COMPARISON OF SOME APPROXIMATELY UNBIASED RATIO ESTIMATORS USING A MONTE CARLO APPROACH UNDER NON NORMAL ERROR REGRESSION MODEL

F. B. ADEBOLA

Department Of Mathematical Sciences, Federal University Of Technology, Akure. Ondo State - Nigeria.
Emails:femi_adebola@yahoo.Com
GSM NO: 08050258756 OR 08038380448

Abstract

This paper compares seven approximately unbiased ratio estimators along side with the classical ratio estimator under a non-normal linear regression model. The variances, bias, mean squared errors and efficiencies of these estimators for the ratio of population means of two characters were compared using Monte Carlo. Our findings revealed that $\hat{R}_A$ estimator is the best.

Keywords: Approximately Unbiased, Mean Square error, Efficiency, Regression model, Ratio Estimator, Bias, Auxiliary variable.

1.0 Introduction

Over the years survey samplers Quenouille [1-10] among many others have been interested in methods of improving the precision of the estimates of population parameters both at the selection and estimation stages by making use of auxiliary information. Ratio estimators are often employed by these samplers to estimating the population mean of the characteristic of interest of the population ratio.

Let y and x be real variates taking y_i and x_i $(1 \leq i \leq N)$ for i^{th} unit of a population of Size N with means μ_y and μ_x respectively. Suppose that a simple random sample of size n units is drawn without replacement from the population. A commonly employed estimator in this context is traditional or classical ratio estimator $r = \frac{\bar{y}}{\bar{x}}$ where $\bar{y}$ and $\bar{x}$ denote the sample means of y and x values, respectively.

2. COMPARISON OF RATIO ESTIMATORS

Several Authors [2, 6, 8-15] and many others carried out some studies on the regression model under the ratio estimator.

They compared different ratio estimators under various assumptions of X or directly with real life data using the model $Y = \alpha + \beta X + e_i$

The comparison of these estimators were done with the assumptions of a simple random sampling (srs) mainly between the approximately unbiased or wholly unbiased ratio type estimators with the classical (biased) type $r = \frac{\bar{y}}{\bar{x}}$.

The comparisons carried out were done with respect to bias, efficiency of variance, approach to normality and mean square errors (MSE)

Durbin [2] compared the classical ratio $\hat{R}$ with Quenouille [1] estimator $\hat{R}_Q$ under the linear regression model $Y = \alpha + \beta X + e$ with two assumed models: X normally distributed (Model 1) and X following a gamma distribution (Model 2).

His conclusions are that estimator $\hat{R}_Q$ maintained smaller variance than that of $\hat{R}$ with the first assumed model but the result is reversed under the second model with $\hat{R}$ having a smaller variance than $\hat{R}_Q$; although $\hat{R}_Q$ still had smaller bias.

It was finally concluded that, generally, Quenouille [1] method will always lead to enhancement of the precision of the ratio estimators.

Lauh and wiliams [11] carried out a Monte Carlo study on the stabilities of V ($\hat{R}$) and V ($\hat{R}_Q$) with the sampling groups g = n for small samples (n = 2 to 9) under the model $Y = \alpha + \beta X + e$ with X following both normal and exponential distributions respectively. It was concluded that Var ($\hat{R}$) = Var ($\hat{R}_Q$) when x is normally distributed and Var ($\hat{R}_Q$) < Var ($\hat{R}$) when X follows gamma distribution.

Beale [5] developed an approximately unbiased ratio estimator $\hat{R}_{B.}$

Tin [6] compared four ratio estimators $\hat{R}$, $\hat{R}_Q$, $\hat{R}_B$ and $\hat{R}_T$ under the linear regression model with respect to bias, efficiency, approach to normality and computational convenience.

He concluded thus: -

$\hat{R}_B$ records the least bias, $\hat{R}_T$ slightly more biased than $\hat{R}_Q$, for both finite and infinite population in which X and Y have bivariate normal distributions. Under the model Y = α + BX + e, he concluded that $\hat{R}_T$ records the least bias when the population involved is infinite.

In terms of efficiency:

$\hat{R}_T$ is the most efficient; $\hat{R}_B$ is more efficient whilst $\hat{R}_Q$ is the least efficient.

$\hat{R}_B$ and $\hat{R}_T$ seem to be more close to normal than $\hat{R}$ and $\hat{R}_Q$ and the computation of $\hat{R}$ or $\hat{R}_Q$ is still much simpler than $\hat{R}_T$ and $\hat{R}_B$.

Rao and Webster [16] made a comparison of $\hat{R}_T$ and $\hat{R}_Q$ assuming Durbin [2] second model. Their comparison shows that the precision of $\hat{R}_T$ and $\hat{R}_B$ with g = n are about the same.

Rao and Beegle [17] investigated the precision of Mickey [3] estimator $\hat{R}_M$ under the model Y = α + BX + e with Durbin [2] assumptions and also compared with four estimators $\hat{R}_{HR}$, of Hartley and Ross [19] , $\hat{R}_Q$, $\hat{R}_T$ and $\hat{R}_P$ They concluded that the variance of $\hat{R}_M$ is monotonically decreasing as the sampling group (g) increases, therefore, the optimum choice of g is n. $\hat{R}_M$ is said to be more efficient than $\hat{R}_{HR}$ for n>2 and g = n and with g = n and n>2, $\hat{R}_Q$ is the most efficient estimator.

Rao and Beegle [12] carried out a Monte Carlo experiment on nine ratio estimators and different sample sizes and concluded $\hat{R}_M$ to be more efficient than either $\hat{R}_{HR}$ or $\hat{R}$.

Rao [17] using some set of fifteen natural populations ,compared $\hat{R}$, $\hat{R}_M$, $\hat{R}_Q$, $\hat{R}_B$ and $\hat{R}_T$ for n = 2, 4, 6 and 8 , concluded that for n = 4, $\hat{R}_Q$ and $\hat{R}_M$ were slightly inferior to $\hat{R}$ and for n = 6 all these estimators have average MSE very close to those of $\hat{R}$.

Rao and Rao [18] worked on the relative efficiencies of five ratio estimators and the stabilities of three variance estimators under a particular model $y_i = \alpha + \beta x_i + e_i$, E $(e_i/x_i) = 0$, Var $(e_i/x_i) = \lambda x_i^g$, E = $(e_i\ e_i\ /\ x_i\ x_j = 0$ with x following a gamma distribution and e_i is said to be normally distributed.

They concluded that both MSE ($\hat{R}_Q$) and MSE ($\hat{R}_M$) are monotonically decreasing function of K number of groups for all g (variance function) and k = n is the optimum choice for $\hat{R}_Q$ and $\hat{R}_M$. They also said that for g = 0, 1, 2, $\hat{R}_M$ is to be preferred to $\hat{R}_{HR}$; at g = 0, there is a considerable gain in efficiency.

Also $\hat{R}_T$ is more efficient than $\hat{R}_Q$ for g = 1, the gain in efficiency is higher for g = 2, $\hat{R}_T$ is said to be more efficient than $\hat{R}_M$ for all g and considerably more efficient than $\hat{R}$ when g = 0.

Hutchison [13] carried out a Monte Carlo comparison of six ratio estimators, Viz; $\hat{R}$, $\hat{R}_{HR}$, $\hat{R}_M$, $\hat{R}_Q$ and $\hat{R}_T$, under two models. He assumed the two models with X following a lognormal distribution (model 1) and X following a chi-square distribution (model 2).

He concluded that for k = n and N infinite $\hat{R}_B$ and $\hat{R}_T$ are the most efficient of all the estimators. Even for very small n, and very large σ and g, $\text{Var}(\hat{R}_B) \leq \text{Var}(\hat{R}_T) < \text{Var}(\hat{R}_Q) < \text{Var}(\hat{R}_M) < \text{Var}(\hat{R}_{HR}) < \text{Var}(\hat{R})$.

For n > 2, $\hat{R}_Q$ follows closely with larger variance but reduces bias more effectively. $\hat{R}_M$ was found to be less efficient than $\hat{R}_B$, $\hat{R}_T$ and $\hat{R}_Q$ but is preferable to $\hat{R}_{HR}$.

Sahoo [7] studied the relative performance of four ratio estimators Viz; $\hat{R}$, $\hat{R}_Q$, $\hat{R}_T$ and $\hat{R}_S$ considering two small populations and concluded that $\hat{R}_B$ and $\hat{R}_T$ are less efficient than $\hat{R}_S$ for both populations.

Dalabehera and Sahoo [14] compared six almost unbiased ratio estimators with respect to bias and efficiency for both finite and infinite populations in which the joint distribution of the characters under study is bivariate normal. They concluded that when the regression line y on x is linear passing through the origin ($C_{11} = C_{20}$), which is of course an optimality condition for ratio estimate to be fruitfully employed in practice, the performance of $\hat{R}_{S2}$ seems to be better than its competitors, But for other situations ($C_{11} \neq C_{20}$) no general conclusions can be obtained and performance of one estimator over the other depends mainly on the population parameters.

Dalabehera and Sahoo [15] further compared the efficiencies of six almost unbiased ratio estimators under a linear regression model using the mean square errors. They concluded that, the estimator $\hat{R}_{S1}$ is more efficient than $\hat{R}_{B}$ for all values of the

Variance function (t), and more efficient than $\hat{R}_{S2}$ if t > 0.5.When t = 0.5

both $\hat{R}_{S1}$ and $\hat{R}_{Q}$ perform equally well but they are more efficient than other estimators.

Sahoo et al [9] carried out a Monte Carlo study to compare the performance of nine almost unbiased ratio estimators, Viz, $\hat{R}_{Q}$, $\hat{R}_{M}$, $\hat{R}_{P}$, $\hat{R}_{B}$, $\hat{R}_{T}$, $\hat{R}_{S}$, $\hat{R}_{S1}$, $\hat{R}_{S2}$, $\hat{R}_{S3}$ with respect to their relative bias, efficiency, achieved coverage rate of the nominal 95% confidence intervals and approach to normality under the linear regression model $y_i = a + bx_i + e_i$.

They concluded that;

i. $\hat{R}_{Q}$, $\hat{R}_{M}$ and $\hat{R}_{P}$ on the ground of relative efficiency, achieved coverage rate and skewness are highly satisfactory when the intercept on the Y- axis (a) $\neq$ 0

ii. The relative bias of $\hat{R}_B$, $\hat{R}_T$, $\hat{R}_S$, $\hat{R}_{S1}$, $\hat{R}_{S2}$ are less than those of $\hat{R}$, $\hat{R}_Q$, $\hat{R}_M$ and $\hat{R}_P$.

iii. They concluded that $\hat{R}_{S1}$ and $\hat{R}_{S2}$ may be preferable to other estimators when a = 0 and a ≠ 0 respectively in respect to relative efficiency and coverage preferable to others when a = 0.

In this work, following the line of Dalabehera and Sahoo [14] , Dalabehera and Sahoo [15] and Sahoo et al [9], we carried out Monte Carlo comparisons of seven approximately ratio estimators $\hat{R}$, $\hat{R}_B$, $\hat{R}_T$, $\hat{R}_S$, $\hat{R}_{S1}$, $\hat{R}_{S2}$ and $\hat{R}_A$.

3.0 EMPIRICAL INVESTIGATIONS

In this section, an empirical study is carried out using a Monte Carlo technique to compare the performance of seven approximately unbiased ratio estimators. These seven estimators above are virtually equivalent in the sense that they have the same approximate mean square error to 0 (n^{-2}). Moreover, the same statistics like y, x, s_{yx} and s^2_x were also used for their computations. Henceforth, they are called approximately unbiased ratio estimators.

Using a Monte Carlo technique to compare the performance of these approximately Unbiased ratio estimators along side with the classical one.

We shall be considering the usual model for ratio estimator $y_i = \beta x_i + e_i$, i = 1,2,3,…,N.

This work shall be viewed from the angle of real life situation which is always encountered in sampling practice by assuming that the auxiliary variable x to follow a gamma distribution (i.e. skewed population).

The approximately unbiased ratio estimators shall be compared under the following assumed model:

the regression of y on x is linear i.e, $y = \alpha + \beta x + e$ with x having a gamma distribution with parameter(2, 1) ,that is, $x_i \sim G(2, 1)$and e having a gamma distribution with parameter (0.25, 1), that is, $e_i \sim G(0.25x_i, 1.0)$. Under varying values of intercept (α) = 0, 0.5, 1.2 choosing from population of size 500.

We shall consider the value of the variance function $t = 2.0$ f or varying sample sizes n = 20, 40, 100, 200 with a gamma parameter, k = 2 under the regression model: $y_i = 0.25x_i + e_i x_i$; $e_i \sim G(0.25, 1.0)$; $x_i \sim G(2, 1)$.

4.0. DESCRIPTION OF MONTE CARLO SIMULATION AND RESULTS

Our Monte Carlo experiment involves repeated draws of simple random sampling without replacement. We employed the use of R software packaged to draw 10000 independent samples each of size 20, 40,100 and 200 at different values of α = 0, 0.5, and 1.2 with β =1,for each sample. The values of variance, bias, Mean square errors, and efficiency were calculated for the seven approximately unbiased ratio estimators. The outcomes of the performance of these estimators with respect to variance, bias mean squared errors and efficiency are presented in the tables 4.0.1 to 4.0.5. For these estimators, for different sample sizes (n = 20(small), n = 40(moderate), n = 100(large) and n = 200(extremely large) when α = 0, 0.5, 1.2 and β =1.0 under the simple model $y = \alpha + \beta x + e$ and x assumed a gamma (500, 2, 1) with e assuming a gamma (500, 0.25, 1).

It is clear from Tables 4.0.1 to 4.0.5, that for α = 0 describes the true ratio estimator with the regression line passing through the origin at the value of β = 1.

Figures 1 to 6 are those of scatter plots of the estimates of the bias against the sample sizes and MSE against the sample sizes for α = 0, 0.5, 1.2 and β = 1.0 for the classical ratio estimator $\hat{R}$ and the best two approximately unbiased ratio estimators, $\hat{R}_{\mathbf{s1}}$ **and** $\hat{R}_{\mathbf{A}}$ respectively, Adebola [10].

TABLE 4.0.1:- ESTIMATES OF VARIANCE, BIAS, MSE AND EFFICIENCY OF ALL THE EIGHT ESTIMATORS WHEN x~ gamma (2, 1) AND e ~ gamma (0.25, 1) for α = 0, β =1.0 and n=20, 40,100,200 samples.

ESTIMATOR	SAMPLE SIZE	**VARIANCE**	BIAS	MSE	EFFICIENCY	
R	20	0.01029076	1.17027	1.379823	-	
RB	20	0.009901867	1.165361	1.367968	1.008666	
RT	20	0.009892338	1.165238	1.367672	1.008884	
RS	20	0.0098948	1.164104	1.365033	1.010835	
RS1	20	0.00988662	1.164104	1.365025	1.010841	
RS2	20	0.009884895	1.165127	1.367406	1.009081	
RS3	20	0.009901635	1.166385	1.370356	1.006909	
RA	20	0.01053025	1.07387	1.163727	1.185693	
R	40	0.005769206	1.098343	1.212127	-	
RB	40	0.005695733	1.096034	1.206986	1.004259	
RT	40	0.005695068	1.096012	1.206937	1.0043	
RS	40	0.00569524	1.096015	1.206944	1.004294	
RS1	40	0.005695225	1.095846	1.206574	1.004603	
RS2	40	0.005694566	1.095993	1.206895	1.004335	
RS3	40	0.005695084	1.09618	1.207306	1.003993	
RA	40	0.005734991	1.060607	1.130622	1.072088	
R	100	0.002574001	1.097364	1.206782	-	
RB	100	0.002559164	1.096333	1.204505	1.00189	
RT	100	0.002559105	1.096329	1.204496	1.001898	
RS	100	0.002559119	1.096329	1.204496	1.001898	
RS1	100	0.0025591	1.096297	1.204426	1.001956	
RS2	100	0.002559059	1.096325	1.204488	1.001905	
RS3	100	0.002559123	1.096361	1.204567	1.001839	
RA	100	0.002556986	1.050968	1.10709	1.090049	
R	200	0.000952518	1.096661	1.203618	-	
RB	200	0.000950442	1.096272	1.202763	1.000711	
RT	200	0.000950439	1.096271	1.202761	1.000713	

RS	200	0.00095044	1.096271	1.202761	1.000713	
RS1	200	0.000950439	1.096267	1.202752	1.00072	
RS2	200	0.000950436	1.09627	1.202758	1.000715	
RS3	200	0.000950439	1.096276	1.202772	1.000704	
RA	200	0.000950019	1.043491	1.089824	1.104415	

FIGURE 1: GRAPH OF ESTIMATES OF BIAS OF ESTIMATORS: $\hat{R}$**,** $\hat{R}_{s1}$ **and** $\hat{R}_A$ **WHEN x~** gamma (2, 1) AND e ~ gamma (0.25, 1) for $\alpha = 0$, $\beta = 1.0$ and n=20, 40,100,200 samples and k = 2.

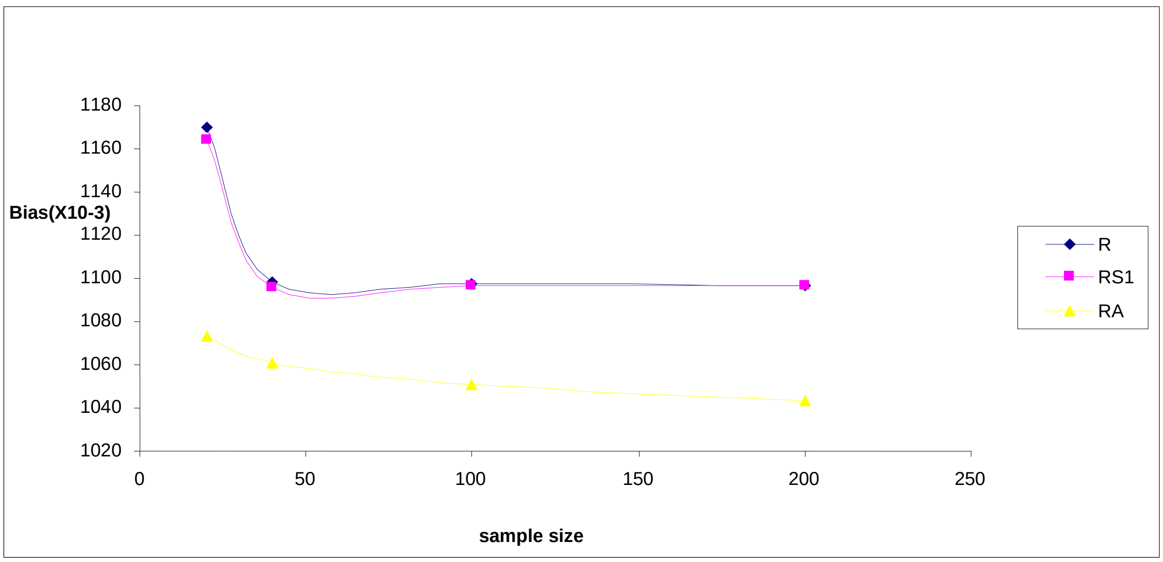

FIG 1

FIGURE 2: GRAPH OF ESTIMATES OF MSE OF ESTIMATORS: $\hat{R}$**,** $\hat{R}_{s1}$ **and** $\hat{R}_A$ **WHEN x~** gamma (2, 1) AND e ~ gamma (0.25, 1) for $\alpha = 0$, $\beta = 1.0$ and n=20, 40,100,200 samples and k = 2.

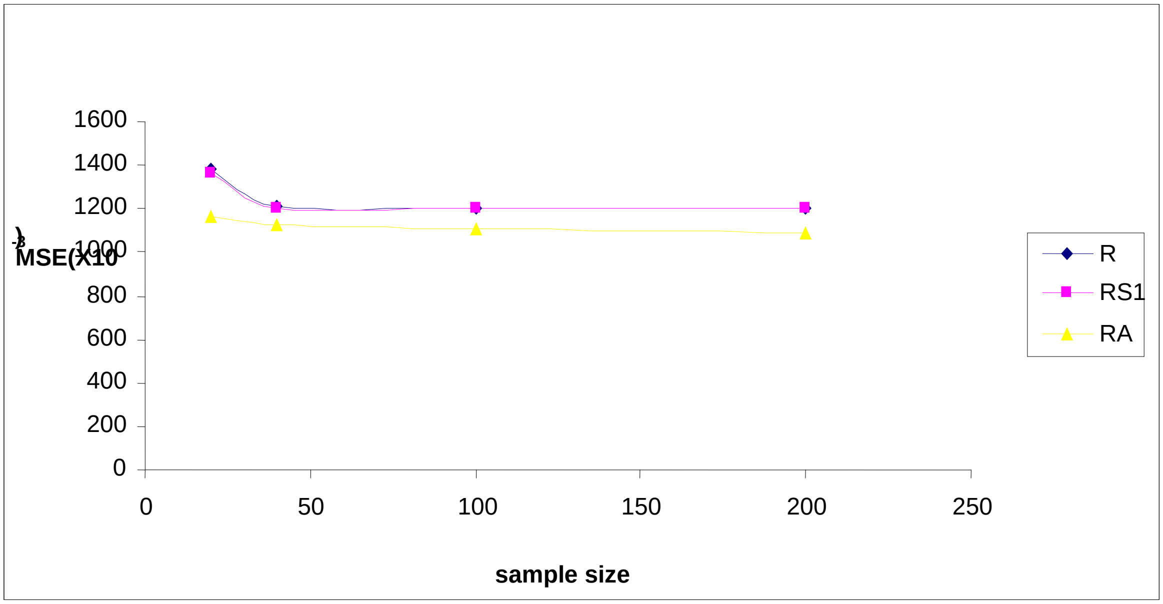

FIG 2

TABLE 4.0.2:- ESTIMATES OF VARIANCE, BIAS, MSE AND EFFICIENCY OF ALL THE EIGHT ESTIMATORS WHEN x~ gamma (2, 1) AND e ~ gamma (0.25, 1) for α = 0.5, β =1.0 and n=20, 40,100,200 samples.

ESTIMATORS	SAMPLE SIZE	**VARIANCE**	BIAS	MSE	EFFICIENCY	
R	20	0.02923144	1.361784	1.883687	-	
RB	20	0.0281799	1.34382	1.834032	1.027074	
RT	20	0.02815429	1.343342	1.832722	1.027808	
RS	20	0.02815429	1.343488	1.833114	1.027588	
RS1	20	0.0281525	1.342104	1.829396	1.029677	
RS2	20	0.02814214	1.342989	1.831762	1.028347	
RS3	20	0.028171	1.344706	1.836405	1.025747	
RA	20	0.02866918	1.249445	1.589782	1.184871	

R	40	0.01356829	1.352171	1.841935	-	
RB	40	0.01330607	1.343461	1.818194	1.013058	
RT	40	0.01330274	1.343355	1.817905	1.013218	
RS	40	0.01330432	1.343386	1.81799	1.013171	
RS1	40	0.01330121	1.33082	1.784383	1.032253	
RS2	40	0.01330095	1.343278	1.817697	1.013335	
RS3	40	0.01330585	1.343658	1.818723	1.012763	
RA	40	0.01322394	1.218674	1.498391	1.229275	
R	100	0.00469212	1.345324	1.814589	-	
RB	100	0.00462779	1.342291	1.806373	1.004548	
RT	100	0.00462766	1.342279	1.806341	1.004566	
RS	100	0.00462772	1.342282	1.806349	1.004562	
RS1	100	0.00462759	1.342246	1.806252	1.004616	
RS2	100	0.00462758	1.34227	1.806316	1.00458	
RS3	100	0.00462779	1.342314	1.806435	1.004514	
RA	100	0.00460824	1.208162	1.464264	1.239250	
R	200	0.00172609	1.343689	1.807226	-	
RB	200	0.00172169	1.342547	1.804154	1.001703	
RT	200	0.00172168	1.342546	1.804151	1.001704	
RS	200	0.00172169	1.342546	1.804151	1.001704	
RS1	200	0.00172168	1.342541	1.804138	1.001712	
RS2	200	0.00172068	1.342544	1.804145	1.001708	
RS3	200	0.00172169	1.342551	1.804165	1.001697	
RA	200	0.00171873	1.177530	1.388296	1.301758	

FIGURE 3: GRAPH OF ESTIMATES OF BIAS OF ESTIMATORS: $\hat{R}$**,** $\hat{R}_{s1}$ **and** $\hat{R}_A$ **WHEN X~** gamma (2, 1) AND e ~ gamma (0.25, 1) for α = 0.5, β = 1.0 and n=20, 40,100,200 samples and k = 2.

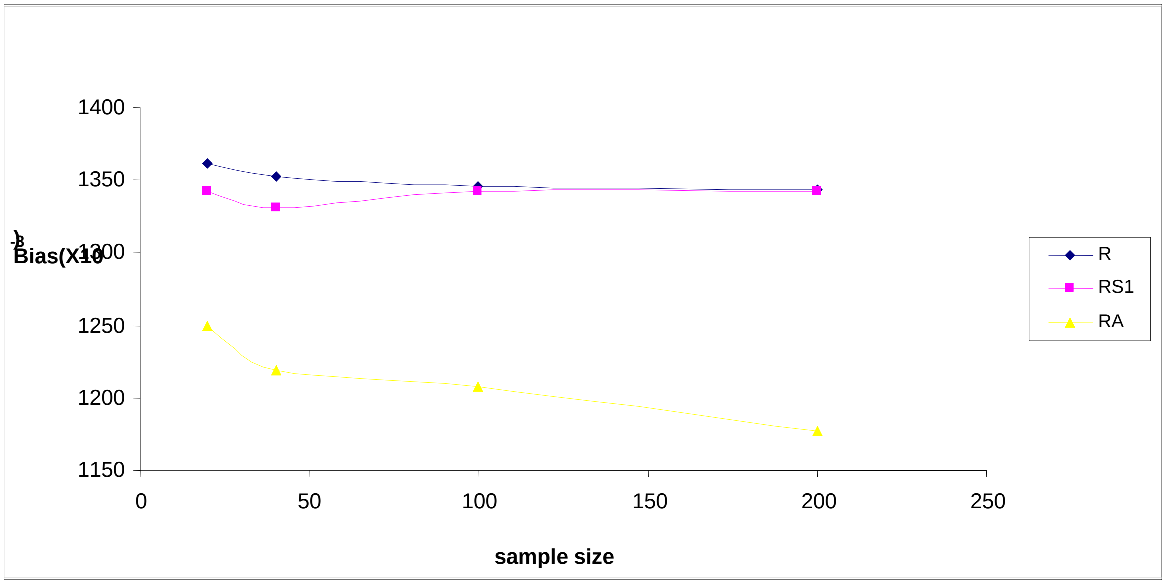

FIG.3

FIGURE 4: GRAPH OF ESTIMATES OF MSE OF ESTIMATORS: $\hat{R}$**,** $\hat{R}_{s1}$ **and** $\hat{R}_A$ **WHEN X~** gamma (2, 1) AND e ~ gamma (0.25, 1) for α = 0.5, β = 1.0 and n=20, 40,100,200 samples and k = 2.

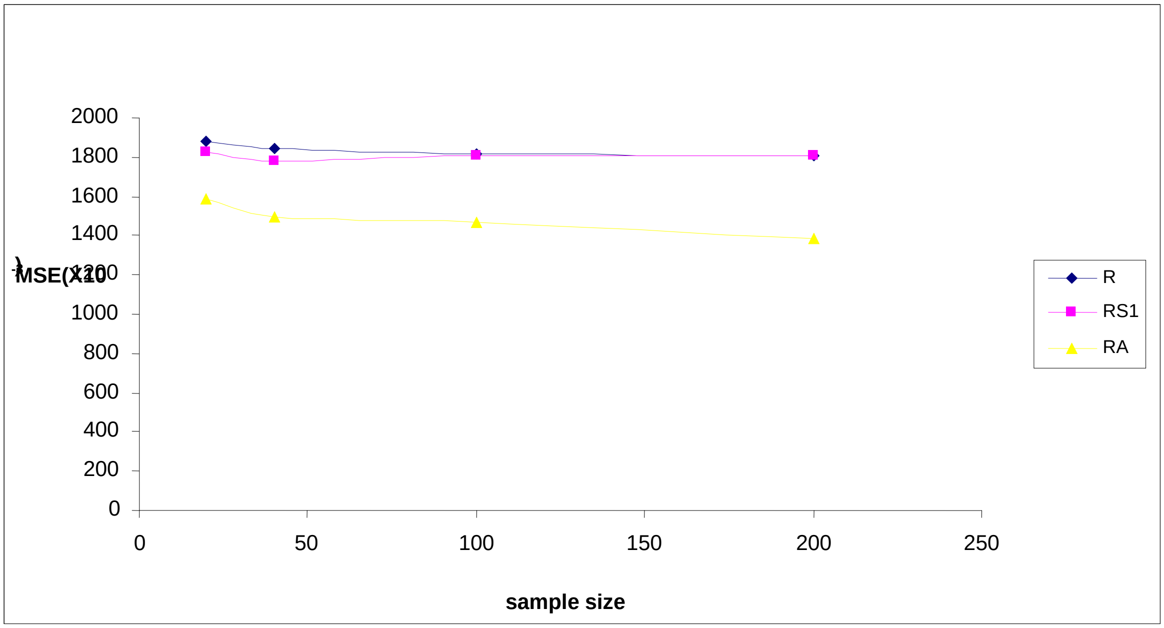

FIG 4

TABLE 4.03:- ESTIMATES OF VARIANCE, BIAS, MSE AND EFFICIENCY OF ALL THE EIGHT ESTIMATORS WHEN X~ gamma (2, 1) AND e ~ gamma (0.25, 1) for α = 1.2, β =1.0 and n=20, 40,100,200 samples.

ESTIMATORS	SAMPLE SIZE	VARIANCE	BIAS	MSE	EFFICIENCY
R	20	0.06995395	1.724352	3.043344	-
RB	20	0.06694677	1.689658	2.921891	1.041567
RT	20	0.06687135	1.688736	2.918701	1.042705
RS	20	0.0669226	1.689149	2.920147	1.042189
RS1	20	0.06686051	1.687499	2.914513	1.044203
RS2	20	0.06684632	1.688196	2.916852	1.043366
RS3	20	0.06693401	1.690356	2.924237	1.040731
RA	20	0.06696598	1.594839	2.610477	1.165819
R	40	0.03237292	1.68966	2.887324	-
RB	40	0.03163735	1.68856	2.882872	1.001544
RT	40	0.03162765	1.68856	2.882863	1.001548
RS	40	0.03163373	1.688356	2.88218	1.001785

RS1	40	0.03162433	1.688083	2.881249	1.002109	
RS2	40	0.03162393	1.688239	2.881775	1.001926	
RS3	40	0.03163699	1.688716	2.883399	1.001361	
RA	40	0.03138409	1.581112	2.5313	1.140649	
R	100	0.01094651	1.692469	2.875398	-	
RB	100	0.0108594	1.686632	2.855587	1.006938	
RT	100	0.01085901	1.686609	2.855509	1.006965	
RS	100	0.01085924	1.686619	2.855543	1.006953	
RS1	100	0.01085885	1.686576	2.855397	1.007004	
RS2	100	0.01085885	1.686595	2.855462	1.006982	
RS3	100	0.0108594	1.686651	2.855651	1.006915	
RA	100	0.01081537	1.578530	2.502572	1.148977	
R	200	0.00406305	1.689528	2.858568	-	
RB	200	0.00405774	1.687333	2.85115	1.002602	
RT	200	0.00405075	1.68733	2.851133	1.002608	
RS	200	0.00405765	1.687331	2.851144	1.002604	
RS1	200	0.00405744	1.687325	2.851123	1.002611	
RS2	200	0.00405744	1.687328	2.851133	1.002608	
RS3	200	0.00405774	1.687336	2.851161	1.002598	
RA	200	0.00404425	1.514508	2.483120	1.151200	

FIGURE 5: GRAPH OF ESTIMATES OF BIAS OF ESTIMATORS: $\hat{R}$, $\hat{R}_{s1}$ **and** $\hat{R}_A$ **WHEN X~** gamma (2, 1) AND e ~ gamma (0.25, 1) for α = 1.2, β = 1.0 and n=20, 40,100,200 samples and k = 2.

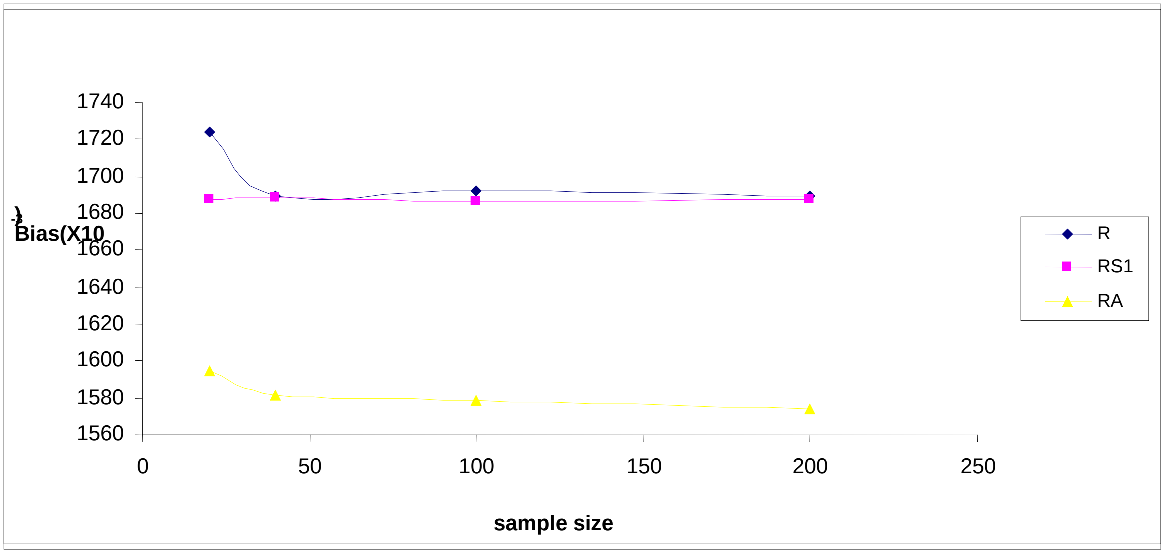

FIG.5

FIGURE 6: GRAPH OF ESTIMATES OF MSE OF ESTIMATORS: $\hat{R}$, $\hat{R}_{s1}$ **and** $\hat{R}_A$ **WHEN X~** gamma (2, 1) AND e **~** gamma (0.25, 1) for α = 1.2, β = 1.0 and n=20, 40,100,200 samples and k = 2.

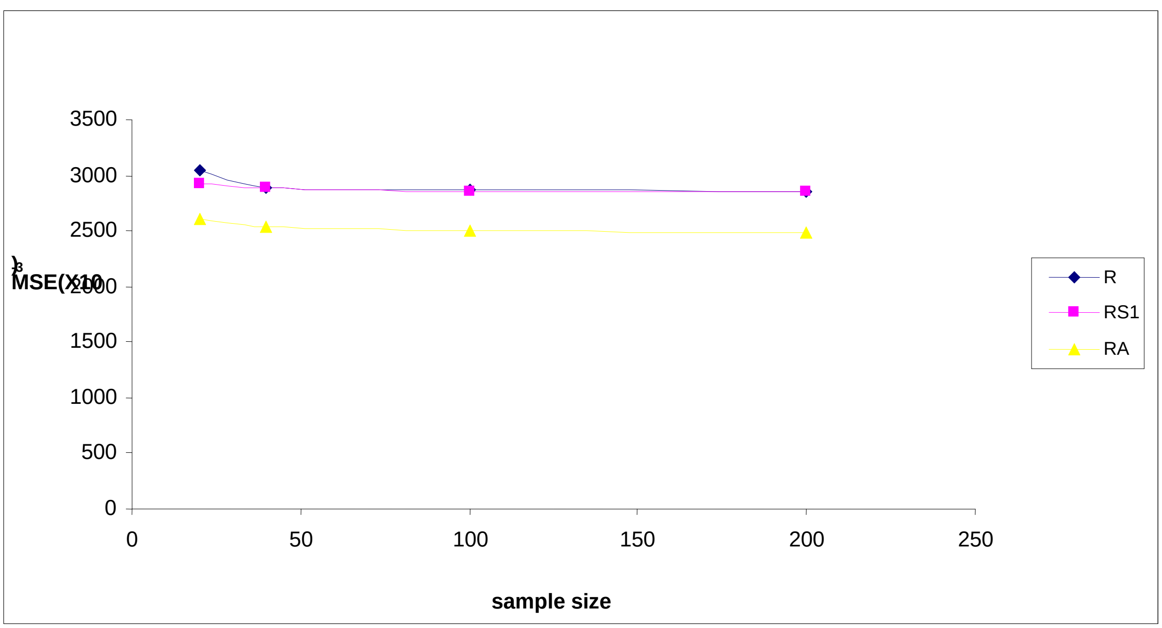

FIG .6

5.0. DISCUSSIONS OF RESULTS

From the observations on the Tables 4.0.1 to 4.0.3 and the graphs on figures 1 to 6, the following observations can be made about the model for α =0, 0.5,1.2 with respect to all the approximately unbiased estimators compared

(i) R_A estimator produced the least bias for all the sample sizes considered..

(ii) R_A estimator produced the least MSE for all the sample sizes considered

(iii) R_A estimator is more efficient than the other estimators for all the sample sizes considered

(iv). In terms of variance there exists no specific estimator that can be said to be the best.

6.0. CONCLUSION

The simulations results above confirmed that R_A estimator is better than the existing ones in terms of the bias and MSE whenever $1 \le t \le 2.0$ and $n\ k > 8$

REFERENCES

[1] Quenouille , M.H. (1956) : Notes on bias in estimation .Biometrika , 43 , 353 – 360.

[2] Durbin,J.(1959) : A note on the application of Quenouille`s method of bias reduction to estimation of ratios. Biometrika , 46, 447 – 480.

[3] Mickey, M.R. (1959): Some finite population unbiased ratio and regression estimators, JASA 54, 594 – 612.

[4] Pascual, J.N. (1961): Unbiased ratio estimators in stratified sampling, JASA, 56, 70 – 87.

[5] Beale, E.M.L.(1962): Some uses of computers in operational research. Industrielle

[6] Tin, M (1965): comparison of some ratio estimators JASA. P.294-307.

[7] Sahoo, L. N (1983): On a method of bias reduction in Ratio estimation. Journal of Statistical Research. 17 , 1-6.

[8] Oshungade I. O. (1988): Non response and Ratio estimation Problems in Sample Survey. Unpublished Ph D Thesis, University of Essex, United Kingdom.

[9] Sahoo, L. N, Das B.C and sahoo, J (2006): A Note on the performance of several almost unbiased ratio estimators. Pakistan Journal of Statistics. Vol. 22 p, 209 – 215.

[10] Adebola F.B. (2010): Development of an approximately unbiased Ratio Estimator.Unpublished Ph D Thesis, University of Ilorin, Ilorin, Nigeria.

[11] Lauh, E. and Williams, W.H. (1963): "Some small sample results for the variance of a ratio". Proceedings of American Statistical Association (Social Statistics section), 273 – 283.

[12] Rao, J. N. K and Beegle ,L.D (1967) A monte carlo study of some Ratio estimators. Sankhya B.29, 47 - 56.

[13] Hutchison, M.C. (1971): A Monte Carlo Comparison of some ratio estimators. Biometrika , 58, 313 – 321.

[14]Dalabehera M and Sahoo, L. N (1994): Comparison six almost unbiased Ratio Estimators. Questio, 18, 3, 369-375.

[15] Dalabehera M and Sahoo, L. N (1995): Efficiencies of six almost unbiased ratio estimators under a particular model. Statistical Papers. 18, 61 – 67.

[16] Rao, J. N. K and Webster, J.T.(1966): On two methods of bias reduction in the estimation of ratios. Biometrika 53, 571 – 577.

[17] Rao, J.N.K. (1969): Ratio and regression estimators .New Development in survey sampling, N.L. Johnson and H. Smith (eds) , New York: John Wiley and Sons. 213 – 234.

[18] Rao, P.S.R.S., and Rao, J.N.K. (1971): Small Sample results for ratio estimators.Biometrika, 58(3):625-630.

[19] Hartley, H.O. and Ross, A. (1954): Unbiased ratio estimates. Nature, 174, 270 – 271.