt) and distance (r) between the coordinate point and the receiving station computation. Here we focused mostly on estimating the Geometrics range delay time which we regard as the time error ($d\tau$).

We have the following parameters in the receive GPS data: frequency ($l_1\, l_2$), time of data reception and GPS receiver coordinate. Therefore the parameters we need to compute are:

i. GPS signal arrival time conversion into total seconds
ii. Corresponding wavelength of the frequencies
iii. Distance and time error

The RINEX format was used for this purpose. MATLAB scripts was written and used to compute the parameters i – iii above.

Step1: script for computing signal arrival time in HH: MM:SS and total seconds
T_0 = [42925:5:49570]: GPS signal arrival time in seconds, at 5 seconds interval.
T_0': to display above result in a single column form
Time In sec = [T_0']:
Disp (datestr (datnum(0,0,0,0 Time In sec): HH:MM:SS)) corresponding HH:MM:SS:
Time In sec = [4292542930
\- -
\- -
49570]

Step2: scripts for computing the corresponding GPS signal wavelength (λ) of frequency (l_1) and the approximate distance (pseudorange).
C = 300000000: speed of light 3.0 x 10^8m/s
l_1 = [40642316.1020: frequency l_1 generated from GPS signal
128204356.5680
110657670.3090
\- -
\- -
129543305.6710
129561031.0310]

$\lambda = c^{*}l_1$: corresponding wavelength computation
Format long g: display λ (meters)without approximation
Wavelength (λ) display the answer
λ=[2.711439985795610
2.340014084005630
- - -
- - -
2.315827888180560
2.315511057705450]

A_0 = 100000000: approximate total wavelength from GPS satellite to receivers.
Distance (D) = $\lambda^{*}A_0$: compute the approximate corresponding distance from the GPS satellite to receivers.
Format long g.
Distance (D) = [271143998.5795610
234001408.4005630
- - -
- - -
231582788.8180560
231551105.7705450]

In order to obtain the time the signal used to travel along curved lines in space we use equation (11) carrying out the necessary computation we obtain

dt =[42924.0961866703
42929.2199953042
- - -
- - -
49564.2280573695
49569.2281629296]

Time error = $d\tau - dt$: compute the time error i.e the difference between($d\tau$) , proper time and (dt) coordinate time.
Time error = [0.903813329745302
0.780004695806608
- - -
- - -
0.771942630530971
0.771837020372593]

Taking equation (11) into consideration for computing coordinate time for the trip, it may be noted that we have obtained the desired time error which is in confirmation of the theoretical background.

Summary and Conclusion

In this paper we applied, Schwarzschild's time dilation equation derived, in the GPS data set collated in the month of February, 2012 and found a difference between a elapsed coordinate time (dt) and the elapsed proper time ($d\tau$) as measured at the orbit of the satellite which we regard as time error. It is well known that for a static observer at a radius r outside a gravitating body the proper time will be dilated as given in equation (11). The stationary GPS receiver for this research paper work is used to acquire and record a real life data in RINEX format at regular specified intervals of 5secs as configured by the receiver user the scientific computing software, MATLAB was used to write the scripts needed for the computation of the following parameters

i. GPS signal arrival time conversion into total seconds
ii. Corresponding wavelengths and frequencies
iii. Distances and time error

This computational results are in agreement with the theoretical concept . in principle, Einstein consider gravitation to be a geometrical theory of space time coordinates, which is determined by the presences of matter that curves the space time. It is interesting to say that the time dilated values obtained were due to the fact that the signals pass through the lines of curved space (geometrics) and therefore takes a longer time thereby given a time delay as stated by Albert Einstein in the theory of General Relativity.

References

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