Time Series Forecasting by Fourier Series Analysis (FSA); Constant Level Versus Trend Line Seasonal – Effects Models

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Abstract

In this paper, the Fourier series Models of seasonal data are derived for forecasting outputs that have constant-level, linear trend line and 12th difference trend-line seasonal-effects. A practical case of the Nigerian Defence Academy (NDA) Kaduna cadet's sick parade showed that results obtained from the 12th difference trend line and linear trend linear seasonal- effect models were reasonably better than the constant or mean model in terms of forecast values and relative accuracies of Mean absolute Deviation (MAD), Sum of squares of errors (SSE)

Key words: Fourier series analysis, forecasting, sick-parade, seasonal-effects, amplitudes, fitted values, fitted errors.

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1. **Introduction.**

Joseph Fourier showed that any periodic observation can be represented by sine and cosine terms. We considered the works of Garnett [1], Hintz et al [2], Herman [3], Newton [4], Morse et al [5] Bajpai et al [6] Dean [7] Fourier series analysis has always been done using a mean or constant level but 1998, Durlrigo [8], and other researchers showed that a (FSA) model can have trend line models and this study considers such models.

The study aims at developing the constant level, least square linear and 12^{th} difference trend line seasonal -effect models from a given time series data with a view to examining the forecast and relative accuracies of these models [9]

The objective is to use available data to derive spectral models using computer packages.

In order to achieve the aim above, the following additional objectives will be followed through: Determine the Fourier coefficients ak and bk up to the highest permissible harmonic, determine the significant frequencies or harmonics, determine an equation each for the constant level, least square linear trend and 12^{th} difference trend line seasonal-effects model, determine the frequencies that minimize the Sum of Squares of Errors (SSE), make forecast with the models developed and compare the forecast accuracies of the developed models

Based on the stated objectives, the following questions now arise:

(a) In the developed models, are the harmonics statistically significant?

(b) Among these three types of models are the forecasting powers the same?

The work will not be concerned with complex Fourier series and double Fourier series. The monthly sick parade report of cadets of the full (NDA) Medical Centre Kaduna, from 2000-2009, constitutes the data set. This is contained in Table 1.

Table 1 Actual Values for Cadets Sick Parade 2002-2008

Year/Month	2002	2003	2004	2005	2006	2007	2008	2009
Jan	124	100	129	94	93	100	137	126
Feb	111	105	97	99	87	129	160	132
Mar	175	158	180	184	195	182	107	24
Apr	125	230	100	120	142	236	210	160
May	190	180	283	184	152	300	150	319
Jun	185	204	159	221	144	215	136	270
Jul	151	178	143	202	110	145	73	154
Aug	220	254	231	231	297	376	169	264
Sep	200	298	194	200	209	145	169	128
Oct	250	300	350	210	206	244	179	192
Nov	170	123	167	172	236	250	205	305
Dec	90	80	87	82	130	160	156	209

2. **Methodology**

The NCSS and EXCELL packages will be employed to obtain results.

The Fourier synthesis equation is expressed as

$$\hat{Y}_t = a_o + \sum_{K=1}^{n} a_k Cos(kwt) + b_k Sin(kwt) \quad (1)$$

For a constant level seasonal – effect model, it is given by

$$\hat{Y}_t = a_0 + b_o t + Y\sum_{K=1}^{n} a_k Cos(kwt) + b_k Sin(kwt) \quad (2)$$

It will be used for the 12th difference trend level model and for the least squares trend line model we have

$$\hat{Y}_t = \alpha + \beta t + \sum_{K=1}^{n} a_k Cos(kwt) + b_k Sin(kwt) \quad (3)$$

However for a discrete data of seasonal length Y

$$\hat{Y}(t) = a_0 + \sum_{k=1}^{\gamma 1} C_k Cos(kwt), \text{ and } y(t) = a_o + \sum_{K=1}^{n} a_k Cos(kwt) + b_k Sin(kwt) \quad (4)$$

with $c_k = a^2_k + b^2_k$ (5)

For the 12th difference trend line model, we determine b_0, the monthly trend by taking seasonal differences, then sum it and divide by N-12 to get

$$\sum_{t=12}^{N} \frac{(y_t - y_{t-12})}{N-12},$$

This gives the annual trend, and dividing that by 12 we obtain the monthly trend, which is given by

$$b_0 = \frac{\sum_{t=12}^{N} y_t - y_{t-12}}{(N-12)12} \quad (6)$$

Since seasonal length is Y, then

$$b_0 = \frac{\sum_{t=12}^{N} y_t - y_{t-\gamma}}{(N-\gamma)\gamma}. \quad (7)$$

Next the mean,

$$Y = \frac{\sum_{t=1}^{N} y_t}{N}. \quad (8)$$

The mean value of time that is needed for computation is given by

$$t = \left(\frac{t_1 + t_n}{2}\right) = t_0. \quad (9)$$

At time t, line of best fit has the form

$T_t = a_0 + b_{0t}$ (10)

where, a_0 is the point of intersection of the mean value $\bar{Y}$ and Tt. At time t_0 the trend becomes $T_{t0} = a_0 + b_{0t}$. (11)

The constants α and β for the least squares trend line model were obtained by solving the systems of normal equations

$\Sigma Y_t = \alpha N + \beta \Sigma Xt$ (12)

$\Sigma Y_t X_t = \alpha \Sigma X_t + \beta \Sigma X_t^2$ (13)

The Fourier series synthesis equations are;

(a) Constant level model: $\tilde{Y}_t = a_0 + \sum_{K=1}^{n} a_k Cos(kwt) + b_k Sin(kwt)$. (14)

(b) For the trend line model $\tilde{Y}_t = a_0 + b_0 t + \sum_{K=1}^{n} a_k Cos(kwt) + b_k Sin(kwt)$ (15)

(c) Least squares trend line model now becomes

$$\tilde{Y}_t = \alpha + \beta t + \sum_{K=1}^{n} a_k Cos(kwt) + b_k Sin(kwt) \quad (16)$$

The corresponding analysis equations for obtaining coefficient values are given by

$$a_o = \frac{1}{N}\sum_{t=1}^{n} y_t ,$$

$$a_k = \frac{2}{N}\sum_{k=1}^{\gamma 1} y_t Cos(wkt) ,$$

$$b_k = \frac{2}{N}\sum_{k=1}^{\gamma 1} y_t \sin(wkt) , \quad (17)$$

$$\omega = \frac{2\pi f}{N} \quad (18)$$

Computation of the Fourier coefficients a_k and b_k. was from equation (17)

Note that to obtain a_k and b_k the Y_t used are as follows:

For the 12th difference model = Y_t - $(a_0 + b_t,)$ (19)

For the level model = Y_t - $\tilde{Y}_t$ and 20

For the least squares trend line = $Y_t - (\alpha + \beta t)$ (21)

3. **Results**

a. Trend and Seasonal Amplitudes. The mean or trend seasonal amplitude for time t was obtained for each model by the addition of the Figure 1 $a_k\cos(\omega t)$ and $b_k\sin(\omega t)$. Figure 1 shows the seasonal amplitudes for the 12th difference linear seasonal-effects model.

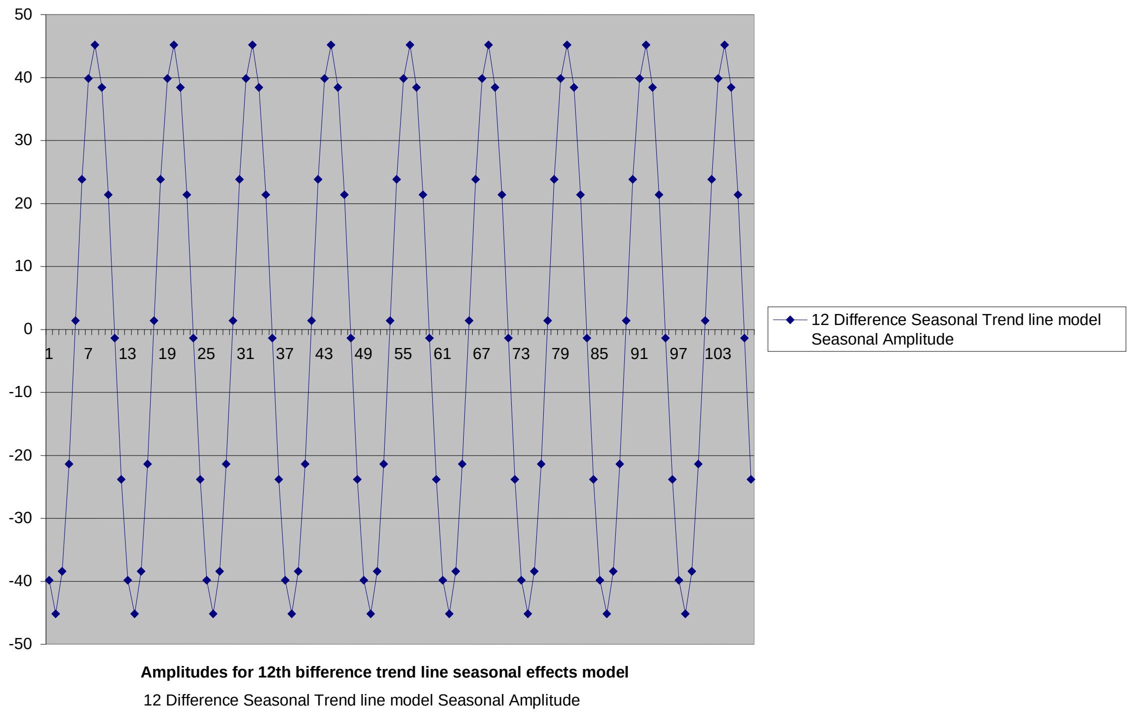


Fig.1.Seasonal Amplitudes for 12th term difference trend line seasonal-effects model.

b. **Fitted Values.**

Fitted values ($\hat{Y}_t$) were obtained at time (t) by the addition of the mean or trend value to the various seasonal amplitudes from section (1) above, Fitted values for the accepted model are contained in table 2.

Table3. Fitted values for 12th difference trend line seasonal-effects model.

Table 2. Fitted Values 12th Difference Trend Line Seasonal Effects Model for Cadets Sick Parade 2002-2008

Year/Month	2002	2003	2004	2005	2006	2007	2008	2009
Jan	122.8261	126.3019	129.7778	133.2536	136.7294	140.2053	143.6811	147.157
Feb	117.7521	121.2283	124.7044	128.1806	131.6568	135.1333	138.6092	142.0853
Mar	124.7907	128.2672	131.7437	135.2203	138.6968	142.1733	145.6498	149.0853
Apr	142.1336	145.604	149.0871	152.5639	156.0406	159.5174	162.9941	166.4708

May	165.2114	168.6882	172.165	175.6418	179.1187	182.5955	186.0723	189.5491
Jun	187.9179	191.3946	194.8713	198.348	201.8248	205.3016	208.7782	212.2549
Jul	204.2465	207.723	216.1995	214.676	218.1524	221.6289	225.1054	228.5819
Aug	209.8997	231.3759	216.852	220.3281	223.8043	227.2804	230.7566	234.2327
Sep	203.4403	206.9161	210.3919	213.8677	217.3435	220.8193	224.2951	227.7709
Oct	186.6766	190.1526	193.6277	197.1033	200.5789	204.0545	207.53	211.0056
Nov	164.1782	167.6537	171.1292	174.6047	178.0802	181.5557	185.0312	188.5067
Dec	142.0511	145.5267	149.0023	152.4779	155.9535	159.4291	162.9047	166.3803

c. **Fitted Errors (e_t)** The fitted error (e_t) for time t is given as the difference between the actual and the fitted values at time (t):

$$Y_t - \hat{Y}_t = e_t \quad (22)$$

The fitted error for the 12th difference seasonal -effects model is shown in Fig.2

d. **Significant and insignificant amplitude**

An amplitude Ak is termed significant if $\frac{AK}{RSE_k} \geq 0.5$, significant

$\frac{AK}{RSE_k} < 0.5$ insignificant, only significant amplitudes are included in any model. Table 3 shows the various amplitudes for each model and Table 4 is a display of the various components of the 12th difference trend line seasonal-effects mode

Table 3. Fourier coefficients

K		12th Diff.	mean level	Linear Trend
1	a1	-23.802	-23.5123	-23.7163
	b1	-38.4559	-39.537	-38.7757
	A1	45.22601	45.99999	45.45344
2	a2	-12.9007	-12.6109	-12.815
	b2	4.449817	-0.3311	-0.31473
	A2	13.66238	12.61527	12.81883
3	a3	6.48393	0.85329	0.0027

	b3	-10.7244	0.2058	-15.5766
	A3	12.53211	0.8769	15.5766
4	a4	-11.835	-11.5453	-11.7498
	b4	-18.5109	-18.6781	-18.5603
	A4	21.97093	21.95826	21.9665
5	a5	-9.98926	-9.69955	-9.90358
	b5	22.96327	22.8875	22.94034
	A5	25.04191	24.85635	24.98679
6	a6	11.18948	11.47916	11.27516
	b6	0.00615	0.006254	0.006181
	A6	11.18948	11.47917	11.27517
	SSE	322235.3	333753.5	320703.8
	RSE	62.68731	63.79788	62.53816

Table 4 Components of 12th term trend line model

T	Yt	Tt	$\hat{Y}$	$a_1$cosx	$b_1$sinx	Amplitude	e_t	et*et
1	124	166.7386	126.8118	-20.5389	-19.3879	-39.9268	-2.81181	7.906248
2	111	166.9426	121.5037	-11.8581	-33.5808	-45.4389	-10.5037	110.328
3	175	167.1466	128.371	8.71E-05	-38.7757	-38.7756	46.62901	2174.265
4	125	167.3506	145.6282	11.85825	-33.5806	-21.7224	-20.6282	425.5228
5	190	167.5546	168.7059	20.53899	-19.3876	1.151346	21.29405	453.4367
6	185	167.7586	191.4752	23.7163	0.000285	23.71658	-6.47518	41.92802
7	151	167.9626	207.8896	20.53882	19.38814	39.92695	-56.8896	3236.421
8	220	168.1666	213.6055	11.85795	33.58093	45.43888	6.39452	40.88989
9	200	168.3706	207.146	-0.00026	38.7757	38.77544	-7.14604	51.06587
10	250	168.5746	190.2967	-11.8584	33.5805	21.7221	59.7033	3564.484
11	170	168.7786	167.6269	-20.5391	19.3874	-1.15168	2.37308	5.63151
12	90	168.9826	145.2657	-23.7163	-0.00057	-23.7169	-55.2657	3054.301
13	100	169.1866	129.2595	-20.5387	-19.3884	-39.9271	-29.2595	856.1175
14	105	169.3906	123.9517	-11.8578	-33.5811	-45.4389	-18.9517	359.168
15	158	169.5946	130.8193	0.000436	-38.7757	-38.7753	27.18066	738.7885

16	230	169.7986	148.0768	11.85855	-33.5804	-21.7218	81.92321	6711.412
17	180	170.0026	171.1546	20.53917	-19.3872	1.152014	8.845386	78.24085
18	204	170.2066	193.9238	23.7163	0.000855	23.71715	10.07625	101.5307
19	178	170.4106	210.3379	20.53864	19.38863	39.92727	-32.3379	1045.738
20	254	170.6146	216.0535	11.85765	33.58122	45.43886	37.94654	1439.94
21	298	170.8186	209.5937	-0.00061	38.7757	38.77509	88.40631	7815.676
22	300	171.0226	192.7441	-11.8587	33.58022	21.72152	107.2559	11503.82
23	123	171.2266	170.0743	-20.5393	19.3869	-1.15235	-47.0743	2215.985
24	80	171.4306	147.7132	-23.7163	-0.00114	-23.7174	-67.7132	4585.072
25	129	171.6346	131.7072	-20.5386	-19.3889	-39.9274	-2.70717	7.328752
26	97	171.8386	126.3997	-11.8575	-33.5814	-45.4389	-29.3997	864.345
27	180	172.0426	133.2677	0.000784	-38.7757	-38.7749	46.73232	2183.909
28	100	172.2466	150.5254	11.85885	-33.5801	-21.7212	-50.5254	2552.814
29	283	172.4506	173.6033	20.53934	-19.3867	1.152682	109.3967	11967.64
30	159	172.6546	196.3723	23.7163	0.001424	23.71772	-37.3723	1396.691
31	143	172.8586	212.7862	20.53847	19.38912	39.92759	-69.7862	4870.113
32	231	173.0626	218.5014	11.85735	33.5815	45.43885	12.49855	156.2138
33	194	173.2666	212.0413	-0.00096	38.7757	38.77474	-18.0413	325.49
34	350	173.4706	195.1915	-11.859	33.57993	21.72093	154.8085	23965.66
35	167	173.6746	172.5216	-20.5394	19.38641	-1.15302	-5.52158	30.4879
36	87	173.8786	150.1606	-23.7163	-0.00171	-23.718	-63.1606	3989.26
37	94	174.0826	134.1548	-20.5384	-19.3894	-39.9278	-40.1548	1612.412
38	99	174.2866	128.8478	-11.8572	-33.5816	-45.4388	-29.8478	890.8889
39	184	174.4906	135.716	0.001132	-38.7757	-38.7746	48.28397	2331.342
40	120	174.6946	152.974	11.85916	-33.5798	-21.7206	-32.974	1087.282
41	184	174.8986	176.0519	20.53951	-19.3862	1.153349	7.948051	63.17151
42	221	175.1026	198.8209	23.7163	0.001994	23.71829	22.17911	491.9127
43	202	175.3066	215.2345	20.53829	19.38962	39.92791	-13.2345	175.1523
44	231	175.5106	220.9494	11.85704	33.58179	45.43883	10.05057	101.014
45	200	175.7146	214.489	-0.00131	38.7757	38.77439	-14.489	209.9309
46	210	175.9186	197.6389	-11.8593	33.57965	21.72034	12.36106	152.7957

47	172	176.1226	174.9689	-20.5396	19.38592	-1.15368	-2.96892	8.814467
48	82	176.3266	152.608	-23.7163	-0.00228	-23.7186	-70.608	4985.493
49	93	176.5306	136.6025	-20.5382	-19.3899	-39.9281	-43.6025	1901.18
50	87	176.7346	131.2958	-11.8569	-33.5819	-45.4388	-44.2958	1962.116
51	195	176.9386	138.1644	0.001481	-38.7757	-38.7742	56.83562	3230.288
52	142	177.1426	155.4226	11.85946	-33.5795	-21.72	-13.4226	180.1649
53	152	177.3466	178.5006	20.53969	-19.3857	1.154017	-26.5006	702.2827
54	144	177.5506	201.2695	23.7163	0.002564	23.71886	-57.2695	3279.791
55	110	177.7546	217.6828	20.53812	19.39011	39.92823	-107.683	11595.59
56	297	177.9586	223.3974	11.85674	33.58207	45.43881	73.60259	5417.341
57	209	178.1626	216.9366	-0.00166	38.7757	38.77404	-7.93664	62.99033
58	206	178.3666	200.0864	-11.8596	33.57936	21.71976	5.913644	34.97119
59	236	178.5706	177.4162	-20.5398	19.38542	-1.15435	58.58375	3432.056
60	130	178.7746	155.0555	-23.7163	-0.00285	-23.7191	-25.0555	627.7756
61	100	178.9786	139.0502	-20.538	-19.3904	-39.9284	-39.0502	1524.919
62	129	179.1826	133.7438	-11.8566	-33.5822	-45.4388	-4.7438	22.5036
63	182	179.3866	140.6127	0.001829	-38.7757	-38.7739	41.38727	1712.906
64	236	179.5906	157.8711	11.85976	-33.5792	-21.7195	78.12886	6104.119
65	300	179.7946	180.9493	20.53986	-19.3852	1.154685	119.0507	14173.07
66	215	179.9986	203.718	23.7163	0.003133	23.71943	11.28197	127.2828
67	145	180.2026	220.1312	20.53795	19.3906	39.92855	-75.1312	5644.69
68	376	180.4066	225.8454	11.85644	33.58236	45.4388	150.1546	22546.41
69	145	180.6106	219.3843	-0.002	38.7757	38.7737	-74.3843	5533.024
70	244	180.8146	202.5338	-11.8599	33.57908	21.71917	41.46623	1719.448
71	250	181.0186	179.8636	-20.5399	19.38493	-1.15502	70.13642	4919.117
72	160	181.2226	157.5029	-23.7163	-0.00342	-23.7197	2.497118	6.2356
73	137	181.4266	141.4979	-20.5379	-19.3909	-39.9287	-4.49789	20.23102
74	160	181.6306	136.1918	-11.8563	-33.5825	-45.4388	23.80819	566.8298
75	107	181.8346	143.0611	0.002178	-38.7757	-38.7735	-36.0611	1300.401
76	210	182.0386	160.3197	11.86006	-33.5789	-21.7189	49.68028	2468.13
77	150	182.2426	183.398	20.54004	-19.3847	1.155352	-33.398	1115.423

78	136	182.4466	206.1666	23.7163	0.003703	23.72	-70.1666	4923.352
79	73	182.6506	222.5795	20.53777	19.3911	39.92887	-149.579	22374.02
80	169	182.8546	228.2934	11.85614	33.58264	45.43878	-59.2934	3515.705
81	169	183.0586	221.8319	-0.00235	38.7757	38.77335	-52.8319	2791.215
82	179	183.2626	204.9812	-11.8602	33.57879	21.71858	-25.9812	675.0218
83	205	183.4666	182.3109	-20.5401	19.38444	-1.15569	22.68909	514.7946
84	156	183.6706	159.9503	-23.7163	-0.00399	-23.7203	-3.95031	15.60497
85	126	183.8746	143.9456	-20.5377	-19.3913	-39.929	-17.9456	322.0435
86	132	184.0786	138.6398	-11.856	-33.5828	-45.4388	-6.63983	44.08735
87	24	184.2826	145.5094	0.002526	-38.7757	-38.7732	-121.509	14764.54
88	160	184.4866	162.7683	11.86036	-33.5787	-21.7183	-2.76831	7.663545
89	319	184.6906	185.8466	20.54021	-19.3842	1.15602	133.1534	17729.82
90	270	184.8946	208.6152	23.7163	0.004273	23.72057	61.38483	3768.097
91	154	185.0986	225.0278	20.5376	19.39159	39.92919	-71.0278	5044.947
92	264	185.3026	230.7414	11.85584	33.58292	45.43876	33.25864	1106.137
93	128	185.5066	224.2796	-0.0027	38.7757	38.773	-96.2796	9269.761
94	192	185.7106	207.4286	-11.8605	33.57851	21.718	-15.4286	238.0416
95	305	185.9146	184.7582	-20.5403	19.38394	-1.15635	120.2418	14458.08
96	209	186.1186	162.3977	-23.7163	-0.00456	-23.7209	46.60226	2171.77

e. **Residual Plots**

In each model, the plot of the errors appears random and confirms the absence of autocorrelations and model misspecification. [9]. Fig.2 shows the residual errors for the 12^{th} difference trend line seasonal effects model Fitted values. Actual values and 12^{th} difference trend lines is displayed in Fig.3

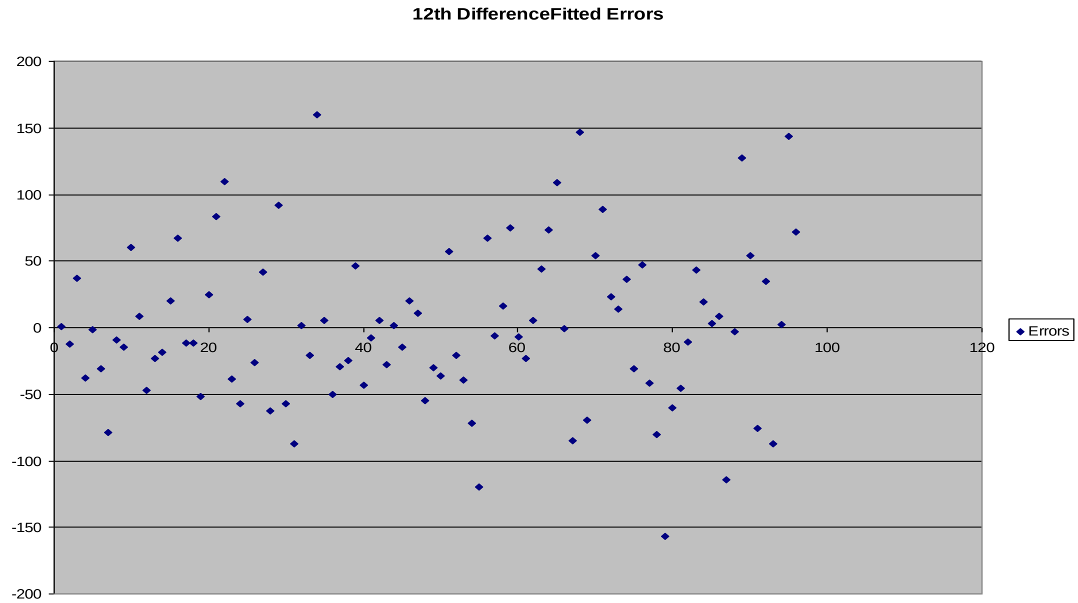
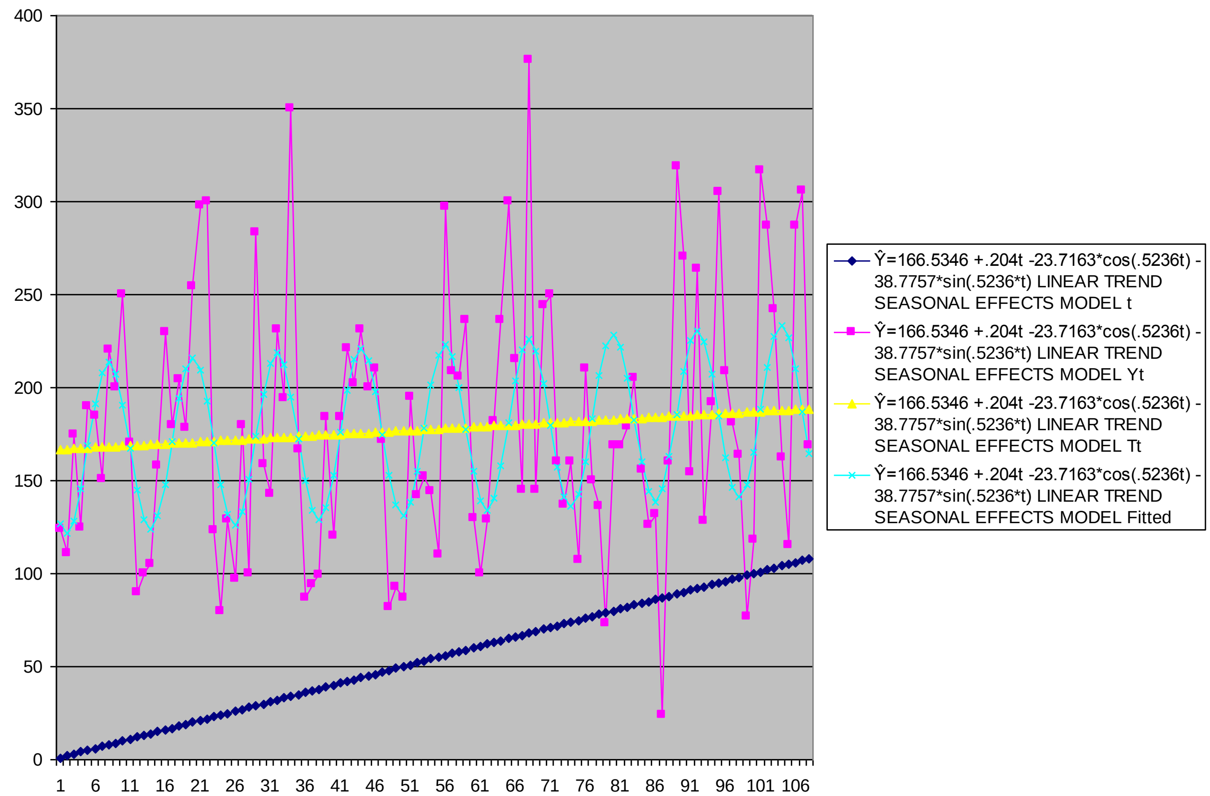


Fig.2 Fitted Errors of 12th difference Seasonal-effects model



. Fig.3.Actual and fitted values for 12th difference Trend line model

f. **Out of Sample Forecasts.**

Using the models developed, we made a one- year ahead out- of -sample forecast for each model. Table 5 shows the out of sample forecasts.

Table 5. Out of sample forecasts

		12th Diff.	Mean level	Linear Trend
T	Actual	Tt	Fitted	
1	181	146.3933	122.9598	126.8118
2	164	141.0878	123.3556	121.5037
3	77	147.9578	137.972	1.28E+02
4	118	165.21`69	162.8928	145.6282
5	317	188.2953	191.4402	168.7059
6	287	211.0637	215.9651	191.4752
7	242	227.4761	229.8959	207.8896
8	162	233.1893	229.4999	213.6055
9	115	226.7273	214.8833	207.146
10	287	209.876	189.9624	190.2967
11	306	187.2056	161.415	167.6269
12	169	164.8452	136.8902	145.2657
Total Error		760.8882	854.3729	777.8483
MAD		63.40735	71.1977	64.82069

The first harmonic is statistically significant for all the models considered. The 12^{th} difference trend line model has better fit statistics as shown in Table 6

Table 6. Fit statistics for the models.

Statistics	12^{th} difference. trend line model	Least squares linear model	mean level model
SSE	322235.1	332073.8	33375.5
RSE	62.69	71.56	71.82
Amplitude(A1)	45.46	45.45	45.6
Out of Sample MAD	63.41	64.82	71.2

4. **Summary and Conclusion**

The work showed that the data set is a seasonal time series with seasonality modelling and by using Fourier series method it gives a better result than using other forecasting methods. It also showed that in each model the first harmonic is significant and those with the lowest amplitude value performed better than others. The following models were obtained from the data set:

For the Linear trend line seasonal-effects model the equation is giving by

$$\hat{Y}_t = 166.54 + 0.2t - 23.72\cos(.5236t) - 38.78\sin(.5236t) \qquad (23)$$

For the Mean level seasonal-effects model the equation is giving by

$$\hat{Y}_t = 176.43 - 23.51\cos(.5236t) - 39.54(.5236t) \qquad (24)$$

For the 12^{th} difference trend line seasonal-effects model the equation is given by

$$\hat{Y}_t = 162.38 + 0.29t - 23.8\cos(.5236t) - 38.46\sin(.5236t) \qquad (25)$$

Findings made from this work:

1. Cadets' sick parade reports are a seasonal time series data set.
2. Among the significant amplitudes, those with smaller values have greater forecast accuracy.
3. Period outside examinations are periods when more cadets report sick at the NDA medical centre.
4. From Table 5 the Linear Trend under scored throughout.

5. Underscoring and over scoring fluctuations were common in both 12th Difference Trend Line and Mean Level models.
6. The 12th Difference Trend Model has the least Root Mean Square (RSE) (62.69) which makes it the best derived model.

The 12th difference trend line seasonal-effects model has better forecast and fitted accuracies than the least squares linear seasonal –effects and the mean or constant level model but these models have their first harmonics been statistically significant.

The model equation for the accepted 12th difference trend line seasonal –effects model is:

$$\hat{Y}_t = 162.38 + 0.29t - 23.8\cos(0.5236t) - 38.46\sin(0.5236t) \quad (26)$$

or in terms of phase angle, we have

$$\hat{Y}_t = 162.38 + 0.29t + 45.2\sin(0.5236t - 0.555) \quad (27)$$

References

1. Garnnet P.W, *Chaos_Theory_Tamed*, *Joseph* Henery Press(1997)
2. Hintz J Number *Cruncher Statistical System*(NCSS) Package (2009)
3. Herman R.L. (2002) *Fourier Analysis of Time Series*; Work in progress UNC Willington.
4. Newton H.J, *Times Lab. A Times series Analysis* Laboratory. Books Cole, Pacific Grove. (1998)
5. Morse P.M, Boden, R.H and Scheter H (1938) Acoustic Vibrations and external combustion engine performance/standing waves in the intake pipe system, *J. Appl. Phys., 9,16-23*
6. Bajpai A.C, Mustoe L.R, and Walker D, *Advanced Engineering Mathematics*. John Wiley and Sons (1998).
7. Dean M.G. *Advanced Engineering Mathematics*__pdf down load of CRC press (27Oct 2009)
8. DurLrigo S, *Forecasting Principles and Applications* MacGraw-Hill (1998)
9. Garnet Amir A and Jayavel S, *Complete Business Statistics* MacGraw-Hill/Irwin (2002).