AN ORDER LEVEL INVENTORY REPLENISHMENT MODEL WITH TIME DEPENDENT DEMAND, FINITE PRODUCTION RATE AND PARTIALLY BACKLOGGED SHORTAGE.

by

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ABSTRACT

In this paper, we examine the inventory replenishment problem for deteriorating items with exponential and constant demand trend when shortage is partially backlogged with finite production rate. The backlogging rate is variable and dependent on the waiting time for the next replenishment. Further, an optimal model are developed which minimizes the total average cost for the system. Numerical example is used to illustrate the proposed model. Sensitivity analysis of the optimal solution with respect to the major parameters is carried out.

Keywords: Inventory; exponential distribution; finite production; partial backlogging.

1.0 INTRODUCTION

The development of an optimal replenishment policy for an inventory system is one of the greatest challenge confronting supply managers of any modern organisation. Unfortunately, this is so as the cost of inventory contributes to the overall cost of the entire supply chain process. Demand play a major role in inventory management and an understanding of the various forms of demand is necessary for efficiency in the development of optimal replenishment models. In inventory modelling constant demand, time-dependent demand, probabilistic demand and stock-dependent demand are usually assumed. It is important to note that demand is now a function of many factors in today's research.

Donaldson [1] proposed an analytic solution for items with linear increasing demand. Ritchie [2] discussed the solution of a linear increasing time-dependent demand, which was obtained by Donaldson [1].Silver and Meal [3] developed a model for deterministic time varying demand, which also gives an approximate solution procedure termed as Silver-Meal Heuristic. Goswami and Chaudhuri [4] developed an Economic Order Quantity (EOQ) model by assuming a linear trend in demand, finite rate of replenishment with shortages. Further, deteriorating items with exponential demand developed by Aggarwal and Bahari-Kashani [5]. Ouyang et al [6] proposed an EOQ model for deteriorating items with exponentially decreasing demand where shortages are allowed and partially backlogged. Here, the backlogging rate is variable and dependent for the next replenishment on the waiting time. Bhunia and Maiti [7] developed a model in which the production rate is variable. Shortages are not allowed and the production rate was dependent on either the on hand inventory or on demand.

Su et al [8] presented a production inventory model for deteriorating items with an exponentially declining demand over a fixed time horizon. Begum et al [9] examined an order-level inventory model with finite production rate depending on demand. In that paper, shortages are allowed and completely backlogged. Ghosh et al [10] developed a model with time-dependent demand, finite production and shortages by assuming a two-parameter Weibull demand rate. Here shortages are allowed and completely backlogged. Hollter and Mak [11] developed inventory replenishment policies for deteriorating items. They considered replenishment problems with declining demand. Again Aggarwal and Bahari-Kashani [5] developed a model assuming flexible production rate for deteriorating items.

In this paper, we develop an inventory model for an item having both constant and exponential demand trends. This paper is an extension of the work of Begum et al [9] by assuming both constant and exponential demand trend and partially backlogged shortage. The assumption of constant demand over the shortage time period is taken from the fact that only loyal customers will place orders when they are not sure of instant supply of order (Customer loyalty). The production rate is finite and proportional to the demand rate. Here shortages are allowed and partially backlogged with the partial backlogging rate depending on the waiting time. A numerical example is considered to illustrate the developed model.

2.0 NOTATIONS AND ASSUMPTIONS

The model is developed on the basis of the following assumptions and notations. However, the assumptions and notations used here are those of Begum et al (2009) except for assumption (a), (e) and C_3 which is the per unit cost due to lost sales.

NOTATIONS

C_1: shortage cost per unit

C_2: Holding cost per unit

C_3: Cost of lost sales per unit

C_4: Ordering cost per unit

C: The average total cost of the system.

S: The maximum level of stock in the system

P: The maximum shortage accumulated.

SC: Shortage Cost

HC: Holding Cost

CLS: Cost due to lost Sales

OC: Operating Cost

ASSUMPTIONS

a) The demand rate for the shortage period $[0, t_2]$ is constant given by D.

b) The demand rate for the time interval $[t_3, t_4]$ is given by $D(t) = \alpha e^{\beta t}$

c) The production rate k(t) at anytime depends on the demand and is constant and is given by $K(t) = \lambda D(t)$, $\lambda > 1$ and $K(t) > \lambda D(t)$.

d) sThe lead-time is zero.

e) Shortage is allowed and partially backlogged at the rate $e^{-\delta t}$, $0 < \delta < 1$

3.0 MATHEMATICAL FORMULATION OF THE MODEL AND SOLUTION.

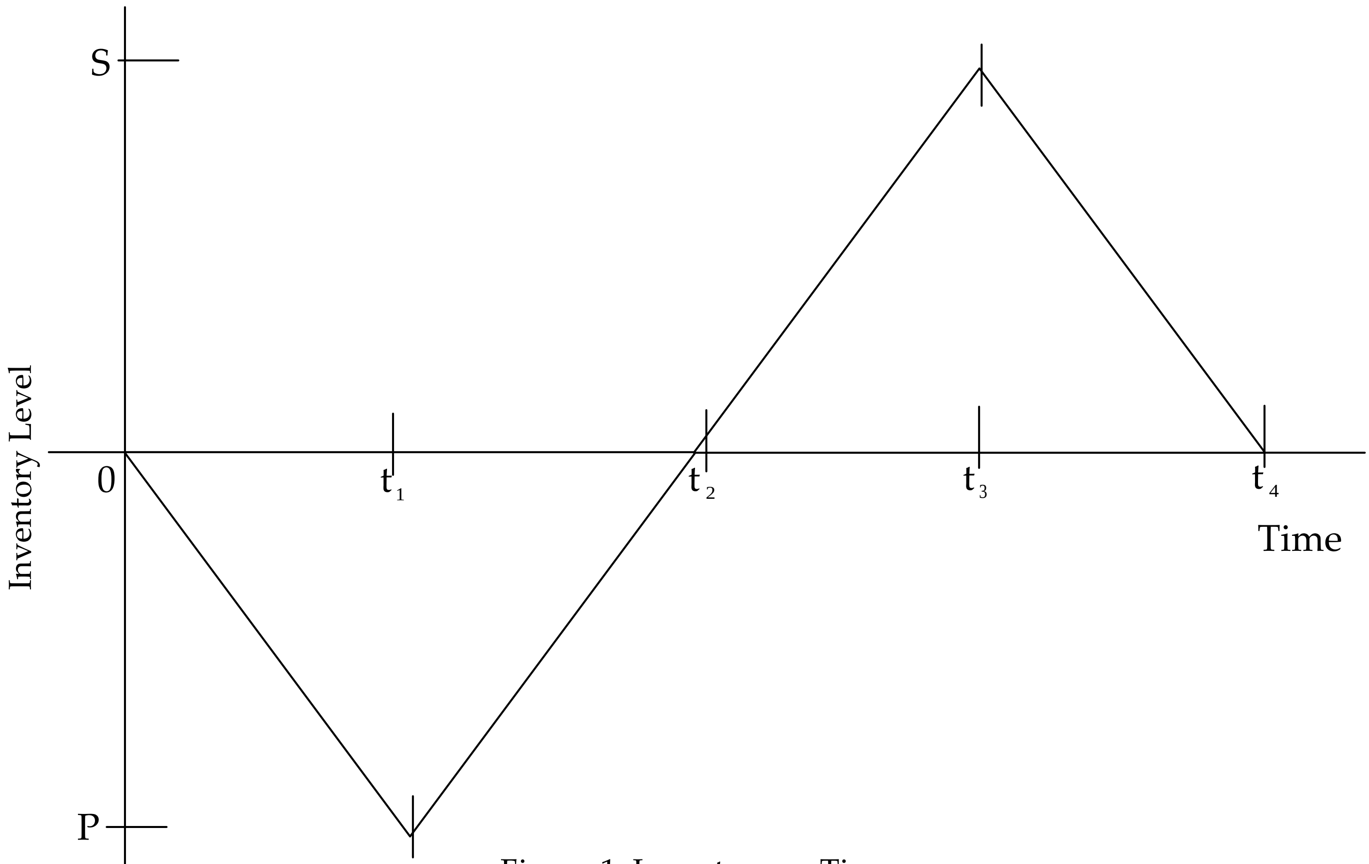

Figure 1: Inventory vs Time

The stock level is zero from the beginning, i.e at time t = 0. Now, at this point, shortages start to build up to the level P at t = t_1. Shortages are allowed and partially backlogged at the rate $e^{-\delta t}$. Note that the longer the waiting time the fewer the number of customers who will be willing to wait. The production inventory starts at t = t_1. The partially backlogged shortage starts to clear at t = t_2 and production stops at t = t_3 at which time stock level is now S. The inventory level becomes zero at point t = t_4. This decrease is due to demand. After time t_4, the cycle repeat itself.

Let $Z_i(t),\ i=1,2,3,4$ represent the instantaneous inventory level at any time t, $0 \le t \le t_4$. Thus, the differential equation governing the instantaneous states of $Z(t)$ in the interval $(0, t_4)$ are as follow:

$$\frac{dZ_1(t)}{dt} = -De^{-\delta t}, \qquad 0 \le t \le t_1 \qquad (1.1)$$

$$\frac{dZ_2(t)}{dt} = \lambda De^{-\delta t} - De^{-\delta t}, \qquad t_1 \le t \le t_2 \qquad (1.2)$$

$$\frac{dZ_3(t)}{dt} = \lambda \alpha e^{\beta t} - \alpha e^{\beta t}, \qquad t_2 \le t \le t_3 \qquad (1.3)$$

$$\frac{dZ_4(t)}{dt} = -\alpha e^{\beta t} \qquad t_3 \le t \le t_4 \qquad (1.4)$$

with initial conditions that $Z(0)=0,\ Z(t_1)=-P,\ Z(t_2)=0,\ Z(t_3)=S\ \ and\ \ Z(t_4)=0$. Solving the above equations and substituting the above boundary conditions, we obtain the following results:

$$Z_1(t) = \frac{D}{\delta}\left(e^{-\delta t} - 1\right), \qquad 0 \le t \le t_1 \qquad (1.5)$$

$$Z_2(t) = \frac{D}{\delta}(\lambda - 1)\left[e^{-\delta t_2} - e^{-\delta t}\right], \qquad t_1 \le t \le t_2 \qquad (1.6)$$

$$Z_3(t) = (\lambda - 1)\frac{\alpha}{\beta}\left[e^{\beta t} - e^{-\beta t_2}\right], \qquad t_2 \le t \le t_3 \qquad (1.7)$$

$$Z_4(t) = \frac{\alpha}{\beta}\left[e^{\beta t_4} - e^{-\beta t}\right], \qquad t_3 \le t \le t_4 \qquad (1.8)$$

Now, Substituting the initial condition $Z(t_1) = -P$ into equation (1.5) and (1.6) we have

$$-P = \frac{D}{\delta}\left(e^{-\delta t_1} - 1\right) \qquad (1.9)$$

$$-P=\frac{D}{\delta}(\lambda-1)\left[e^{-\delta t_2}-e^{-\delta t_1}\right] \qquad (1.10)$$

Subtracting (1.10) from (1.9) we have.

$$\frac{D}{\delta}e^{-\delta t_1}-\frac{D}{\delta}-\frac{D}{\delta}(\lambda-1)e^{-\delta t_2}+\frac{D}{\delta}(\lambda-1)e^{-\delta t_1}=0$$

$$\frac{D}{\delta}e^{-\delta t_1}-\frac{D}{\delta}-\frac{D}{\delta}\lambda e^{-\delta t_2}+\frac{D}{\delta}e^{-\delta t_2}+\frac{D}{\delta}e^{-\delta t_1}-\frac{D}{\delta}e^{-\delta t_1}=0$$

Dividing through by $\frac{D}{\delta}$ and collecting like terms we have

$$\lambda e^{-\delta t_1}=e^{-\delta t_2}+1-e^{-\delta t_2}$$

$$\lambda e^{-\delta t_1}=\left[\frac{e^{-\delta t_2}(\lambda-1)+1}{\lambda}\right]$$

Taking the natural log of both sides we have

$$-\delta t_1=\log\left[\frac{e^{-\delta t_2}(\lambda-1)+1}{\lambda}\right]$$

$$t_1^*=-\left(\frac{1}{\delta}\right)\log\left[\frac{e^{-\delta t_2}(\lambda-1)+1}{\lambda}\right] \qquad (1.11)$$

Also, substituting the initial condition $Z(t_3)=S$ into the equations (1.7) and (1.8) we have

$$S=\frac{\alpha}{\beta}(\lambda-1)\left[e^{\beta t_3}-e^{\beta t_2}\right] \qquad (1.12)$$

$$S=\frac{\alpha}{\beta}\left[e^{\beta t_4}-e^{\beta t_3}\right] \qquad (1.13)$$

Subtracting (1.13) from (1.12) we have

$$\frac{\alpha}{\beta}(\lambda-1)e^{\beta t_3}-\frac{\alpha}{\beta}(\lambda-1)e^{\beta t_2}-\frac{\alpha}{\beta}e^{\beta t_4}+\frac{\alpha}{\beta}e^{\beta t_3}=0$$

$$e^{\beta t_3}=\left[\frac{(\lambda-1)e^{\beta t_2}+e^{\beta t_4}}{\lambda}\right]$$

Taking the natural log of both side of the above and dividing through by β we have

$$t_3^* = \left(\frac{1}{\beta}\right)\log\left[\frac{(\lambda-1)e^{\beta t_2} + e^{\beta t_4}}{\lambda}\right] \tag{1.14}$$

The average total cost ATC of this inventory system is given by

$$ATC = \frac{1}{t_4}\left[SC + HC + CLS + OC\right] \tag{1.15}$$

Where

Now, calculating the various cost components we have,

$$SC = C_1\left[-\int_0^{t_1} Z_1(t)dt - \int_{t_1}^{t_2} Z_2(t)dt\right]$$

$$= C_1\left[-\int_0^{t_1} \frac{D}{\delta}(\ell^{-\delta t} - 1)dt - \int_{t_1}^{t_2} \frac{D}{\delta}(\lambda-1)(\ell^{-\delta t_2} - \ell^{-\delta t})dt\right]$$

$$SC = C_1\left[\frac{D}{\delta^2}\ell^{-dt} + \frac{D}{\delta}t\Big|_0^{t_1} - \left(\frac{D}{\delta}(\lambda-1)\right)\ell^{-\delta t_2}t + \frac{D}{\delta^2}\ell^{-\delta t}(\lambda-1)\Big|_{t_1}^{t_2}\right]$$

Let $\frac{D}{\delta}(\lambda-1) \;=\; V$

$$SC = C_1\left[\frac{D}{\delta^2}\left(\ell^{-\delta t_1} - 1\right) - \left(V\ell^{-\delta t_2}(t_2 - t_1)\right) + \frac{V}{\delta}\left(\ell^{-\delta t_2} - \ell^{-\delta t_1}\right)\right] \tag{1.16}$$

$$HC = C_2\left[\int_{t_2}^{t_3} Z_3(t)dt - \int_{t_3}^{t_4} Z_4(t)dt\right]$$

$$HC = C_2\left[\frac{\alpha}{\beta}(\lambda-1)\left(\frac{\ell^{\beta t}}{\beta} - t\ell^{\beta t_2}\Big|_{t_2}^{t_3}\right) + \frac{\alpha}{\beta}\left[t - \ell^{\beta t_4} - \frac{\ell^{\beta t}}{\beta}\right]_{t_3}^{t_4}\right]$$

$$HC = C_2\left[\frac{\alpha}{\beta}(\lambda-1)\left(\frac{\ell^{\beta t_3}}{\beta} - t_3\ell^{\beta t_2} - \frac{\ell^{\beta t_2}}{\beta} - t_2\ell^{\beta t_2}\right) + \frac{\alpha}{\beta}\left(t_4\ell^{\beta t_4} - \frac{\ell^{\beta t_4}}{\beta} - t_3\ell^{\beta t_4} - \frac{\ell^{\beta t_3}}{\beta}\right)\right]$$

$$HC = C_2\left[\frac{\alpha}{\beta}(\lambda-1)\left(\frac{\ell^{\beta t_3} - \ell^{\beta t_2}}{\beta} + \ell^{\beta t_2}(t_2 - t_3)\right) + \frac{\alpha}{\beta}\left(\ell^{\beta t_4}(t_4 - t_3) + \frac{\ell^{\beta t_3} - \ell^{\beta t_4}}{\beta}\right)\right] \tag{1.17}$$

$$CLS = C_3\left[\int_0^{t_1} (1 - \ell^{-\delta t})Ddt + \int_{t_1}^{t_2} (1 - \ell^{-\delta t})Ddt\right]$$

$$= C_3\left[Dt + \frac{D}{\delta}\ell^{-\delta t}\Big|_0^{t_1} + \left(Dt + \frac{D}{\delta}\ell^{-\delta t}\Big|_{t_1}^{t_2}\right]$$

$$=C_3\left[Dt_2-\frac{D}{\delta}+\frac{D}{\delta}\ell^{-\delta t_2}\right] \tag{1.18}$$

Substituting equation (1.16), (1.17) and (1.18) into (1.15) we have the following:

$$ATC=\frac{1}{t_4}\left[\begin{array}{l} C_1\left[\frac{D}{\delta^2}\left(\ell^{-\delta t_1}-1\right)-\left(V\ell^{-\delta t_2}\left(t_2-t_1\right)\right)+\frac{V}{\delta}\left(\ell^{-\delta t_2}-\ell^{-\delta t_1}\right)\right]+ \\ C_2\left[\begin{array}{l}\frac{\alpha}{\beta}(\lambda-1)\left(\frac{\ell^{\beta t_3}-\ell^{\beta t_2}}{\beta}+\ell^{\beta t_2}\left(t_2-t_3\right)\right)+ \\ \frac{\alpha}{\beta}\left(\ell^{\beta t_4}\left(t_4-t_3\right)+\frac{\ell^{\beta t_3}-\ell^{\beta t_4}}{\beta}\right)\end{array}\right]+C_3\left[Dt_2-\frac{D}{\delta}+\frac{D}{\delta}\ell^{-\delta t_2}\right]+C_4 \end{array}\right] \tag{1.19}$$

Again substituting the values of t_1 from equation (1.11) and t_3 from equation (1.14) into equation (1.19) we have that ATC becomes a function of two variables t_2 and t_4 given by

$$ATC=$$

$$\frac{1}{t_4}(C_4+C_3(-\frac{\mathrm{D1}}{\delta}+\frac{\mathrm{D1}e^{-\delta t_2}}{\delta}+\mathrm{D1}t_2)+C_1(\frac{\mathrm{D1}(-1+e^{\frac{\mathrm{Log}(1+e^{-\delta t_2}(-1+\lambda))}{\lambda}})}{\delta^2}-$$
$$\frac{(-e^{\frac{\mathrm{Log}(1+e^{-\delta t_2}(-1+\lambda))}{\lambda}}+e^{-\delta t_2})V_1}{\delta}-e^{-\delta t_2}(\frac{\mathrm{Log}(1+e^{-\delta t_2}(-1+\lambda))}{\delta\lambda}+t_2)V_1)+$$
$$C_2(\frac{e^{\frac{\mathrm{Log}(e^{\beta t_4}+e^{\beta t_2}(-1+\lambda))}{\lambda}}-e^{\beta t_4}}{\beta}+(\frac{e^{\frac{\mathrm{Log}(e^{\beta t_4}+e^{\beta t_2}(-1+\lambda))}{\lambda}}-e^{\beta t_2}}{\beta}+e^{\beta t_2}(-\frac{\mathrm{Log}(e^{\beta t_4}+e^{\beta t_2}(-1+\lambda))}{\beta\lambda}+$$
$$t_2))V_1W_1+e^{\beta t_4}(-\frac{\mathrm{Log}(e^{\beta t_4}+e^{\beta t_2}(-1+\lambda))}{\beta\lambda}+t_4)Z_1)) \tag{1.20}$$

Where $Z_1=\frac{\alpha}{\beta}$ and $W_1=\lambda-1$. Now our aim is to minimize the ATC using differential calculus. The simplification of ATC was done with mathematics 7.5. The optimal values of t_2 and t_4 for the minimum average total cost are the solution of the equations

$$\frac{\delta ATC}{\delta t_2}=.$$

$$\frac{1}{t_4}((\mathrm{D1}-\mathrm{D1}e^{-\delta t_2})C_3+C_1(-\frac{\mathrm{D1}e^{\frac{\mathrm{Log}(1+e^{-\delta t_2}(-1+\lambda))}{\lambda}-\delta t_2}\mathrm{Log}(-1+\lambda)}{\delta\lambda}-e^{-\delta t_2}(1-$$
$$\frac{e^{-\delta t_2}\mathrm{Log}(-1+\lambda)}{\lambda})V_1-\frac{(-e^{-\delta t_2}\delta+\frac{e^{\frac{\mathrm{Log}(1+e^{-\delta t_2}(-1+\lambda))}{\lambda}-\delta t_2}\mathrm{Log}\delta(-1+\lambda)}{\lambda})V_1}{\delta}+e^{-\delta t_2}\delta(\frac{\mathrm{Log}(1+e^{-\delta t_2}(-1+\lambda))}{\delta\lambda}+$$
$$t_2)V_1)+C_2(\frac{e^{\frac{\mathrm{Log}(e^{\beta t_4}+e^{\beta t_2}(-1+\lambda))}{\lambda}+\beta t_2}\mathrm{Log}(-1+\lambda)}{\lambda}+(e^{\beta t_2}(1-\frac{e^{\beta t_2}\mathrm{Log}(-1+\lambda)}{\lambda})+$$

$$\frac{-e^{\beta t_2}\beta+\frac{e^{\frac{\text{Log}(e^{\beta t_4}+e^{\beta t_2}(-1+\lambda))}{\lambda}+\beta t_2}\text{Log}\beta(-1+\lambda)}{\lambda}}{\beta}+e^{\beta t_2}\beta(-\frac{\text{Log}(e^{\beta t_4}+e^{\beta t_2}(-1+\lambda))}{\beta\lambda}+t_2))V_1W_1-\frac{e^{\beta t_2+\beta t_4}\text{Log}(-1+\lambda)Z_1}{\lambda}))\quad(1.21)$$

$$\frac{\delta ATC}{\delta t_4}=\frac{1}{t_4}C_2(\frac{-e^{\beta t_4}\beta+\frac{e^{\frac{\text{Log}(e^{\beta t_4}+e^{\beta t_2}(-1+\lambda))}{\lambda}+\beta t_4}\text{Log}\beta}{\lambda}}{\beta}+(\frac{e^{\frac{\text{Log}(e^{\beta t_4}+e^{\beta t_2}(-1+\lambda))}{\lambda}+\beta t_4}\text{Log}}{\lambda}-\frac{e^{\beta t_2+\beta t_4}\text{Log}}{\lambda})V_1W_1+e^{\beta t_4}(1-\frac{e^{\beta t_4}\text{Log}}{\lambda})Z_1+e^{\beta t_4}\beta(-\frac{\text{Log}(e^{\beta t_4}+e^{\beta t_2}(-1+\lambda))}{\beta\lambda}+t_4)Z_1)-\frac{1}{t_4^2}(C_4+C_3(-\frac{\text{D1}}{\delta}+\frac{\text{D1}e^{-\delta t_2}}{\delta}+\text{D1}t_2)+C_1(\frac{\text{D1}(-1+e^{\frac{\text{Log}(1+e^{-\delta t_2}(-1+\lambda))}{\lambda}})}{\delta^2}-\frac{(-e^{\frac{\text{Log}(1+e^{-\delta t_2}(-1+\lambda))}{\lambda}}+e^{-\delta t_2})V_1}{\delta}-e^{-\delta t_2}(\frac{\text{Log}(1+e^{-\delta t_2}(-1+\lambda))}{\delta\lambda}+t_2)V_1)+C_2(\frac{e^{\frac{\text{Log}(e^{\beta t_4}+e^{\beta t_2}(-1+\lambda))}{\lambda}}-e^{\beta t_4}}{\beta}+(\frac{e^{\frac{\text{Log}(e^{\beta t_4}+e^{\beta t_2}(-1+\lambda))}{\lambda}}-e^{\beta t_2}}{\beta}+e^{\beta t_2}(-\frac{\text{Log}(e^{\beta t_4}+e^{\beta t_2}(-1+\lambda))}{\beta\lambda}+t_2))V_1W_1+e^{\beta t_4}(-\frac{\text{Log}(e^{\beta t_4}+e^{\beta t_2}(-1+\lambda))}{\beta\lambda}+t_4)Z_1))\quad(1.22)$$

provided they satisfy the following equations. $\frac{\delta^2 ATC}{\delta t_2^2}>0$, $\frac{\delta^2 ATC}{\delta t_4^2}>0$,

$$\frac{\delta^2 ATC}{\delta t_2^2}\frac{\delta^2 ATC}{\delta t_4^2}-\left(\frac{\delta^2 ATC}{\delta t_2\delta t_4}\right)^2>0$$

4.0 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

In this section, we shall give a numerical example to illustrate our model. The parameters used here are extracted from Begum et al [9] except for δ, D and C_3

$\alpha=100;\delta=2;C_2=25;\beta=1.5;\ D=60;\ C_3=40;\lambda=2.5;\ C_1=30;C_4=20$

Result from proposed model is given as

$t_1^*=0.048367;\ t_2^*=0.0834238;\ t_3^*=0.181796;\ t_4^*=0.271169;ATC^*=1234.27;\quad P^*=2.76609;\ S^*=14.7453$

Table 1.1 Sensitivity of optimal times of inventory interval $t_1^*, t_2^*, t_3^*, t_4^*$, average total inventory cost ATC^* , the maximum level of stock in the system S^* and the maximum accumulated shortage P^* to perturbations in the partial backlogging parameter δ .

δ	t_1^*	t_2^*	t_3^*	t_4^*	ATC^*	P^*	S^*
0.2	0.0682178	0.114219	0.862558	1.32858	6425.5	4.06527	245.987
0.5	0.0867617	0.146767	0.469105	0.771973	2631.37	5.0944	77.4867
1	0.0674405	0.115068	0.281312	0.471961	1661.35	3.913	33.6566
1.5	0.0559828	0.0960996	0.206183	0.343281	1368.28	3.2218	20.738
2	0.48367	0.0834238	0.164966	0.271169	1234.27	2.76609	14.7453

Table 1.2: Sensitivity of optimal times of inventory interval $t_1^*, t_2^*, t_3^*, t_4^*$, average total inventory cost ATC^* , the maximum level of stock in the system S^* and the maximum accumulated shortage P^* to perturbations in the the shape parameter (The marginal propensity to consume).

β	t_1^*	t_2^*	t_3^*	t_4^*	ATC^*	P^*	S^*
1	0.0583197	0.101358	0.163269	0.249493	1371.08	3.30282	10.6021
1.5	0.0483672	0.0834238	0.164966	0.271169	1234.27	2.76609	14.7453
1.75	0.0415188	0.0768014	0.159674	0.270981	1200.58	2.3905	16.2504
2	0.0415188	0.0712461	0.159659	0.268467	1111.31	2.3905	16.7283
2.5	0.0365183	0.062435	0.15379	0.260663	1026.75	2.11299	17.995

Table 1.3: Sensitivity of optimal times of inventory interval $t_1^*, t_2^*, t_3^*, t_4^*$, average total inventory cost ATC^* , the maximum level of stock in the system S^* and the maximum accumulated shortage P^* to perturbations in constant demand.

D	t_1^*	t_2^*	t_3^*	t_4^*	ATC^*	P^*	S^*
1	0.0583197	0.101358	0.163269	0.249493	1371.08	3.30282	10.6021
1.5	0.0483672	0.0834238	0.164966	0.271169	1234.27	2.76609	14.7453
1.75	0.0415188	0.0768014	0.159674	0.270981	1200.58	2.3905	16.2504
2	0.0415188	0.0712461	0.159659	0.268467	1111.31	2.3905	16.7283
2.5	0.0365183	0.062435	0.15379	0.260663			
2.5	1026.75	2.11299	17.995				

REMARKS

We observe that the total cost obtained from the proposed model is higher; this is so because of the cost due to lost sales which is a reflection of the partially backlogged demand assumption. We observed also that P which is the total shortage accumulated follows apriori expectation since shortage is measured by unsatisfied demand. We further noticed that the time t_2 at which shortage clears is faster for the proposed model. This is so since the production inventory required to clear P is just a fraction of total demand. From the what if analysis we observe also that average total inventory cost is very sensitive to perturbations in D and the partial backlogging parameter δ. A close look at the times t_1, t_2, t_3 and t_4 show that they follow apriori expectation.

5.0 CONCLUSION

In this paper, we proposed a model for items that have constant and exponential demand trends with shortages partially backlogged. Although this model was previously studied by Begum et al [9], we extended the work by incorporating the idea of constant demand trend during the shortage period and partial backlogging. The backlogged rate is dependent on the duration of the waiting time up to the arrival of the next lot. We carried out sensitivity analysis which indicated that results obtained from the proposed model followed apriori expectation.

The average total cost of the system for the proposed model is higher than that of Begum et al [9] which is due to the addition of cost due to lost sales reflecting the partial backlogging assumption of shortages. The proposed shows that an increase in the partial backlogging parameter is accompanied by an increase in the total cost of inventory. Thus, inventory managers must pay attention to the rate of partial backlogging in a bid to optimize the total cost of inventory.

Work is currently going on by way of incorporating deterioration into the formulation and a more generalized demand trend that fluctuates with inflation.

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