Non Darcy flow effect on the Pressure response of Retrograde Gas Reservoir

Ohirhian, P.U. and Ekaiko, S.

Department of Petroleum Engineering,

University of Benin, Benin City, Nigeria

E-mail: okuopet@gmail.com. Phone: 08023394848

ABSTRACT

An analytical model has been developed in this work to adequately account for the influence of non-Darcy flow on the pressure response of gas-condensate systems. The model was developed using the pseudo-time function introduced by Penuela and Civan and the common pseudo-pressure function with non-Darcy effect. The model is applied to a typical retrograde gas well, and the result obtained shows that it offers a better understanding of the pressure behavior of similar.gas condensate systems It can therefore serve as a very good alternative to the rather expensive compositional numerical methods.

1.0 INTRODUCTION

The pressure response of retrograde gas reservoirs to non-darcy flow effects around the well bore region has been a subject of concern over the years. Unfortunately most existing literatures on the subject used numerical and compositional simulations which are always costly. Also numerical and compositional simulation can be very rigid because they are within limits of certain assumptions which sometimes are complex for practical standards according to Penuel and Civan [1]

The essence of this study is to derive an analytical pressure transient model (similar to that used in the Honer's plots) which will incorporate the Forchheimer's equation (non-darcy flow model) into the multiphase diffusivity equation. The model was used in analysing the pressure buildup characteristics (well test analysis) of a retrograde gas reservoir. The effect of neglecting and incorporating non-darcy flow in estimating flow rate from a retrograde gas reservoir was also studied.

In the flow of gas condensate fluids through porous media at high velocities, there seems to be two phenomena which cause the effective gas permeability to be rate dependent. First is an increase in relative permeability with velocity which has been demonstrated by flood experiments. Second is the inertial (non-darcy) flow effect which reduces the effective gas permeability at high velocity.

Gas condensates related topics (well deliverability, well test interpretation, flow in reservoir in general) have been long standing problems. O'Dell and Miller [2] presented the first gas rate equation using a pseudo-pressure to describe the effect of condensate blockage.

The method of Hector H.G. et al [3] is used in the derivation of the model and the final derived equation is given as

$$p_p = p_{pi} - \frac{q}{4\pi C_1 kh}\left[\ln\left(\frac{C_1 k t_p}{\emptyset {r_w}^2}\right) + 0.80907 - 2s\right] \quad (1.1)s$$

The parameters in eqn. (1.1) will be defined in the next section.

The equation (1.1) was used to accurately study the pressure transient of retrograde gas reservoirs, hence well bore characteristics such as skin factor, non-darcy coefficient can be estimated.

2.0 DERIVATION OF THE EQUATION

The mathematical bases of this work are the multiphase diffusivity equation and the Forchheimer modified equation. The modified Forchheimer equation is incorporated into the two-phase diffusivity equation and the resulting complex partial differential equation solved under the following assumptions:

- Constant formation porosity and absolute permeability
- Constant fluid viscosity
- Steady-state fluid flow velocity (high)
- Infinite reservoir of uniform thickness with a centrally located well
- Constant molar flow rate

With these assumptions the model is obtained as follows:

2.1 TWO-PHASE DIFFUSIVITY:

$$\frac{1}{r}\frac{\partial}{\partial_r}\left[r\left(\rho_o u_0 x_i + \rho_g u_g y_i\right)\right] = \emptyset\frac{\partial}{\partial_t}\left(\rho_o S_o x_i + \rho_g S_g y_i\right) \quad (2.1)$$

where,

r = radius of reservoir

ρ_o = density of condensate

u_0 = velocity of condensate

ρ_g = density of associated gas

u_g = velocity of associated gas

$\emptyset$ = porosity

S_o = saturation of condensate

S_g = saturation of condensate

t = time

x_i = mole fraction of condensate

y_i = mole fraction of gas

For $1 \leq i \leq n_c$

Forchheimer equation as presented by Abiodun. [4] is :

$$-\frac{\partial p}{\partial r} = \frac{\mu}{k}u + \beta\rho u^2 \quad (2.2)$$

This equation is often written in the form

$$u = -\frac{kk_r}{\mu}\delta\frac{\partial p}{\partial r} \quad (2.3)$$

where δ is known as the laminar, inertia, turbulent (LIT) correction factor defined as:

$$\delta = \frac{1}{1 + \beta \frac{k k_r}{\mu} \rho \mu}$$

Substituting (2.3) in (2.1) yields

$$-\frac{C_1}{r}\frac{\partial}{\partial r}\left[k_r \left(\rho_o \frac{k_{ro}}{\mu_o}\delta_o x_i + \rho_g \frac{k_{rg}}{\mu_g}\delta_g y_i\right)\frac{\partial p}{\partial r}\right] = \emptyset \frac{\partial}{\partial t}\left(\rho_o S_o x_i + \rho_g S_g y_i\right) \quad (2.4)$$

where,

k = absolute permeabilty

k_r = relative permeability

and other terms remain as defined in previous equations and C_1 is a unit-conversion constant equal to 0.00633cuft/dy [3]

Multiphase pseudo-pressure takes the form[3]:

$$p_p = \int_{p_{ref}}^{p_{wf}} \left(\rho_o \frac{k_{ro}}{\mu_o}\delta_o + \rho_g \frac{k_{rg}}{\mu_g}\delta_g\right)\partial p \quad (2.5)$$

The Penuela and Civian [1] pseudo-time function is:

$$t_p = \int_{t_{ref}}^{t} \left[\frac{\partial(\rho_o S_o + \rho_g S_g)}{\partial p_p}\right]^{-1} \partial t \quad (2.6)$$

Equations (2.5) and (2.6) transform (2.4) into a more compact form given as

$$\frac{C_1}{r}\frac{\partial}{\partial r}\left[k_r \frac{\partial p_p}{\partial r} = \emptyset \frac{\partial p_p}{\partial t_p}\right] \quad (2.7)$$

Now for an infinite reservoir subject to the following boundary conditions

$$p_p = p_{pi} \text{ at } t_p = 0 \text{ for all r}$$
$$r\frac{\partial p_p}{\partial r} = \frac{q}{2\pi kh} \text{ for } t_p > 0$$
$$p_p = p_{pi} \text{ as r} \rightarrow \infty$$

The solution to (2.7) is obtained by putting

$$y = \frac{\emptyset\mu C_1 r^2}{4kt_p} \tag{2.8}$$

$$\frac{\partial y}{\partial r} = \frac{\emptyset\mu C_1 r}{2kt_p} \tag{2.9}$$

$$\frac{\partial y}{\partial t_p} = -\frac{\emptyset\mu C_1 r^2}{4kt_p} \tag{2.10}$$

Using chain rule

$$\frac{\partial p_p}{\partial r} = \frac{\partial p_p}{\partial y}\frac{\partial y}{\partial r} = \frac{\partial p_p}{\partial y}\left[\frac{\emptyset\mu C_1 r}{2kt_p}\right] \tag{2.11}$$

Then,

$$\frac{\partial^2 p_p}{\partial r^2} = \partial\left(\frac{\partial p_p}{\partial y}\frac{\partial y}{\partial r}\right) = \left(\frac{\partial y}{\partial r}\right)^2\left(\frac{\partial^2 p_p}{\partial y^2}\right) + \frac{\partial p_p}{\partial y}\frac{\partial^2 y}{\partial r^2} = \left(\frac{\emptyset\mu C_1 r}{2kt_p}\right)^2\left(\frac{\partial^2 p_p}{\partial y^2}\right) + \frac{\partial p_p}{\partial y}\left(\frac{\emptyset\mu C_1}{2kt_p}\right) \tag{2.12}$$

Also ,

$$\frac{\partial p_p}{\partial t_p} = \frac{\partial p_p}{\partial y}\frac{\partial y}{\partial t_p} = \frac{\partial p_p}{\partial y}\left(-\frac{\emptyset\mu C_1 r^2}{4k{t_p}^2}\right) \tag{2.13}$$

But the diffusivity equation in pseudo-pressure and pseudo-time form is given as :

$$\frac{\partial^2 p_p}{\partial r^2} + \frac{1}{r}\left(\frac{\partial p_p}{\partial r}\right) = \frac{\emptyset \mu C_1}{k}\frac{\partial p_p}{\partial t_p} \tag{2.14}$$

Substitution of (2.11), (2.12), and (2.13) in (2.14) gives

$$\left[\left(\frac{\emptyset \mu C_1 r}{2kt_p}\right)\left(\frac{\partial^2 p_p}{\partial y^2}\right) + \frac{\partial p_p}{\partial y}\left(\frac{\emptyset \mu C_1}{2kt_p}\right)\right] + \frac{1}{r}\left[\frac{\partial p_p}{\partial y}\left(\frac{\emptyset \mu C_1 r}{2kt_p}\right)\right] = \frac{\emptyset \mu C_1}{k}\frac{\partial p_p}{\partial y}\left(-\frac{\emptyset \mu C_1 r^2}{4k{t_p}^2}\right) \tag{2.15}$$

Equation (2.15) simplifies to:

$$\left(\frac{\emptyset \mu C_1 r^2}{4kt_p}\right)\frac{\partial p_p}{\partial y^2} + \frac{1}{2}\frac{\partial p_p}{\partial y} = \frac{\partial p_p}{\partial y}(-y) \tag{2.16}$$

Substituting (2.8) into (2.16) gives:

$$y\frac{\partial^2 p_p}{\partial y^2} + \frac{\partial p_p}{\partial y} = -y\frac{\partial p_p}{\partial y}$$

i.e.

$$y\frac{\partial^2 p_p}{\partial y^2} + (1+y)\frac{\partial p_p}{\partial y} = 0 \tag{2.17}$$

Equation (2.17) can be written as:

$$y\frac{\partial p^/}{\partial y} + (1+y)p^/ = 0 \tag{2.18}$$

Where $p^/ = \frac{\partial p_p}{\partial y}$

By separating variables and integrating, (2.18) becomes:

$$\ln p^{/} \quad = -\ln y - y + C_2 \tag{2.19}$$

Where C_2 is the constant of integration.

Now from boundary conditions ,

$$r\frac{\partial p_p}{\partial r} = \frac{q}{2\pi kh} \text{ for } t_p > 0 \text{ we have } \lim_{y\to 0} 2y\frac{\partial p_p}{\partial y} = \frac{q\mu}{2\pi kh}$$

Then,

$$\lim_{y\to 0} 2C_2 e^{-y} = C_2 = \frac{q\mu}{4\pi kh} \tag{2.20}$$

Then,

$$\frac{\partial p_p}{\partial y} = \frac{q\mu}{4\pi kh}\frac{e^{-y}}{y} = \int_0^y \frac{e^{-y}}{y} dy + C_3 = p_p \tag{2.21}$$

Or

$$p_p = -\frac{q\mu}{4\pi kh}\int_y^\infty \frac{e^{-y}}{y} dy + C_3 \tag{2.22}$$

Using the condition , $C_3 = p_{pi}$ using this and (2.10) in (2.22), then:

$$p_{pi} - p_p = \frac{q\mu}{4\pi kh}\left[-E_i\left(-\frac{\emptyset\mu C_1 {r_w}^2}{4kt_p}\right)\right] \tag{2.23}$$

where the Euler's integral is:

$$\int_y^\infty \frac{e^{-y}}{y} dy = -E_i(-y)$$

Then,

$$p_{pi} - p_p = \frac{q}{4\pi C_1 kh}\left[\ln\left(\frac{C_1 kt_p}{\emptyset {r_w}^2}\right) + 0.80907\right] \tag{2.24}$$

Re-arranging and introducing a skin factor s:

$$p_p = p_{pi} - \frac{q}{4\pi C_1 kh}\left[\ln\left(\frac{C_1 k t_p}{\emptyset r_w{}^2}\right) + 0.80907 - 2s\right] \qquad (2.25)$$

Equation (2 25) is similar to that used in the Horner's method (Craft and Hawkins [5])

A study of (2.25) reveals that a plot of pseudo-pressure (p_p) versus log of time gives a straight line of negative slope which is equal to $\frac{q}{4\pi C_1 kh}$

Equation (2.25) can be used to accurately study the pressure transient of retrograde gas reservoirs, hence well bore characteristics such as skin factor and non-Darcy factor can be estimated.

The non darcy factor can be evaluated by flowing the well at two different rates and solving the simultaneous equation to obtain the rate dependent skin factor and the skin factor without turbulence.

$$s = s^* + Dq \qquad (2.26)$$

where,

s= total skin factor

s^*= skin factor without turbulence

Dq= rate dependent skin factor

D= non-darcy or turbulent factor

q= flow rate

3.0 TESTING OF THE DERIVED EQUATION

The derived model was applied to a retrograde gas well. The build-up characteristics were estimated using the model.

Due to the absence of relative permeability data, which is always the case for new wells, the relative permeability curve and saturation profile was generated using the following relation. Penuela G., C and Civian [1]).

$$S_o = \frac{\frac{L}{\rho_O}}{\frac{L}{\rho_O}+\frac{V}{\rho_g}} \ldots.(2.27a) \qquad S_g = \frac{\frac{V}{\rho_O}}{\frac{L}{\rho_O}+\frac{V}{\rho_g}} \ldots.\ldots.(2.27b)$$

$$K_{ro} = \frac{\mu_O S_o}{\mu_o s_O + \mu_g s_g} \ldots\ldots\ldots(2.27c) \qquad K_{rg} = \frac{\mu_g S_g}{\mu_o s_O + \mu_g s_g} \ldots\ldots\ (2.27d)$$

The Honer pseudo-time equivalent takes the form:

$$t_p = \frac{t+\Delta t}{\left[\frac{(\mu_g c_g)_i}{\mu_g c_g}\right]\Delta t} \qquad (2.28)$$

The well and reservoir fluid properties are shown in Table 1

Table 1: Reservoir fluid properties

Initial pressure P_i	6750psia
Dew point pressure P_d	6750psia
γ_g(to air)	0.94
L_{max} vol %	8.7
Temperature T	354°F
Gas flow rate q_g	75.4Mscf/day
Condensate flow rate q_c	2.8b/day
Molar flow rate q_t	200lb-mol/day
Thickness, h	216.5ft
Perforated thickness h_{perf}	36ft
Porosity Ø	0.062
Wellbore radius r_w	0.54ft
Drainage radius r_e	600ft

The well was flowed for 103 hours and was then subjected to a 141 hours buildup. The flow during the test was 75.4Mscf/d of gas and 2.8b/d of condensate. Data for the flow test are shown in Table 2

Table 2: Data for the flow test

Time (days)	Pressure (psia)	Time (days)	Pressure (psia)
0	1083.1	0.9167	6161.0
0.0070	1174.5	1.1667	6336.5
0.0139	1226.7	1.4167	6406.1

0.0208	1303.6	1.7500	6452.5
0.0417	1490.6	2.0833	6487.3
0.0833	1751.6	2.4167	6507.3
0.1250	2046.0	2.8333	6526.5
0.1667	2279.4	3.4167	6556.9
0.2500	2759.4	4.0417	6574.3
0.3333	3246.5	4.6667	6587.3
0.500	4221.0	5.8750	6601.8
0.6667	5162.0		

The gas condensate properties during the flow(pressure build up) test are shown in Table 3

Table 3: **Gas Condensate Fluid Properties During Pressure Build up**

T days	Pws psia	ρ_o lbmole/ ft^3	ρ_g lbmole/ ft^3	μ_o cp	μ_g cp	L fraction	c_g psi^{-1}
0	1083.1	0.3214	0.1274	0.0184	0.0170	0.0556	0.0009037
0.0070	1174.5	0.3248	0.1383	0.0192	0.0172	0.0567	0.0008329
0.0139	1226.7	0.3268	0.1445	0.0196	0.0172	0.0573	0.0007971
0.0208	1303.6	0.3296	0.1537	0.0203	0.0173	0.0582	0.0007494
0.0417	1490.6	0.3366	0.1759	0.0219	0.0176	0.0604	0.0006534
0.0833	1751.6	0.3465	0.2067	0.0240	0.0180	0.0634	0.0004678
0.1250	2046.0	0.3578	0.2411	0.0240	0.0185	0.0666	0.0004155

0.1667	2279.4	0.3669	0.2680	0.0264	0.0189	0.0692	0.0003338
0.2500	2759.4	0.3861	0.3219	0.0282	0.0200	0.0742	0.0002739
0.3333	3246.5	0.4065	0.3745	0.0316	0.0212	0.0789	0.0001924
0.500	4221.0	0.4499	0.4717	0.0347	0.0240	0.0860	0.0001405
0.6667	5162.0	0.4964	0.5540	0.0399	0.0273	0.0857	0.0001001
0.9167	6161.0	0.5530	0.6267	0.0436	0.0315	0.0610	0.0000940
1.1667	6336.5	0.5640	0.6376	0.0460	0.0323	0.0497	0.0000916
1.4167	6406.1	0.5686	0.6418	0.0462	0.0327	0.0440	0.0000900
1.7500	6452.5	0.5716	0.6444	0.0463	0.0331	0.0398	0.0000888
2.0833	6487.3	0.5740	0.6464	0.0463	0.0332	0.0363	0.0000881
2.4167	6507.3	0.5753	0.6476	0.0464	0.0333	0.0342	0.0000875
2.8333	6526.5	0.5766	0.6486	0.0464	00335	0.0322	0.0000864
3.4167	6556.9	0.5787	0.6503	0.0465	0.0335	0.0287	0.0000858
4.0417	6574.3	0.5798	0.6512	0.0465	0.0336	0.0266	0.0000854
4.6667	6587.3	0.5807	0.6520	0.0465	0.0336	0.0250	0.0000850
5.8750	6601.8	0.5818	0.6527	0.0465	0.0037	0.0230	0.0000849

The calculation of the pseudo reduced pressure (P_p) and the pseudo reduced time (tp) are shown in Table 4

Table 4: Calculation Of The Pseudo Reduced Pressure

(P_p) and the pseudo reduced time (t_p)

P_p	t_p	P_p	t_p
-	-	1647001384	1.165
14446091	573.2	1711981603	0.913

23374177	276.64	1737653465	0.7842
36807828	175.3	1754336853	0.6652
73064625	77.73	1767344842	0.5849
129448473	33.97	1774656702	0.5286
202005860	19.91	1781558930	0.477
265123605	13.67	1792920022	0.419
409010342	7.885	1799407981	0.387
570279624	5.238	1804271987	0.359
920393199	2.882	1809539420	0.3223
1272765923	1.86		

A semi log plot of pseudo-pressure versus pseudo-time is use to analyse the pressure build up test. This semi log plot is shown Figure 1

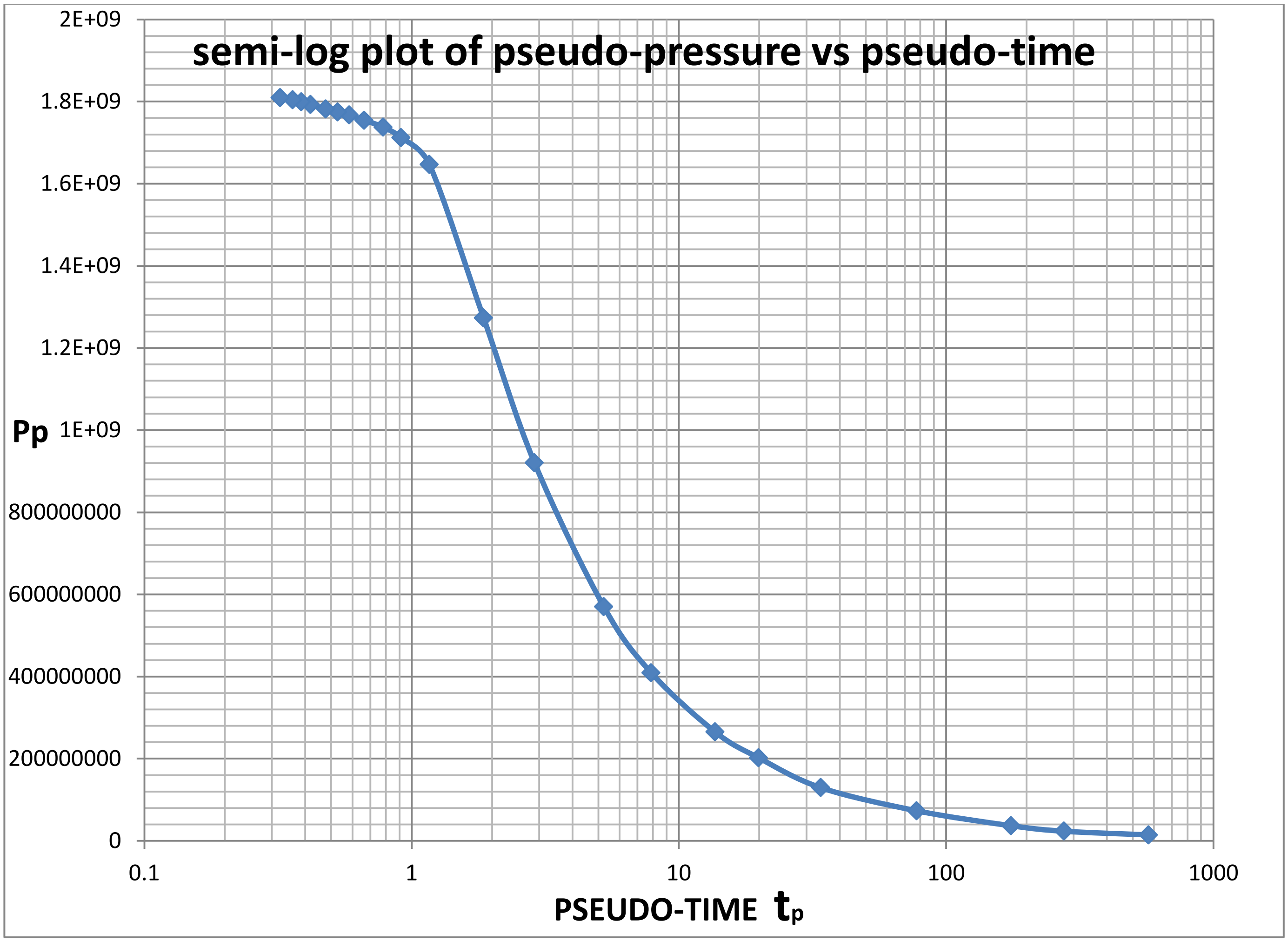

Figure 1: Semi-Log Plot Of Pseudo-Pressure Versus Pseudo-Time

From the semi-log plot the straight line gives a slope equal to

$1.5 \times 10^8 psi^2/cp$. The permeability is estimated from

$$k = \frac{q_t}{4\pi C_1 h(-m)} \qquad (3.1)$$

The q_t of the well is 200lbmol/day is converted to scf/day using the relation

1000scf/d =0.011441lb-mol/d

The permeability of the well is calculated to be equal to 0.00667md.

Rearranging (2.25) and solving for skin factor gives

$$S = \frac{1}{2}\left[\frac{p_{p_{(\Delta t=0)}} - p_{p_{(\Delta t=1)}}}{-m} - ln\left(\frac{C_1 k}{\emptyset r_w^2}\right) - 0.80907\right] \quad (3.2)$$

$$p_{p_{(\Delta t=0)}} = 1.72 \times 10^9 psi^2/cp$$

$$p_{p_{(\Delta t=1)}} = 1.406 \times 10^9 psi^2/cp$$

This gives a skin factor of 4.5.

4.0 ESTIMATION OF NON-DARCY COEFFICIENT AND ITS' EFFECT ON WELL DELIVERABILITY

One of the aims of this work is to estimate the effect of non-darcy flow on well deliverability. The non darcy coefficient can be estimated from pressure transient test. This is done by flowing the well at two different rates and solving the simultaneous equations as given in ion (2.26). Due to time constraints of performing two different test, especially when dealing with tight gas and retrograde gas reservoirs, and the revenue lost during pressure buildup test especially in high producing wells, several authors have come up with ways of estimating non darcy coefficient in the absence of field measurements.

An empirical relationship proposed by Economides et al [6]is:

$$D = \frac{6\times10^{-5}\gamma k_s^{-0.1} h}{\mu r_w h_{perf}^2} \tag{4.1}$$

From (4.1) the non-darcy coefficient is equal to 1.715×10^{-3}

Using (2.26) the non-darcy skin factor is calculated as 4.367.

To estimate the effect of non-darcy flow on well deliverability the general deliverability equation which incorporates Forchheimer non-darcy term is used. The equation is:

$$q(Mscf/d) = \frac{kh\left[\bar{p}_p - p_{p_{(pwf)}}\right]}{1424T\left[ln\left(0.427\frac{r_e}{r_w}\right) + s^* + Dq\right]} \qquad (4.2)$$

From (4.2) the inflow performance relation (IPR) for the well can be generated.

If non-darcy coefficient is neglected the IPR for the well is given as

$$\bar{p}_p - p_{p_{wf}} = 3.68 \times 10^6 q \qquad (4.3)$$

When bottom hole flowing pressure (P_{wf}) is at atmospheric pressure and d = 0 the flow rate in (4.3) is called the absolute open flow (AOF). It is calculated as $503 Mcf/d$

If non-darcy effect is incorporated the IPR for the well is given as

$$\bar{p}_p - p_{p_{wf}} = 3.68 \times 10^6 q + 610.9 q^2 \qquad (4.4)$$

The absolute open flow (AOF) when bottom hole flowing pressure is at atmospheric is calculated and is equal to $467 Mcf/d$.

CONCLUSIONS

The analytical model presented here was developed to account for the influence of non-darcy flow on the pressure response of retrograde gas systems. The model was developed using the pseudo-time function introduced by Penuela and Civian [1] and the common pressure function with non-darcy effect.

The answers obtained by applying the model to a particular retrograde gas well show that (the model offers a better understanding of the pressure behavior of retrograde systems. If condensate saturation is very low as in our case study, it can be conveniently assumed that its effect on the effective permeability to gas is negligible. Hence the more general single phase analogy can be used.

When the common Horner’s plot analysis was used in analyzing the build up data, it shows a different result with that obtained from the model. The Horner method does not consider the influence of non-darcy effect Using Horner’ plot analysis, the well permeability was calculated as 0.0056md and a skin factor of 4.21 was obtained. The model derived in this work can therefore serve as a very good alternative to the popular and rather expensive compositional and numerical simulations often used.

The influence of non-darcy effect on well deliverability was also studied. From the analysis it can be seen that when non-darcy effect is neglected, the well potential was 503Mcf/d. This was contrary to the lower flow rate (467Mcf/d) obtained when non-darcy effect was incorporated in estimating the well deliverability.

It can therefore be concluded that the well deliverability is overestimated when non-darcy effect is neglected especially for well producing at high rates. Hence for proper well deliverability analysis, the non-darcy effect should be incorporated.

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