SECOND DERIVATIVE PARALLEL MULTI-BLOCK METHODS FOR STIFF ODEs

By

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Abstract

A general theoretical background for second derivative multi–block methods and off-node form of block method developed in a previous study are presented herein. The proposed off-node block methods are L-stable for $k \leq 7$. Numerical results are included to justify their application on stiff IVPs in ODEs.

Keywords: Block methods, Off-node methods, Stiff IVPs.

1. INTRODUCTION

Numerical methods for integrating stiff initial value problems (IVPs) in ordinary differential equations (ODEs) of the form

$$y' = f(y(x)) \quad , f : R^n \to R^n ,\ y(x_0) = y_0 \qquad (1)$$

on parallel computers are currently receiving huge research interest. Among these methods are the parallel block methods developed in [1-7]. Chu and Hamilton [1] defined r-block k-point block methods as:

$$Y_m = \sum_{i=1}^{r} A_i Y_{m-i} + h \sum_{i=0}^{r} B_i F_{m-i} \qquad (2)$$

where $A_i, B_i, i = 0,1,\ldots,r$ are k x k matrices, Y_m, F_m are vectors of the solution and its derivatives respectively (see [1]). If k =1, then (2) is the classical linear multi-step method (LMM) of r-step method.

Sommeijer et al [6] developed methods of the form (2) for r=1, such methods are called One-block k-point methods. Classical r-step methods for integrating stiff IVP (1) are necessarily implicit. However, Dahlquist order barrier places severe restriction on the order an A-stable LMM can attain (see [8-11]). In [12], Dahlquist order barrier is circumvented by developing LMMs that incorporate the second derivative component ($y'' = \frac{d^2y}{dx^2}$) directly into their formulas.

Methods whose formulas contain second derivative components are called second derivative LMM ([2], [3], [9], [10], [11], [12]). In [12], the second derivative LMM is thus given as:

$$y_{n+1} = \sum_{i=1}^{r} a_i y_{n+1-i} + h\sum_{i=0}^{r} b_i f_{n+1-i} + h^2 \sum_{i=0}^{r} d_i f'_{n+1-i}. \qquad (3)$$

In this paper a straight forward generalization of (3) to multi-block methods is given. This paper is organized as follows: section 2 is on the theory of second derivative multi–block methods for the IVP (1); Off-node second derivative parallel block backward differentiation type formulas are developed in Section 3. Section 4, is on the stability of off-node second derivative parallel block backward differentiation type formulas. Numerical experiment is performed in section 5, in section 6, is the conclusion.

2. Theory of Second Derivative Multi-Block Methods.

Let y_{n+i} denote the numerical approximate to the solution value $y(x_{n+i})$ of (1). By introducing the k-vectors

$$Y_{m-i} = \begin{pmatrix} y_{n-ik+c_1} \\ y_{n-ik+c_2} \\ \vdots \\ y_{n-k(i-c_k)} \end{pmatrix},\ F(Y_{m-i}) = \begin{pmatrix} f_{n-ik+c_1} \\ f_{n-ik+c_2} \\ \vdots \\ f_{n-k(i-c_k)} \end{pmatrix} \text{ and } F'(Y_{m-i}) = \begin{pmatrix} f'_{n-ik+c_1} \\ f'_{n-ik+c_2} \\ \vdots \\ f'_{n-k(i-c_k)} \end{pmatrix}, \tag{4a}$$

$i = 0,1,2,\cdots,r.$ The (3) can be generalized into the second derivative r-block k–point block. The block formalism of (3) is given by the finite difference equation

$$\sum_{i=0}^{r} A_i Y_{m-i} = h\sum_{i=0}^{r} B_i F_{m-i} + h^2 \sum_{i=0}^{r} D_i F'_{m-i} \tag{4b}$$

where $A_i, B_i\ and\ D_i,\quad i = 0,1,\ldots,r$ are carefully chosen k-by-k matrices. A_0 is an k-by-k unit matrix and h the step-length. The second derivative r–block, k–point method (4b) is explicit; if the coefficient matrices $B_0\ and\ D_0,$ are null or strictly lower triangular otherwise it is implicit.

DEFINITION 1

Method (4b) is said to be parallel if the matrices $B_0\ and\ D_0$ are diagonal matrices.

DEFINITION 2

Let $Z_{m-i} = \left(y(x_{n-ik+1}) \quad y(x_{n-ik+2}) \quad \cdots \quad y(x_{n-k(i-1)}) \right)^T, \quad i = 0,1,\cdots,r$, be the theoretical solution to (1). The local truncation error (l.t.e) of (4b) is given by the vector E_m :

$$\mathrm{E}_m = \mathrm{Z}_m - \sum_{i=1}^{r} A_i Y_{m-i} - h\sum_{i=0}^{r} B_i F_{m-i} - h^2 \sum_{i=0}^{r} D_i F'_{m-i} \tag{5}$$

DEFINITION 3

The second derivative block method (4b) has error order $p \geq 1$ provided there exist a constant C such that the local truncation error E_m satisfies:

$$\left\| \mathrm{E}_m \right\| = Ch^{p+1} Y^{(p+1)}(x_n^*) + O(h^{p+2}), \quad x_n \leq x^* \leq x_{n+1} \tag{6}$$

where $\|.\|$ may be the maximum norm, the C is called the error constant of (4b).

DEFINITION 4

The second derivative block method (4b) is zero stable if the roots $R_j,\ j=1,2,\ldots,r$ of the first characteristics polynomial

$$\rho(R)=\det\left(\sum_{i=1}^{r}A_iR^{r-i}\right)=0 \tag{7}$$

satisfies $|R_j|\le 1$, with $|R_j|=1$ is simple.

When (4b) is applied to the test equation

$$y'=\lambda y,\ \ with\quad \mathrm{Re}(\lambda)<0 \tag{8}$$

yields the characteristic equation

$$\pi(\mathrm{R},\mu)=\det(\sum_{i=0}^{r}A_iR^{r-i}-\mu\sum_{i=0}^{r}B_iR^{r-i}-\mu^2\sum_{i=0}^{r}D_iR^{r-i})=0 \tag{9a}$$

where $\mu=\lambda h$. By Setting $\rho(R)=\sum_{i=0}^{r}A_iR^{r-i}$, $\sigma(R)=\sum_{i=0}^{r}B_iR^{r-i}$ and $\gamma(R)=\sum_{i=0}^{r}D_iR^{r-i}$,

we rewrite (9a) as

$$\pi(R,\mu)=\det(\rho(R)-\mu\sigma(R)-\mu^2\gamma(R))=0 \tag{9b}$$

The stability region associated with (4b) is the set

$$Z=\{\mu: all\ roots\ R_j(\mu);\ j=1(1)k\ of\ (9b)\, are\ such\ that\ |R_j(\mu)|\le 1\}$$

DEFINITION 5

The second derivative block method (4b) is A–stable if the stability region Z contains the entire left half plane $\mathbb{C}^-=\{\mu\in C;\ \mathrm{Re}(\mu)<0\}$.

DEFINITION 6

The second derivative block method (4b) is L–stable, if it is A–stable and in addition (9b) has vanishing roots as $\mu\to-\infty$.

3. Off-node Block Method.

In [2], second derivative parallel block backward differentiation type formulas (SDBBDF) which is given as:

$$\mathrm{Y}_{\mathrm{m}} = A_1 Y_{m-1} + hB_0 F(Y_m) + h^2 D_0 F'(\mathrm{Y}_m), \tag{10}$$

where $B_0, and\ D_0$ are diagonal matrices and k-vectors as specified in (4a). The SDBBDF (10) is a generalization of one-step second derivative backward differentiation formulas (SDBDF) developed in [10]. In this paper, we present off-node SDBBDF a variant of SDBBDF (10). The $c_i's$ in (4a) for SDBBDF (10) are given as $c_i = i, i = 1(1)k;$ by specifying the k-vectors as

$$Y_{m-1} = \begin{pmatrix} y_{n-(k-1)} \\ y_{n-(k-2)} \\ \vdots \\ y_n \end{pmatrix}, Y_m = \begin{pmatrix} y_{n+c_1} \\ y_{n+c_2} \\ \vdots \\ y_{n+c_k} \end{pmatrix}, \ F(Y_m) = \begin{pmatrix} f_{n+c_1} \\ f_{n+c_2} \\ \vdots \\ f_{n+c_k} \end{pmatrix} \text{ and } F'(Y_m) = \begin{pmatrix} f'_{n+c_1} \\ f'_{n+c_2} \\ \vdots \\ f'_{n+c_k} \end{pmatrix}, c_i = \frac{i}{k}, \ i = 1(1)k; \tag{11}$$

an off-node variant of SDBBDF (10) is developed. Substituting (11) into (10) and using methods of undetermined coefficients and Taylor's series expansion, elements of $A_1, B_0, and\ D_0$ are determined. In what follows, we present coefficient matrices $A_1, B_0, and\ D_0$ for proposed off-node SDBBDF for block sizes $k \leq 7$.

Two Point Block Method

$$A_1 = \begin{pmatrix} -\frac{1}{26} & \frac{27}{26} \\ -\frac{1}{7} & \frac{8}{7} \end{pmatrix}, \ B_0 = \begin{pmatrix} \frac{6}{13} & 0 \\ 0 & \frac{6}{7} \end{pmatrix}, \ D_0 = \begin{pmatrix} -\frac{9}{104} & 0 \\ 0 & -\frac{2}{7} \end{pmatrix}, \ C_4 = \begin{pmatrix} \frac{9}{1664} & \frac{1}{21} \end{pmatrix}^T, \ p = 3. \tag{12}$$

Three Point Block Method

$$A_1 = \begin{pmatrix} \frac{32}{10665} & -\frac{343}{10665} & \frac{10976}{10665} \\ \frac{125}{7101} & -\frac{1024}{7101} & \frac{8000}{7101} \\ \frac{4}{85} & -\frac{27}{85} & \frac{108}{85} \end{pmatrix}, \quad B_0 = \begin{pmatrix} \frac{364}{1185} & 0 & 0 \\ 0 & \frac{440}{789} & 0 \\ 0 & 0 & \frac{66}{85} \end{pmatrix}, \quad D_0 = \begin{pmatrix} -\frac{392}{10665} & 0 & 0 \\ 0 & -\frac{800}{7101} & 0 \\ 0 & 0 & -\frac{18}{85} \end{pmatrix},$$

$$C_5 = \begin{pmatrix} \frac{2267}{4483350} & \frac{36346}{9436905} & \frac{9}{425} \end{pmatrix}^T, \; p = 4 \qquad (13)$$

Four Point Block Method

$$A_1 = \begin{pmatrix} -\frac{30375}{65376512} & \frac{274625}{65376512} & -\frac{1601613}{65376512} & \frac{66733875}{65376512} \\ -\frac{1125}{351136} & \frac{9261}{351136} & -\frac{42875}{351136} & \frac{385875}{351136} \\ -\frac{456533}{46606592} & \frac{3472875}{46606592} & -\frac{13476375}{46606592} & \frac{57066625}{46606592} \\ -\frac{9}{415} & \frac{64}{415} & -\frac{216}{415} & \frac{576}{415} \end{pmatrix}, \quad B_0 = \begin{pmatrix} \frac{118755}{510754} & 0 & 0 & 0 \\ 0 & \frac{4620}{10973} & 0 & 0 \\ 0 & 0 & \frac{211365}{364114} & 0 \\ 0 & 0 & 0 & \frac{60}{83} \end{pmatrix},$$

$$D_0 = \begin{pmatrix} -\frac{342225}{16344128} & 0 & 0 & 0 \\ 0 & -\frac{11025}{175568} & 0 & 0 \\ 0 & 0 & -\frac{1334025}{11651648} & 0 \\ 0 & 0 & 0 & -\frac{72}{415} \end{pmatrix}$$

$$C_6 = \begin{pmatrix} \frac{4448925}{33472774144} & \frac{25725}{22472704} & \frac{102719925}{23862575104} & \frac{24}{2075} \end{pmatrix}^T, \; p = 5. \qquad (14)$$

Five Point Block Method

$$A_1 = \begin{pmatrix} \frac{144}{12019} & -\frac{1125}{12019} & \frac{4000}{12019} & -\frac{9000}{12019} & \frac{18000}{12019} \\ \frac{60665724}{74990603125} & -\frac{525926016}{74990603125} & \frac{2242946629}{74990603125} & -\frac{7533161856}{74990603125} & \frac{8074607644}{74990603125} \\ \frac{205006464}{78201353125} & -\frac{1710777536}{78201353125} & \frac{6811962624}{78201353125} & -\frac{19486825371}{78201353125} & \frac{92381986944}{78201353125} \\ \frac{63521199}{10373884375} & -\frac{512096256}{10373884375} & \frac{1920081024}{10373884375} & -\frac{4818200576}{10373884375} & \frac{13720578984}{10373884375} \\ \frac{144}{12019} & -\frac{1125}{12019} & \frac{4000}{12019} & -\frac{9000}{12019} & \frac{18000}{12019} \end{pmatrix}$$

$$B_0 = \begin{pmatrix} \frac{3736656}{19895065} & 0 & 0 & 0 & 0 \\ 0 & \frac{40982172}{119984965} & 0 & 0 & 0 \\ 0 & 0 & \frac{58792968}{125122165} & 0 & 0 \\ 0 & 0 & 0 & \frac{9662184}{16598215} & 0 \\ 0 & 0 & 0 & 0 & \frac{8220}{12019} \end{pmatrix}$$

$$D_0 = \begin{pmatrix} -\frac{6830208}{497376625} & 0 & 0 & 0 & 0 \\ 0 & -\frac{123370632}{2999624125} & 0 & 0 & 0 \\ 0 & 0 & -\frac{231727392}{3128054125} & 0 & 0 \\ 0 & 0 & 0 & -\frac{45849888}{414955375} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1800}{12019} \end{pmatrix}$$

$$C_7 = \begin{pmatrix} \frac{300529152}{7771509765625} & \frac{15380205456}{46869126953125} & \frac{415718941248}{341230919921875} & \frac{20907548928}{6483677734375} & \frac{600}{84133} \end{pmatrix}^T, \; p = 6 \qquad (15)$$

Six Point Block Method

$$A_1 = \begin{pmatrix} -\frac{16152323403125}{475805888841671424} & \frac{153982037280799}{475805888841671424} & -\frac{350775525953125}{237902944420835712} & \frac{1095115763546875}{237902944420835712} & -\frac{7014487849890625}{475805888841671424} & \frac{4811938665024968 75}{475805888841671424} \\ -\frac{18839275}{72771950421} & \frac{175616000}{72771950421} & -\frac{771656704}{72771950421} & \frac{2249728000}{72771950421} & -\frac{6028568000}{72771950421} & \frac{77165670400}{72771950421} \\ -\frac{6251175}{7253380864} & \frac{57066625}{7253380864} & -\frac{121287375}{3626690432} & \frac{332812557}{3626690432} & -\frac{1540798875}{7253380864} & \frac{8320313925}{7253380864} \\ -\frac{2921811200}{1415916119601} & \frac{26156812000}{1415916119601} & -\frac{107850176000}{1415916119601} & \frac{280368328625}{1415916119601} & -\frac{574194337024}{1415916119601} & \frac{1794357303200}{1415916119601} \\ -\frac{1940449395472489}{469116106139167488} & \frac{17056207271901875}{469116106139167488} & -\frac{34189515834175625}{234558053069583744} & \frac{84670026288299375}{234558053069583744} & -\frac{312534815292573125}{469116106139167488} & \frac{665574142647063727}{469116106139167488} \\ -\frac{100}{13489} & \frac{864}{13489} & -\frac{3375}{13489} & \frac{8000}{13489} & -\frac{13500}{13489} & \frac{21600}{13489} \end{pmatrix}$$

$$B_0 = \begin{pmatrix} \frac{402399852400}{2549542871451} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{28836080}{99824349} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{11275110}{28333519} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{956041240}{1942271769} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1449228745580}{2513696556387} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1260}{1927} \end{pmatrix}$$

$$D_0 = \begin{pmatrix} -\frac{1795533000625}{183567086744472} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{26499200}{898419141} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{12006225}{226668152} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1370784800}{17480445921} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{19057721215225}{180986152059864} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1800}{13489} \end{pmatrix}$$

$$C_8 = \begin{pmatrix} \frac{68741980928928125}{49331554555104493240 32} & \frac{688979200}{5894527984101} & \frac{396205425}{928432750592} & \frac{128168378800}{114689205687681} & \frac{11885252547268995125}{48637957884508885155 84} & \frac{450}{94423} \end{pmatrix}^T, P = 7$$

(16)

Seven Point Block Method

$$A_1 = \begin{pmatrix} \frac{50484527596800}{3979138943535030331} & -\frac{516187414562600}{3979138943535030331} & \frac{2468658002976000}{3979138943535030331} & -\frac{7539206115024000}{3979138943535030331} & \frac{17839437047283456}{3979138943535030331} & -\frac{47037578152016925}{3979138943535030331} & \frac{4013873335638777600}{3979138943535030331} \\ \frac{239615515828800}{2433789106808380069} & -\frac{2417792600678400}{2433789106808380069} & \frac{11339671166866944}{2433789106808380069} & -\frac{33552080381952000}{2433789106808380069} & \frac{74748808961281125}{2433789106808380069} & -\frac{167995128398028800}{2433789106808380069} & \frac{2551426012545062400}{2433789106808380069} \\ \frac{128499849241216}{386983472658061993} & -\frac{1280385124884000}{386983472658061993} & \frac{5895848794320000}{386983472658061993} & -\frac{16940898093324375}{386983472658061993} & \frac{35750708616240000}{386983472658061993} & -\frac{70257292572634848}{386983472658061993} & \frac{433686991189104000}{386983472658061993} \\ \frac{9095453452800000}{11310800211882887401} & -\frac{89547874099200000}{11310800211882887401} & \frac{405265071387515625}{11310800211882887401} & -\frac{1133203273320628224}{11310800211882887401} & \frac{2277044900416000000}{11310800211882887401} & -\frac{3990901835980800000}{11310800211882887401} & \frac{13833047770027200000}{11310800211882887401} \\ \frac{5776455114144000}{3544427205628291423} & -\frac{56224584310853673}{3544427205628291423} & \frac{250325110324640000}{3544427205628291423} & -\frac{682440713832240000}{3544427205628291423} & \frac{1311551755319520000}{3544427205628291423} & -\frac{2082392011513099000}{3544427205628291423} & \frac{4797831194526180096}{3544427205628291423} \\ \frac{13558009322803725}{4589917074183198541} & -\frac{130532699789107200}{4589917074183198541} & \frac{572234645466432000}{4589917074183198541} & -\frac{1523555725273088000}{4589917074183198541} & \frac{2811388813176580416}{4589917074183198541} & -\frac{4094876741995929600}{4589917074183198541} & \frac{6941700773275507200}{4589917074183198541} \\ \frac{3600}{726301} & -\frac{34300}{726301} & \frac{148176}{726301} & -\frac{385875}{726301} & \frac{686000}{726301} & -\frac{926100}{726301} & \frac{1234800}{726301} \end{pmatrix}$$

$$B_0 = \begin{pmatrix} \frac{4607507927880}{33822122955019} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{5192751144240}{20686866074581} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1140884824680}{3289305244057} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{41265001720800}{96140215487449} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{15129404638920}{30127134150127} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{22176741684240}{39013651405309} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{457380}{726301} \end{pmatrix}$$

$$D_0 = \begin{pmatrix} -\frac{12192514567200}{1657284024795931} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{22713260803200}{1013656437654469} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{6496858951200}{161175956958793} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{280390144320000}{4710870558885001} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{117410649127200}{1476229573356223} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{191526894748800}{1911668918860141} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{88200}{726301} \end{pmatrix}$$

$$C_9 = \left(\frac{557482407727760800}{95539126034276078247331}, \frac{283491779171673600}{58435276454469205456669}, \frac{162632621695914000}{92914731785200684 5193}, \frac{1229604246224640000 0}{27157231308730812649801}, \right.$$

$$\left. \frac{832956804605239600 0}{851016972071352770662 3}, \frac{208251023198265216 00}{11020390895113859696941}, \frac{2450}{726301} \right)^T \qquad p = 8. \tag{17}$$

4. Stability Analysis of Proposed Off-node Block Methods.

The roots R of characteristic polynomial equation $\rho(R) = \det(I - A_1 R) = 0$, are eigenvalues of matrix A_1. Eigenvalues of A_1 for methods with coefficient matrices in (12)-(17) are $R \leq 1$ and eigenvalues with $R = 1$ being simple. Thus proposed off-node SDBBDF with coefficient matrices in (12)-(17) are zero stable. An A-stable block method of the form (10) is L-stable, see [2]. Therefore, A-stable off-node SDBBDF implies L-stable method. Applying our proposed off-node SDBBDF block methods (12)-(17) to the test equation (8) yield the characteristic polynomial defined by

$$\pi(\mathrm{R}, \mu) = \det(IR - A_1 - \mu B_0 R - \mu^2 D_0 R) = 0. \tag{18}$$

The boundary locus of the characteristics roots for block sizes $k \leq 7$ are shown in figures (1)-(6)

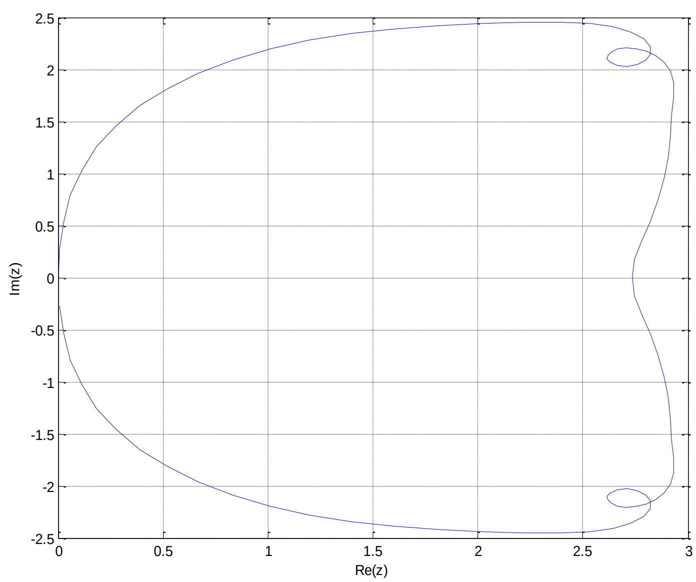

Fig. (1): Stability Region of Off-node SDBBDF (12)

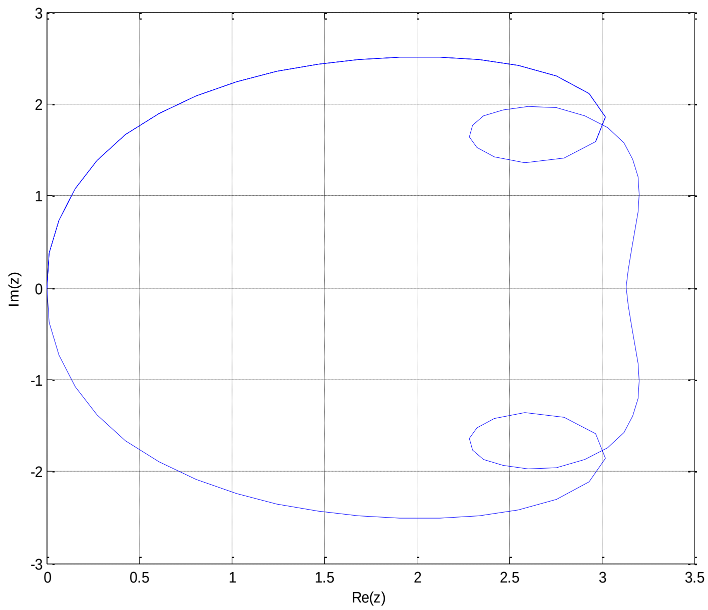

Fig. (2): Stability Region of Off-node SDBBDF (13)

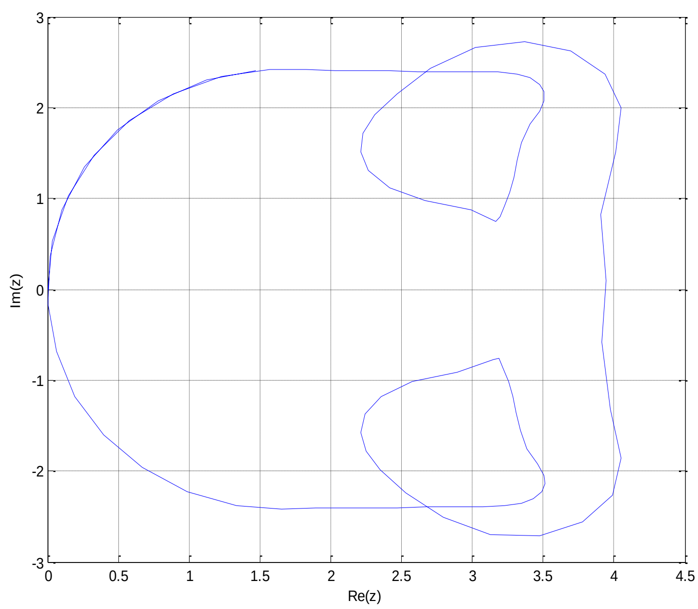

Fig. (3): Stability Region of Off-node SDBBDF (14)

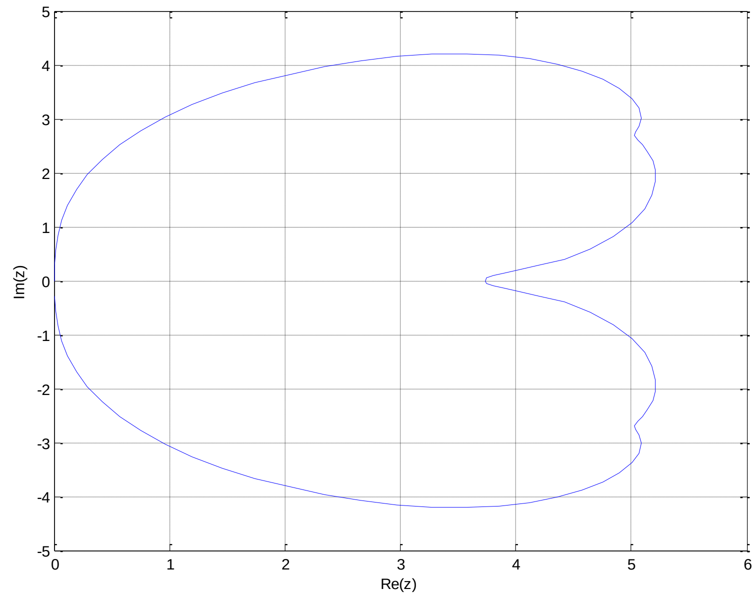

Fig. (4): Stability Region of Off-node SDBBDF (15)

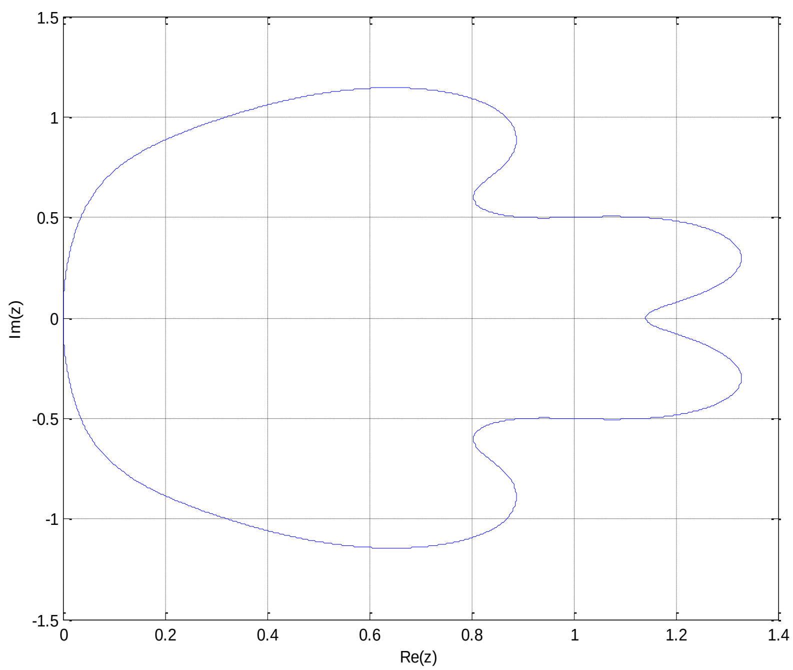

Fig. (5): Stability Region of Off-node SDBBDF (16)

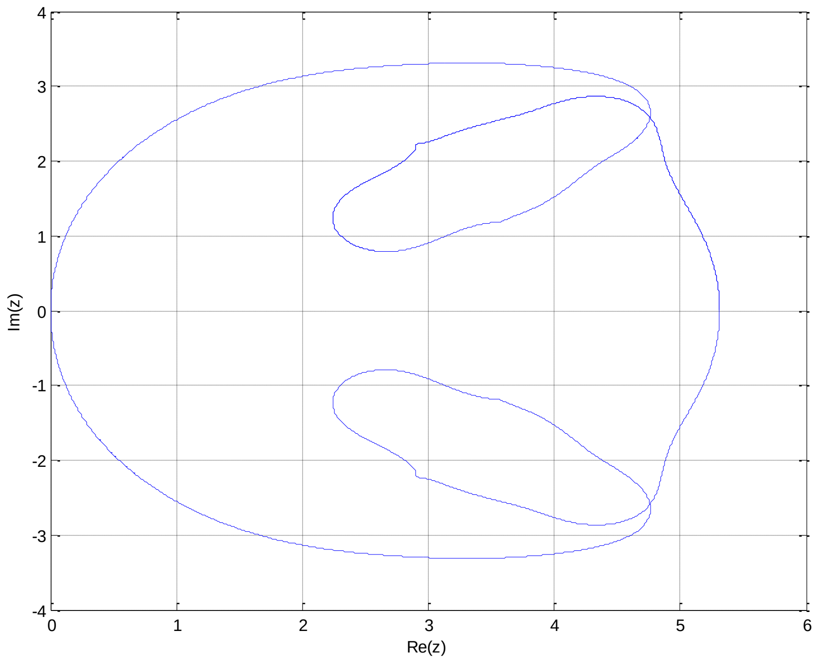

Fig. (6): Stability Region of Off-node SDBBDF (17)

Observe from figures (1)–(6), that the regions of absolute stability for block sizes $k \le 7$ include the entire left of the complex plane, thus our proposed off-node SDBBDF are A-stable and hence L-stable for block sizes $k \le 7$. In [4], construction and prove for L-stable off-node SDBBDF of block size k=8 is shown.

5. Numerical Experiments

In this section, we tested the proposed off-node block size k=2 on the stiff problem (see [13])

$$y' = \begin{pmatrix} -10 & \alpha & 0 & 0 & 0 & 0 \\ -\alpha & -10 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.1 \end{pmatrix} y, \quad y(x) = \begin{pmatrix} e^{-10x}(\cos(\alpha x) + \sin(\alpha x)) \\ e^{-10x}(\cos(\alpha x) - \sin(\alpha x)) \\ e^{-4x} \\ e^{-x} \\ e^{-0.5x} \\ e^{-0.1x} \end{pmatrix}, \quad y(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, 0 \le x \le 3.$$

Using a fixed step-size h=0.01 and inverse Euler's method in [9] to generate starting values. The plots of numerical solution generated by proposed off-node SDBBDF compared with numerical solutions generated by SDBBDF of block size k=2 and SDBDF of step-size k=2 are shown in figure 7.

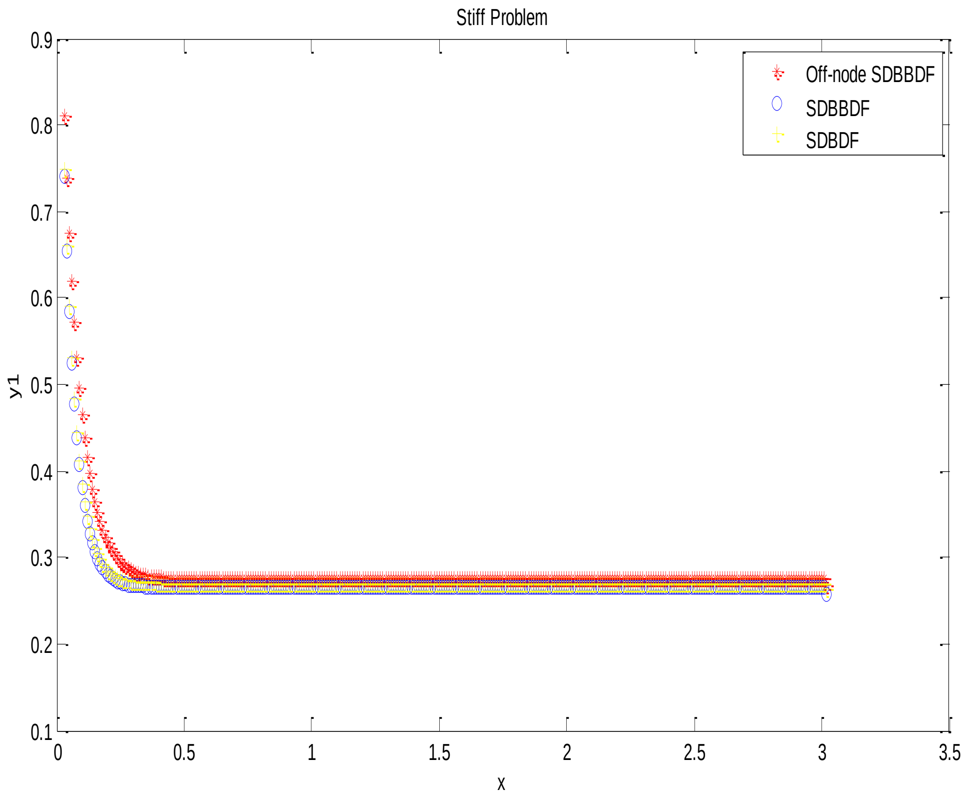

Fig. (7): y_1 Component Generated by Off-node SDBBDF, SDBBDF and SDBDF

From figure (7), the numerical solution of stiff problem generated by proposed off-node SDBBDF compares favourable with SDBBDF and SDBDF.

6. Conclusion

Theory on second derivative multi-block methods is developed, in addition off-node SDBBDF a variant of SDBBDF developed in [2] is proposed. The family of off-node SDBBDF has higher order L-stable methods compared to family of SDBBDF. The numerical result shows that the proposed off-node SDBBDF is suitable for integrating stiff IVPs in ODEs (1).

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