A Mathematical Model and Simulation of In-Situ Combustion in Porous Media

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Abstract

We study the continuity, momentum and coupled nonlinear energy and species convection-diffusion equations describing the in-situ combustion process in porous media. We assume the fuel depends on the space variable x. We prove the existence and uniqueness of solutions of equations. We show that temperature is a non-decreasing function of time. The time-dependent temperature and concentration profiles are obtained using finite difference method.

Key words: In-situ combustion, porous media, heavy oil, flow simulation, reservoir

1. Introduction

Flow simulations in porous media have a very wide range of environmental and industrial applicability. They are an important tool in fields such as ground water hydrology, civil engineering, petroleum production, ceramic engineering, the automotive industry and textile engineering. For instance, engineers simulate underground flow through porous rocks to predict the movement of contaminated fluid from solid waste landfills into drinking water supplies. In industrial applications, harmful particles can be filtered from a fluid stream by flow through a porous medium, whose small pores do not permit the passage of the larger particles [1].

Many researchers have studied the oxidation of crude oil with air injected in porous media. These include Ayeni [2] who studied thermal runaway phenomena while investigating the reaction of oxygen and hydrogen. He provided useful theorems on such flows. Marchesin and Schecter [3] constructed a two-phase model for oxidation, involving air or oxygen and oil that include heat loss to the rock formation. De Souza et al. [4] studied the Riemann problem with forward combustion due to injection of air into a porous medium containing solid fuel. Olayiwola and Ayeni [5, 6] presented a mathematical model of in-situ combustion using high activation energy asymptotics. Redl [7] considered multi channel geometry to show the ability of the Lattice Boltzmann method to deal with fluid flow and heat transfer problems occurring in combustion processes.

In this paper we extend the model investigated by Redl to include the continuity and momentum equations. We assume the fuel depends on the space variable x. We investigate the existence and uniqueness of solution. We also examine the properties of solution. To simulate the flow, we assume that the other end of reservoir is at infinity.

2. Mathematical Model

We consider an underground reservoir contained heavy oil. We assume that air is injected at the leftmost part of the reservoir, so that all propagation is one dimensional. One end of the reservoir is assumed kept at $x = 0$ while the other end is assumed far away (i.e kept at $x = \infty$). We also assume the fuel depends on space variable x. Then the primary dependent variables are the temperature, $T(x,t)$, the oxygen concentration, $C_{ox}(x,t)$, the solid fuel concentration, $C_{fuel}(x,t)$, and the gas product concentration, $C_p(x,t)$. Under these assumptions, the unsteady equations that describe the in-situ combustion process are

The continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \tag{2.1}$$

The momentum equation

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial x^2} \tag{2.2}$$

The energy equation

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \frac{\lambda}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2}\right) + \frac{Q\omega}{\rho c_p} \tag{2.3}$$

The conservation equation for the oxidizer

$$\frac{\partial(\rho C_{ox})}{\partial t} + u\frac{\partial(\rho C_{ox})}{\partial x} = D_{ox}\left(\frac{\partial^2(\rho C_{ox})}{\partial x^2}\right) - \omega s_{ox} \tag{2.4}$$

The conservation equation for the solid fuel

$$\frac{\partial(\rho_{fuel} C_{fuel})}{\partial t} + u\frac{\partial(\rho_{fuel} C_{fuel})}{\partial x} = D_{fuel}\left(\frac{\partial^2(\rho_{fuel} C_{fuel})}{\partial x^2}\right) - \omega\,(1 - s_{sf}) \tag{2.5}$$

The conservation equation for the gas product

$$\frac{\partial(\rho C_p)}{\partial t} + u\frac{\partial(\rho C_p)}{\partial x} = D_p\left(\frac{\partial^2(\rho C_p)}{\partial x^2}\right) + \omega s_p \tag{2.6}$$

It is simple to eliminate the continuity and momentum equations by means of streamline function,

$$\eta(x,t) = \int_0^x \rho(s,t)ds \tag{2.7}$$

The coordinate transformation becomes,

$$\frac{\partial}{\partial x} \to \frac{\partial}{\partial \eta}\frac{\partial \eta}{\partial x} = \rho \frac{\partial}{\partial \eta} \tag{2.8}$$

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial \eta}\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial t} = -\rho u \frac{\partial}{\partial \eta} + \frac{\partial}{\partial t} \tag{2.9}$$

We make the additional assumptions that $\rho = \rho_{fuel}$, c_p, $\rho\lambda$, $\rho^2 D_{ox}$, $\rho^2 D_{fuel}$ and $\rho^2 D_p$ are constant. Although these assumptions could be relaxed in the future, they considerably simplify the equations. The equations can be simplified as

$$\frac{\partial T}{\partial t} = \frac{\rho\lambda}{c_p}\frac{\partial^2 T}{\partial \eta^2} + \frac{1}{\rho c_p} Q\omega \tag{2.10}$$

$$\frac{\partial C_{ox}}{\partial t} = \rho^2 D_{ox}\frac{\partial^2 C_{ox}}{\partial \eta^2} - \frac{1}{\rho}\omega s_{ox} \tag{2.11}$$

$$\frac{\partial C_{fuel}}{\partial t} = \rho^2 D_{fuel}\frac{\partial^2 C_{fuel}}{\partial \eta^2} - \frac{1}{\rho}\omega(1 - s_{sf}) \tag{2.12}$$

$$\frac{\partial C_p}{\partial t} = \rho^2 D_p \frac{\partial^2 C_p}{\partial \eta^2} + \frac{1}{\rho}\omega s_p, \tag{2.13}$$

where T is the local gas temperature, ρ is the density, u is the velocity along x - axis, t is the time, x is the position, p is the pressure, μ is the dynamic viscosity, λ is the thermal conductivity, c_p is the specific heat at constant pressure, Q is the heat of reaction, D are the diffusion coefficients, s_{ox} is the stoichiometric coefficient of oxidizer,

s_{sf} is the stoichiometric coefficient of fuel, s_p is the stoichiometric coefficient of gas product. A general form of the reaction rate is

$$\omega = A\, k_{ov}\, C_{ox}^{\alpha}\, C_{fuel}^{\beta}\, e^{-\frac{E}{RT}}, \tag{2.14}$$

where A is a pre exponential factor, k_{ov} is the overall reaction coefficient, C are the concentrations, α and β are the orders of reaction, E is the activation energy, R is the ideal gas constant.

The source term represents the consumption of fuel during the combustion process.

The initial and boundary conditions were formulated as

$$\left.\begin{array}{lll}
T(\eta,0) = 0, & T(0,t) = T_1, & \lim_{\eta\to\infty} T(\eta,t) = 0,\ t > 0 \\
C_{ox}(\eta,0) = 0, & C_{ox}(0,t) = C_0, & \lim_{\eta\to\infty} C_{ox}(\eta,t) = 0,\ t > 0 \\
C_{fuel}(\eta,0) = 0, & C_{fuel}(0,t) = C_{f0}, & \lim_{\eta\to\infty} C_{fuel}(\eta,t) = 0,\ t > 0 \\
C_p(\eta,0) = 0, & C_p(0,t) = 0, & \lim_{\eta\to\infty} C_p(\eta,t) = 0,\ t > 0
\end{array}\right\} \tag{2.15}$$

3. Method of Solution

3.1 Existence and Uniqueness of Solution

Theorem 1

There exists a unique solution of problem (2.10) – (2.13).

Proof:

Let $D_{ox} = \frac{\lambda}{\rho c_p} = 1$, $D_f = D_p = 1$, $\phi = \left(s_{ox} T + \frac{Q}{c_p} C_{ox} \right)$ and

$\varphi = \left(s_p \rho_{fuel} C_{fuel} + (1 - s_{sf}) \rho C_p \right)$

Then (2.10)-(2.13) become

$$\frac{\partial \phi}{\partial t} - \frac{\partial^2 \phi}{\partial \eta^2} = 0 \tag{3.1}$$

$$\frac{\partial \varphi}{\partial t} - \frac{\partial^2 \varphi}{\partial \eta^2} = 0 \tag{3.2}$$

$$\phi(\eta,0) = 0, \quad \phi(0,t) = s_{ox}T_1 + \frac{Q}{c_p}C_0, \quad \phi(\eta,t) \to 0 \ as \ \eta \to \infty \tag{3.3}$$

$$\varphi(\eta,0) = 0, \quad \varphi(0,t) = s_p \rho_{fuel} C_{f0}, \quad \varphi(\eta,t) \to 0 \ as \ \eta \to \infty \tag{3.4}$$

Using the Fourier sine transform (see Myint-U and Debnath [8], p. 333 - 335), we obtain the solution of problem (3.1) in compact form as

$$\phi(\eta,t) = \frac{2}{\pi}\left(s_{ox}T_1 + \frac{Q}{c_p}C_0 \right)\int_0^\infty s \sin s\eta \int_0^t e^{-s^2(t-\tau)} d\tau \, ds \tag{3.5}$$

and the solution of problem (3.2) in compact form as

$$\varphi(\eta,t) = \frac{2}{\pi}\left(s_p \rho C_{f0} \right)\int_0^\infty s \sin s\eta \int_0^t e^{-s^2(t-\tau)} d\tau \, ds \tag{3.6}$$

Then, we obtain

$$T(\eta,t) = \frac{1}{s_{ox}}\left[\frac{2}{\pi}\left(s_{ox}T_1 + \frac{Q}{c_p}C_0 \right)\int_0^\infty s \sin s\eta \int_0^t e^{-s^2(t-\tau)} d\tau \, ds - \frac{Q}{c_p}C_{ox}(\eta,t) \right] \tag{3.7}$$

$$C_{ox}(\eta,t) = \frac{c_p}{Q}\left[\frac{2}{\pi}\left(s_{ox}T_1 + \frac{Q}{c_p}C_0 \right)\int_0^\infty s \sin s\eta \int_0^t e^{-s^2(t-\tau)} d\tau \, ds - s_{ox}T(\eta,t) \right] \tag{3.8}$$

$$C_{fuel}(\eta,t) = \frac{1}{s_p \rho}\left[\frac{2}{\pi}\left(s_p \rho C_{f0} \right)\int_0^\infty s \sin s\eta \int_0^t e^{-s^2(t-\tau)} d\tau \, ds - (1 - s_{sf})\rho C_p(\eta,t) \right] \tag{3.9}$$

$$C_p(\eta,t) = \frac{1}{(1 - s_{sf})\rho}\left[\frac{2}{\pi}\left(s_p \rho C_{f0} \right)\int_0^\infty s \sin s\eta \int_0^t e^{-s^2(t-\tau)} d\tau \, ds - s_p \rho C_{fuel}(\eta,t) \right] \tag{3.10}$$

Hence, there exists a unique solution of problem (2.10) - (2.13). This completes the proof.

3.2 Non-dimensionalisation

We make the variables dimensionless by introducing

$$\theta = \frac{E}{RT_0^2}\left(T - T_0\right), \quad C'_{ox} = \frac{C_{ox}}{C_{ox}^0}, \quad C'_f = \frac{C_f}{C_f^0}, \quad C'_p = \frac{C_p}{C_p^0} \tag{3.11}$$

and using (3.8) and (3.9) in simplified form, equations (2.10) - (2.13) (after dropping prime) become

$$\frac{\partial\theta}{\partial t} - \frac{\partial^2\theta}{\partial\eta^2} = \delta\left[a\,erfc\left(\frac{\eta}{2\sqrt{t}}\right) - b\theta(\eta,t) - c\right]^{\alpha}\left[C_{f0}\,erfc\left(\frac{\eta}{2\sqrt{t}}\right) - dC_p(\eta,t)\right]^{\beta} e^{\frac{\theta}{1+\in\theta}} \tag{3.12}$$

$$\theta(\eta,0) = 0, \qquad \theta(0,t) = \theta_*, \qquad \theta(\eta,t) \to 0 \ as\ \eta \to \infty$$

$$\frac{\partial C_{ox}}{\partial t} - \frac{\partial^2 C_{ox}}{\partial\eta^2} = -S\left[a\,erfc\left(\frac{\eta}{2\sqrt{t}}\right) - b\theta(\eta,t) - c\right]^{\alpha}\left[C_{f0}\,erfc\left(\frac{\eta}{2\sqrt{t}}\right) - dC_p(\eta,t)\right]^{\beta} e^{\frac{\theta}{1+\in\theta}} \tag{313}$$

$$C_{ox}(\eta,0) = 0, \qquad C_{ox}(0,t) = C_0, \qquad C_{ox}(\eta,t) \to 0 \ as\ \eta \to \infty$$

$$\frac{\partial C_{fuel}}{\partial t} - \frac{\partial^2 C_{fuel}}{\partial\eta^2} = -S_1\begin{bmatrix} a\,erfc\left(\frac{\eta}{2\sqrt{t}}\right) - b\theta(\eta,t) \\ -c \end{bmatrix}^{\alpha}\left[C_{f0}\,erfc\left(\frac{\eta}{2\sqrt{t}}\right) - dC_p(\eta,t)\right]^{\beta} e^{\frac{\theta}{1+\in\theta}} \tag{3.14}$$

$$C_{fuel}(\eta,0) = 0, \qquad C_{fuel}(0,t) = C_{f0}, \qquad C_{fuel}(\eta,t) \to 0 \ as\ \eta \to \infty$$

$$\frac{\partial C_p}{\partial t} - \frac{\partial^2 C_p}{\partial\eta^2} = S_2\left[a\,erfc\left(\frac{\eta}{2\sqrt{t}}\right) - b\theta(\eta,t) - c\right]^{\alpha}\left[C_{f0}\,erfc\left(\frac{\eta}{2\sqrt{t}}\right) - dC_p(\eta,t)\right]^{\beta} e^{\frac{\theta}{1+\in\theta}} \tag{3.15}$$

$$C_p(\eta,0) = 0, \qquad C_p(0,t) = 0, \qquad C_p(\eta,t) \to 0 \ as\ \eta \to \infty,$$

where

$\delta = \dfrac{QAk_{ov}e^{-\frac{E}{RT_0}}}{\rho c_p \in T_0}$ is the Frank-Kamenetskii parameter

$$S=\frac{s_{ox}Ak_{ov}e^{-\frac{E}{RT_0}}}{\rho C_{ox}^0},\ S_1=\frac{(1-s_{sf})Ak_{ov}e^{-\frac{E}{RT_0}}}{\rho C_{fuel}^0},\ S_2=\frac{s_pAk_{ov}e^{-\frac{E}{RT_0}}}{\rho C_p^0},\ a=\frac{c_p}{Q}\left(s_{ox}T_1+\frac{Q}{c_p}C_0\right),$$

$$b=\frac{c_ps_{ox}}{Q}\in T_0,\ c=\frac{c_ps_{ox}}{Q}T_0,\ d=\frac{(1-s_{sf})C_p^0}{s_p}$$

3.3 Properties of Solution

Theorem 2

Let $\in>0$ *and* $\alpha=\beta=0$ in (3.12). Then $\frac{\partial\theta(\eta,t)}{\partial t}\geq 0$.

In the proof, we shall make use of following Lemma of Kolodner and Pederson [9].

Lemma (Kolodner and Pederson [9])

Let $u(x,t)=0\left(e^{\alpha|x|^2}\right)$ be a solution on $R^n\times[0,t)$ of the differential inequality

$$\frac{\partial u}{\partial t}-\Delta u+K(x,t)u\geq 0$$

where K is bounded from below. If $u(x,0)\geq 0$, then $u(x,t)\geq 0$ for all $(x,t)\in R^n\times[0,t_0)$.

Proof of Theorem 2

Let $\in>0$ *and* $\alpha=\beta=0$ in (3.12). We obtain

$$\frac{\partial\theta}{\partial t}-\frac{\partial^2\theta}{\partial\eta^2}-\delta e^{\frac{\theta}{1+\in\theta}}=0$$

Differentiating with respect to t, we have

$$\frac{\partial}{\partial t}\left(\frac{\partial\theta}{\partial t}\right)-\frac{\partial^2}{\partial\eta^2}\left(\frac{\partial\theta}{\partial t}\right)-\left(\delta\left(\frac{1}{1+\in\theta}\right)^2e^{\frac{\theta}{1+\in\theta}}\right)\frac{\partial\theta}{\partial t}=0$$

Let

$$u=\frac{\partial\theta}{\partial t}$$

Then

$$\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial \eta^2}-\left(\delta\left(\frac{1}{1+\in\theta}\right)^2 e^{\frac{\theta}{1+\in\theta}}\right)u\geq 0$$

This can be written

$$\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial \eta^2}+K(\eta,t)u\geq 0,$$

where

$$K(\eta,t)=-\delta\left(\frac{1}{1+\in\theta}\right)^2 e^{\frac{\theta}{1+\in\theta}}$$

Clearly, K is bounded from below. Hence by Kolodner and Pederson's lemma $u(\eta,t)\geq 0$ i.e., $\frac{\partial \theta}{\partial t}\geq 0$. This completes the proof.

Theorem 3

Let $c=d=0$ *and* $\alpha=\beta=1$ in (3.12). Then $\theta(\eta,t)\geq 0$ for $(\eta,t)\in(0,\infty)\times(0,t_0)$, $t_0>0$.

Proof:

Let $c=d=0$ *and* $\alpha=\beta=1$ in (3.12). We obtain

$$\frac{\partial \theta}{\partial t}-\frac{\partial^2 \theta}{\partial \eta^2}+\delta bC_{f0}erfc\left(\frac{\eta}{2\sqrt{t}}\right)e^{\frac{\theta}{1+\in\theta}}\theta=\delta aC_{f0}\left(erfc\left(\frac{\eta}{2\sqrt{t}}\right)\right)^2 e^{\frac{\theta}{1+\in\theta}}$$

That is

$$\frac{\partial \theta}{\partial t}-\frac{\partial^2 \theta}{\partial \eta^2}+\delta bC_{f0}\,erfc\left(\frac{\eta}{2\sqrt{t}}\right)e^{\frac{\theta}{1+\in\theta}}\theta\geq 0 \qquad \text{since} \quad \delta aC_{f0}\left(erfc\left(\frac{\eta}{2\sqrt{t}}\right)\right)^2 e^{\frac{\theta}{1+\in\theta}}\geq 0$$

This can be written as

$$\frac{\partial\theta}{\partial t}-\frac{\partial^2\theta}{\partial\eta^2}+k(\eta,t)\theta\geq 0 \quad ,$$

where

$$k(\eta,t)=\delta\, bC_{f0}\, erfc\left(\frac{\eta}{2\sqrt{t}}\right)e^{\frac{\theta}{1+\epsilon\theta}}$$

Hence, by Kolodner and Pederson's lemma $\theta(\eta,t)\geq 0$. This completes the proof.

3.4 Numerical Solution

In this section, we solve equation (3.12) - (3.15) numerically using finite difference scheme. The error function $erf\left(\frac{\eta}{2\sqrt{t}}\right)$ is defined as

$$erf\left(\frac{\eta}{2\sqrt{t}}\right)=\frac{2}{\sqrt{\pi}}\int_0^{\frac{\eta}{2\sqrt{t}}}e^{-x^2}\,dx \tag{3.16}$$

By representing the exponential function in the integral by its Maclaurin series we see that

$$erf\left(\frac{\eta}{2\sqrt{t}}\right)=\frac{2}{\sqrt{\pi}}\sum_0^{\infty}\frac{(-1)^n\left(\frac{\eta}{2\sqrt{t}}\right)^{2n+1}}{n!(2n+1)} \tag{3.17}$$

The complementary error function is defined as

$$erfc\left(\frac{\eta}{2\sqrt{t}}\right)=\frac{2}{\sqrt{\pi}}\int_{\frac{\eta}{2\sqrt{t}}}^{\infty}e^{-x^2}\,dx=1-\frac{2}{\sqrt{\pi}}\sum_0^{\infty}\frac{(-1)^n\left(\frac{\eta}{2\sqrt{t}}\right)^{2n+1}}{n!(2n+1)} \tag{3.18}$$

Using finite difference approximations. Then (3.12) becomes

$$\theta_{i,j+1} = \varepsilon\left(\theta_{i+1,j} + \theta_{i-1,j}\right) + (1-2\varepsilon)\theta_{i,j} + k\delta \exp\left(\alpha \ln\left(a\left(1-\frac{2}{\sqrt{\pi}}\sum_{0}^{\infty}\frac{(-1)^n\left(\frac{\eta}{2\sqrt{t}}\right)^{2n+1}}{n!(2n+1)}\right) - b\theta_{i,j} - c\right)\right) \exp\left(\beta \ln\left(C_{f0}\left(1-\frac{2}{\sqrt{\pi}}\sum_{0}^{\infty}\frac{(-1)^n\left(\frac{\eta}{2\sqrt{t}}\right)^{2n+1}}{n!(2n+1)}\right) - dC_{pi,j}\right)\right)\exp\left(\frac{\theta_{i,j}}{1+\in\theta_{i,j}}\right), \tag{3.19}$$

where

$$\varepsilon = \frac{k}{h^2}$$

Equation (3.13) becomes

$$C_{oxi,j+1} = \varepsilon\left(C_{oxi+1,j} + C_{oxi-1,j}\right) + (1-2\varepsilon)C_{oxi,j} - kS \exp\left(\alpha \ln\left(a\left(1-\frac{2}{\sqrt{\pi}}\sum_{0}^{\infty}\frac{(-1)^n\left(\frac{\eta}{2\sqrt{t}}\right)^{2n+1}}{n!(2n+1)}\right) - b\theta_{i,j} - c\right)\right) \exp\left(\beta \ln\left(C_{f0}\left(1-\frac{2}{\sqrt{\pi}}\sum_{0}^{\infty}\frac{(-1)^n\left(\frac{\eta}{2\sqrt{t}}\right)^{2n+1}}{n!(2n+1)}\right) - dC_{pi,j}\right)\right)\exp\left(\frac{\theta_{i,j}}{1+\in\theta_{i,j}}\right) \tag{3.20}$$

Equation (3.14) becomes

$$C_{fuel\,i,j+1} = \varepsilon\left(C_{fuel\,i+1,j} + C_{fuel\,i-1,j}\right) + (1-2\varepsilon)C_{fuel\,i,j} - kS_1 \exp\left(\alpha \ln\left(a\left(1 - \frac{2}{\sqrt{\pi}}\sum_0^\infty \frac{(-1)^n\left(\frac{\eta}{2\sqrt{t}}\right)^{2n+1}}{n!(2n+1)}\right) - b\theta_{i,j} - c\right)\right) \exp\left(\beta \ln\left(C_{f0}\left(1 - \frac{2}{\sqrt{\pi}}\sum_0^\infty \frac{(-1)^n\left(\frac{\eta}{2\sqrt{t}}\right)^{2n+1}}{n!(2n+1)}\right) - dC_{p\,i,j}\right)\right)\exp\left(\frac{\theta_{i,j}}{1+\in\theta_{i,j}}\right) \tag{3.21}$$

Equation (3.15) becomes

$$C_{p\,i,j+1} = \varepsilon\left(C_{p\,i+1,j} + C_{p\,i-1,j}\right) + (1-2\varepsilon)C_{p\,i,j} + kS_2 \exp\left(\alpha \ln\left(a\left(1 - \frac{2}{\sqrt{\pi}}\sum_0^\infty \frac{(-1)^n\left(\frac{\eta}{2\sqrt{t}}\right)^{2n+1}}{n!(2n+1)}\right) - b\theta_{i,j} - c\right)\right) \exp\left(\beta \ln\left(C_{f0}\left(1 - \frac{2}{\sqrt{\pi}}\sum_0^\infty \frac{(-1)^n\left(\frac{\eta}{2\sqrt{t}}\right)^{2n+1}}{n!(2n+1)}\right) - dC_{p\,i,j}\right)\right)\exp\left(\frac{\theta_{i,j}}{1+\in\theta_{i,j}}\right) \tag{3.22}$$

The initial and boundary conditions are

$$\left.\begin{array}{lll}
\theta_{i,0} = 0, & \theta_{0,j} = \theta_*, & \theta_{\frac{2}{h},j} = 0 \\
C_{ox\,i,0} = 0, & C_{ox\,0,j} = C_0, & C_{ox\frac{2}{h},j} = 0 \\
C_{fuel\,i,0} = 0, & C_{fuel\,0,j} = C_{f0}, & C_{fuel\frac{2}{h},j} = 0 \\
C_{p\,i,0} = 0, & C_{p\,0,j} = C_0, & C_{p\frac{2}{h},j} = 0
\end{array}\right\} \tag{3.23}$$

A computer program in Pascal codes was written to perform the iterative computations.

4. Results and Discussion

First we have seen that if $c = d = 0$ *and* $\alpha = \beta = 1$ in (3.12), $\theta(\eta, t)$ is a non-decreasing function of time. In Figures 1 and 2 we display the graphs of $\theta(\eta, t)$ versus η for various values of δ and t. It is easy to see that $\theta(\eta, t)$ increases as δ and t increases. Figure 3 displays the graph of $\theta(\eta, t)$ versus t for various values of δ. It is seen from figure that $\theta(\eta, t)$ increases as δ increases. Figures 4, 5 and 6 display the graphs of $C_{ox}(\eta, t)$, $C_{fuel}(\eta, t)$ and $C_p(\eta, t)$ respectively versus η at various time t. It is seen from the figures that $C_{ox}(\eta, t)$, $C_{fuel}(\eta, t)$ and $C_p(\eta, t)$ increases as t increases.

It is worth pointing out the effect of δ as shown in Figures 1 and 3 indicating that there is increase in heat of reaction Q. When the heat of reaction is high, the rate of conversion of heavy oils into light oils, water and gas is high and consequently, the recovery rate is boosted. This is of great economic importance.

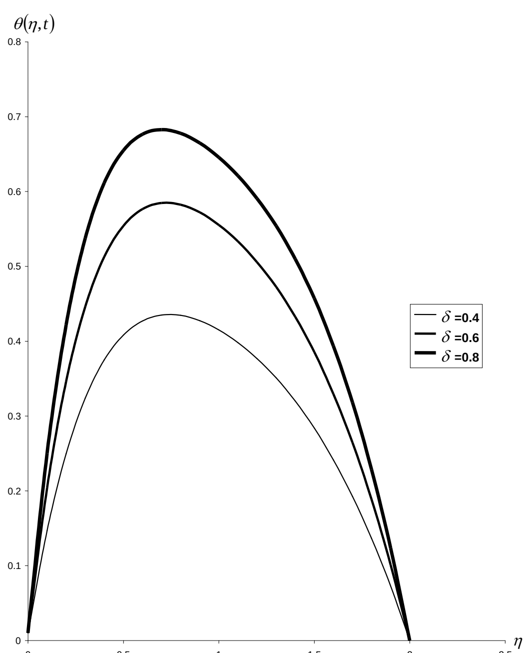

Figure 1: Plots of $\theta(\eta,t)$ against η for equations (3.12) and (3.15) at various values of δ when $\in$=0.01, α=1, β=1, t=0.2.

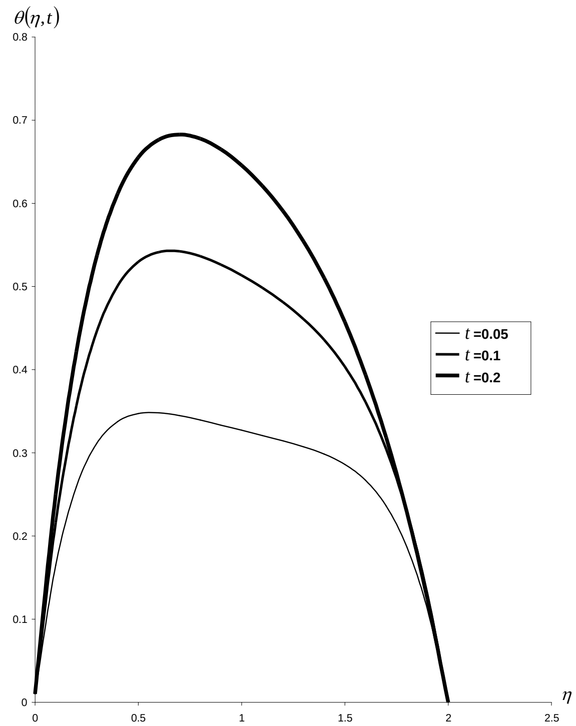

Figure 2: Plots of $\theta(\eta,t)$ against η for equations (3.12) and (3.15) at various time t when $\in$=0.01, δ =0.8, α =1, β =1.

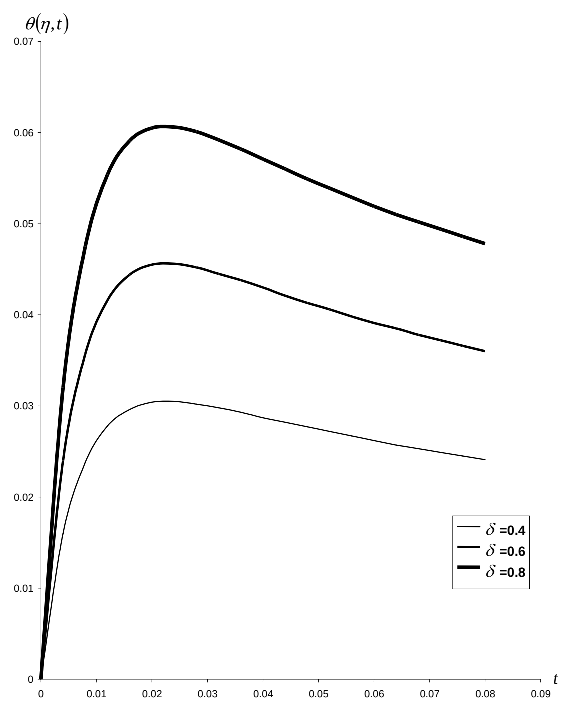

Figure 3: Plots of $\theta(\eta,t)$ against time t for equations (3.12) and (3.15) at various values of δ when $\in$=0.01, α =1, β =1, η =1.5.

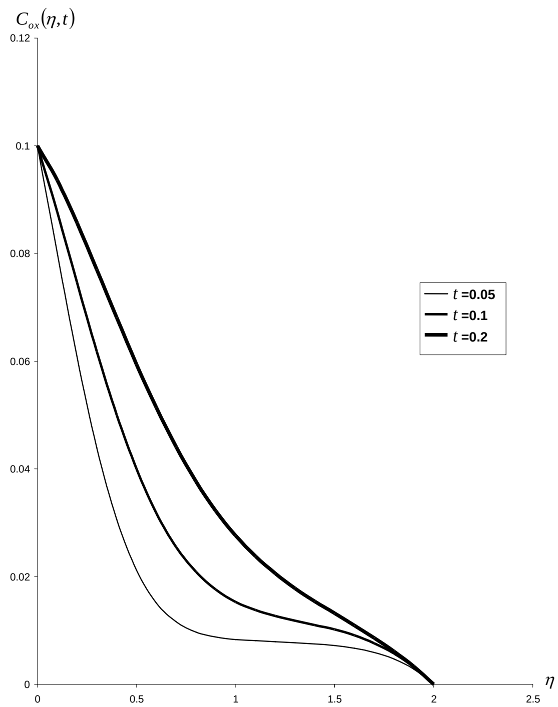

Figure 4: Plots of $C_{ox}(\eta,t)$ against η for equations (3.12), (3.13) and (3.15) at various time t when $\in$=0.01, δ=0.8, α=1, β=1.

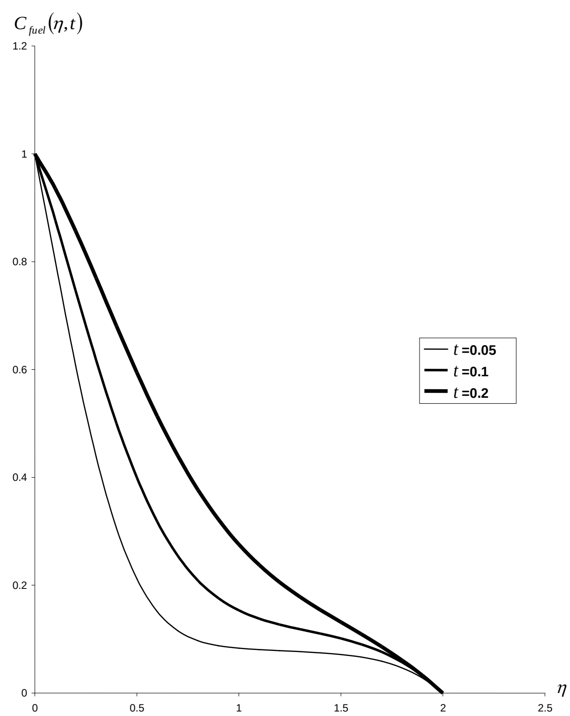

Figure 5: Plots of $C_{fuel}(\eta,t)$ against η for equations (3.12), (3.14) and (3.15) at various time t when $\in$=0.01, δ=0.8, α=1, β=1.

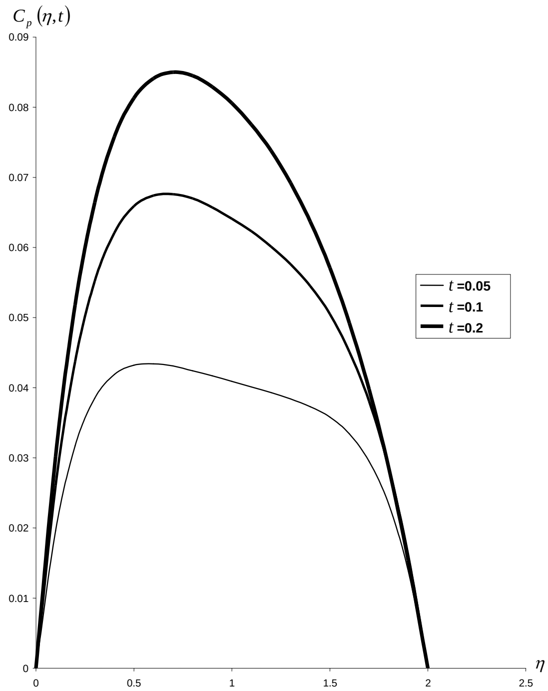

Figure 6: Plots of $C_p(\eta,t)$ against η for equations (3.12) and (3.15) at various time t when $\in$=0.01, δ=0.8, α=1, β=1.

Also the effects observed in Figures 2, 4, 5 and 6 respectively physically means that the local gas temperature, oxygen and solid fuel consumption and gas product production increases as time increases.

5. Conclusion

The equations of in-situ combustion in porous media have been presented by mathematical point of view. The equations have been solved by finite difference method and the results and discussion for that have been described. The graphical summaries of the system responses were provided.

It can be concluded from the simulations that chemical reactions occurring due to injection of air into a reservoir have considerable effects on the phenomena of flow in the medium. The analysis has also shown that the parameters involved in the in-situ combustion model have significant effects on temperature and concentration field of the system.

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