MAGNETIC AND POROSITY EEFECT ON MHD FLOW OF A DUSTY VISCOELASTIC FLUID THROUGH HORIZONTAL PLATES WITH HEAT TRANSFER

By

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Abstract

An analysis has been carried out to study the Magnetohydrodynamics flow of incompressible non-Newtonian dusty viscoelastic fluid between two parallel horizontal non conducting porous plates with heat transfer. The partial differential equations governing the flow and heat transfer are converted into highly non-linear coupled ordinary differential equations which are solved numerically by employing *Adams-Bashford-Moulton* classical method (a "predictor-corrector" method) in a computer algebra package known as Maple.

The effect of magnetic and porosity parameters are examined for both velocity and temperature distribution of the fluid and particles. The analysis reveals that the fluid and particles temperature profile decreases significantly due to increase in magnetic and porosity parameter while the velocity profile increases.

Keywords: Viscoelastic fluid, classical method, particle concentration, magnetic and porosity effect, Magnetohydrodynamics (MHD) flow.

Introduction

The studies on the flow of a viscous incompressible fluid between two horizontal parallel plates and their different transport phenomena has application in devices such as MHD pumps, MHD power generators, accelerators, petroleum industry, purifications of molten metals from non-metallic inclusions and fluid droplets - sprays

The fluids under consideration are electrically conductive and have both viscous and elastic properties. Examples are molten plastics, pulps, emulsion e.t.c. and some variety of industrial products that have viscoelastic behaviour in their motion. Such fluids contain some spherical non-conducting dust particles in form of impurities. The application of these dust particles on viscoelastic fluid flow can be found in the extrusion of plastics in the manufacturing of nylon and rayon, textile industry, purification of crude oil, paper and pulp industry.

In the last decades, several authors have carried out the study of viscoelastic fluid under different physical conditions. Sujit and Emmanuel [1] have studied the flow and heat transfer of viscoelastic fluid on a stretching sheet.

Shyamanta [2] investigated the unsteady flow of density viscous electrically conducting fluid through a channel and discussed the magnetic effect on velocity profiles.

Attia [3] also studied the effect of porosity on unsteady poiseulle flow of a viscoelastic fluid with temperature dependent viscosity.

In the present work, we shall study the movement of a dusty viscoelastic electrically conductive fluid past horizontal parallel plates with heat transfer. Effect of porosity and magnetic field are also investigated. We assume that the plates are maintained at a temperature which decays exponentially with time. The expressions for fluid and dust particle velocity and temperature distribution of the fluid are also obtained.

Formulation of the Problem

The dusty viscoelastic fluid which is electrically conductive is assumed to be flowing between two infinite horizontal plates located at the $y = 0$ and $y = +h$. The plates are mounted at two different temperatures which decay exponentially with time. The central line of the channel is the $x - axis$ and the perpendicular to it is the $y - axis$. B_0 which is uniform magnetic field is applied normal to the plate.

The velocity and magnetic field distributions are $V = [u(y,t), 0, 0]$ and $\mathrm{B} = [0, \mathrm{B}_0, 0]$. The force experienced by the dust particle is inertial force and is equal and opposite to that experienced by the dust particles due to the fluid in motion.

Assumptions

The presence of the dust particle in viscoelastic fluid (under consideration) makes the study of the dynamics fluid complicated.

We will investigate this problem under various simplified assumptions. To write down the governing equation of this dusty viscoelastic fluid flow in a reasonable simple form, certain assumptions were made as follows;

* The fluid is incompressible
* Dust particles are solids, elastic sphere, identical and symmetrical in size, electrically non-conducting and are distributed uniformly within the fluid motion.
* Chemical reactions and mass transfer and other interaction are neglected

* The plates are infinitely long, so that the velocity (u) of the fluid and dust particle velocity (u_p) are function of $y \text{ and } t$ only.
* The density of dusty particle is constant and has small value throughout the fluid motion
* Hall effect, the effect due to buoyancy, polarization effect are negligible.
* Reynold number is small compared to unity so that the induced field is negligible.
* There is no flow initially $(at\ time\ t = 0)$ and plates are at different temperatures $(i.e\ \ when\ \ t = 0,\ \ T = T_0,\ \ at\ y = 0\ \ \ and\ \ \ when\ \ t > 0,\ T = T_1\ at\ y = +h)$
* The heat generation due to elastic deformation was considered because we assume the fluid possess elastic properties than viscous property.

The governing equations under above assumptions are [1]:

$$\rho\left(\frac{\partial u}{\partial t}+v_0\frac{\partial u}{\partial y}\right) = -\frac{dp}{dx} - \overbrace{\frac{\mu}{k_1}u}^{Porosity\ force} + k_2 N\left(u-u_p\right) - \underbrace{\sigma B_0^2 u}_{magnetic\ \ force} + \underbrace{\frac{\partial \tau}{\partial y}}_{viscoelastic\ \ force} \qquad (1)$$

$$m\frac{\partial u_p}{\partial t} - k_2 N\left(u-u_p\right) = 0 \qquad (2)$$

where

ρ is the density of the fluid

μ is the kenetic coefficient of the viscosity of the fluid

u_p is the velocity of dust particle

m is the average mass of dust particles

u is the velocity of the fluid

k_1 is the porosity of the medium (Drag permeability)

p is the pressure acting on the fluid

τ is the component of the shear stress of the viscoelastic fluid

N is the number of the dust particle

σ is the elastic conductivity

k_2 is proportionality constant

v_0 is the velocity along y component which is constant

B_0 is the uniform magnetic field applied in the position $y-$direction

The heat transfer occurs from $y=0$ to $y=+h$ by conduction through the fluid. The dust particles lost heat energy to the fluid by concentration through their spherical surface. The energy equations needed to describe the temperature distribution for both the fluid and the dusty particles are respectively given as [3]:

$$\rho c\frac{\partial T}{\partial t}+\rho c v_0\frac{\partial T}{\partial y}=K\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_0^2 u^2+\frac{\rho_p c_s}{\gamma_T}\left(T_p-T\right)+k_3\left[\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right)^2\right] \quad (3)$$

$$\frac{\partial T_p}{\partial t}=\frac{1}{\gamma_T}\left(T_p-T\right) \quad (4)$$

T *is the termperature of the fluid*

T_p *is the temperature of the particles*

c *is the specific heat capacity of the fluid at constant*

c_s *is the specific heat capacity of the particles*

K *is the thermal conductivity of the fluid*

k_3 *is the kinematic coefficient of viscoelasticity*

$\gamma_T=\dfrac{3P_r\gamma_p c_s}{2c}$ *is the temperature relaxation*

$\gamma_p=\dfrac{2\rho_s D^2}{9\mu}$ *is the velocity relaxation time*

$\rho_s=\dfrac{3\rho_p}{4\pi D^3 N}$ *is the material density of dust particles*

D *is the average radius of dust particles*

Initial and boundary Conditions

The fundamental equations stated in the previous section are to be solved under appropriate initial conditions to determine the flow field of the fluid and the dust particles.

1. There will be no mass transfer at a solid boundary
2. The plates are maintained at two different temperatures which decay exponentially with time.
3. the dust particle may slip at the boundary and the initial and boundary conditions are to be taken at $y=0$

Therefore boundary conditions of the problem are:

$$\begin{aligned} &u=0 \quad u_p=0 \quad T=T_0 e^{-2nt}, \ at \quad y=0 \\ &u=u_0 e^{-nt}, \quad u_p=u_{p_0}e^{-nt} \quad T=T_1 e^{-2nt}, \ at \quad y=+h \end{aligned} \quad (5)$$

where T_0 *and* T_1 are the temperature at the plate $y=0$ *and* $y=+h$ respectively

The component of the shear stress of the viscoelastic fluid is given as [2]:

$$\tau + \lambda \frac{\partial \tau}{\partial t} = \mu \frac{\partial \gamma}{\partial t}$$
$$= \mu \frac{\partial u}{\partial y} \quad (By\ Newton's\ law\ of\ viscosity) \tag{6}$$

$\lambda = \frac{\mu}{k}$ is called the relaxation time, k is the modulus of rigidity and γ represent the shear strain.

Equation (6) can be solved for τ in terms of velocity component u to obtain

$$\frac{\partial \tau}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) - \frac{1}{k}\frac{\partial}{\partial y}\left(\mu \frac{\partial}{\partial t}\left(\mu \frac{\partial u}{\partial y}\right)\right) \tag{7}$$

Substitute equation (7) into (1) to obtain

$$\rho\left(\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y}\right) = -\frac{dp}{dx} - \frac{\mu}{k_1} u + k_2 N\left(u - u_p\right) - \sigma B_0^2 u + \mu \frac{\partial^2 u}{\partial y^2} - k_3\left[\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right)\right] \tag{8}$$

For non dimensional quantities we have,

$$\bar{u} = \frac{u}{u_0},\ \bar{v} = \frac{v}{v_0},\ \bar{y} = \frac{y}{h},\ \bar{t} = \frac{tu_0}{h}, \bar{T} = \frac{T - T_1}{T_2 - T_1},\quad \bar{k} = \frac{h}{\sqrt{k_1}},\quad \bar{p} = \frac{p}{\rho u_0^2},\ \lambda = -\frac{d\bar{p}}{d\bar{x}}$$
$$\bar{T}_p = \frac{T_p - T_1}{T_2 - T_1},\quad \bar{u}_p = \frac{u_p}{u_0},\quad \bar{v}_p = \frac{v_p}{u_0} \tag{9}$$

Substituting (9) in equations (2), (3), (4) and (8) and then removing the caps, we obtain

$$\frac{\partial u}{\partial t} + s\frac{\partial u}{\partial y} = -\frac{dp}{dx} - \frac{M}{R_e}u + \frac{R}{R_e}\left(u - u_p\right) - \frac{H_a^2}{R_e}u + \frac{1}{R_e}\frac{\partial^2 u}{\partial y^2} - \varphi \frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) \tag{10}$$

$$R_p \frac{\partial u_p}{\partial t} - \left(u - u_p\right) = 0 \tag{11}$$

$$\frac{\partial T}{\partial t} + S\frac{\partial T}{\partial y} = \frac{1}{R_e P_r}\frac{\partial^2 T}{\partial y^2} + \frac{\mathrm{E}_c}{R_e}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\mathrm{E}_c H_a^2}{R_e}u^2 + \frac{2R}{3R_e P_r}\left(T_p - T\right) + \frac{\varphi E_c}{R_e P_r}\left[\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right)^2\right] \tag{12}$$

$$\frac{\partial T_p}{\partial t} = \frac{-L_0}{R_e}\left(T_p - T\right) \tag{13}$$

where

$$s=\frac{v_0}{u_0} \quad \text{is the suction parameter}$$

$$R_e=\frac{\rho h u_0}{\mu} \quad \text{is the Reynold number}$$

$$H_a^2=\frac{\sigma B_0^2 h^3}{\mu} \quad \text{is the Hartman number}$$

$$M=\frac{\rho h^3}{k_1} \quad \text{is the porosity parameter}$$

$$R=\frac{k_2 N h^2}{\mu} \quad \text{is the particle concentrate parameter}$$

$$\varphi=\frac{\mu^2}{\rho k_3 h^2} \quad \text{is the viscoelastic parameter}$$

$$R_p=\frac{m u_0}{k_2 h} \quad \text{is the relaxation time parameter for dust particle}$$

$$\mathrm{E}_c=\frac{u_0^2}{c_p T_0}-\text{Eckert number}$$

$$P_r=\frac{\mu}{\alpha} - \text{Prandth number}$$

$$R=\frac{k_2 N h^2}{\mu} \quad \text{is the particle concentrate parameter}$$

$$L_0=\frac{\rho h^2}{\mu \gamma_T} \quad \text{is the temperature relaxation time parameter}$$

The non-dimensional initial and boundary conditions are

$$\begin{aligned} &u=u_p=0, \quad T=e^{-2nt} \quad at \quad y=0 \\ &u=e^{-nt}, \quad u_p=e^{-nt} \quad T=\beta e^{-2nt} \quad at \quad y=+1 \end{aligned} \tag{14}$$

where $\beta=\left(T_1/T_0\right)$, is a constant temperature

In order to solve equation (10)-(13) we consider

$$u=f(y)e^{-nt},\ T=\theta(y)e^{-2nt},\quad u_p=g(y)\,e^{-nt},\quad T_p=\theta_p(y)e^{-2nt} \tag{15}$$

Substitute equation (15) into (10)-(13) to obtain

$$f''(y)-A_6 f'(y)-A_7 f(y)-A_8=0 \tag{16}$$

$$g(y)=\frac{f(y)}{\left(1-nR_p\right)} \tag{17}$$

$$\theta''(y)-A_{10}\,\theta'(y)-A_{11}\,\theta(y)+\left[f'(y)\right]^2-A_{12}\left[f(y)\right]^2=0 \tag{18}$$

$$\theta_p(y) = \left[\frac{L_0}{L_0 - 2nR_e}\right]\theta(y) \tag{19}$$

where

$$A_1 = \frac{1}{R_e} + \varphi n,\ A_2 = S,\ A_3 = \left(\frac{H_a^{\ 2}}{R_e} + \frac{M}{R_e} - \frac{R}{R_e} - n\right),\quad A_4 = \frac{R}{R_e},\quad \lambda = \frac{dp}{dx}$$

$$A_5 = \left[A_3 + \frac{A_4}{1 - nR_p}\right],\ A_6 = \frac{A_2}{A_1},\quad A_7 = \frac{R_e\lfloor A_3\,(1 + nR_p) + A_4\rfloor}{(1 + nR_p)\,(1 + R_e\varphi n)},\ A_8 = \frac{\lambda_1 R_e}{1 + R_e\varphi n} = \frac{\lambda_1}{A_1}$$

$$A_9 = \frac{1}{R_e P_r},\ A_{10} = \frac{S\ R_e}{E_c},\quad A_{11} = \frac{R_e}{E_c}\left[\frac{2R}{3A_9}\left(1 - \frac{L_0}{L_0 - 2nR_e}\right) - 2n\right],\ A_{12} = H_a^{\ 2}$$

The corresponding initial and boundary conditions are now

$$f(0) = g(0) = 0,\ f'(0) = g'(0) = 0 \quad \theta(0) = 1\ ,\theta'(0) = 0 \tag{20}$$

$$f(1) = 1,\ g(1) = A_8\ ,\quad \theta(1) = \beta \tag{21}$$

Equation (16) and (18) represent a coupled differential equation which are solved numerically under the initial and boundary condition (20), using *Adams-Bashford-Moulton* classical method (a "predictor-corrector" method) in a computer algebra package known as Maple.

Results and Discussion

The objective of our study is to investigate the magnetic and porosity effect of MHD viscoelastic dusty fluid flow between two horizontal plates at a different constant values of magnetic field parameters (H_a) and porosity parameter (M). Computations have been made for $\lambda = 1.0$, $R_e = 1.0$, $Q = 0.2$, $n = 0.2$, $S = 1$, $R = 1.0$, $L_0 = 0.1$, $P_r = 0.76$, $E_c = 1.0$. The velocity and temperature distribution for both the fluid and particles are observed from the figures.

The application of uniform magnetic field adds one resistive term to the momentum equation and Joule dissipation term to the energy equation.

Figure 1-3 illustrates the influence of the magnetic parameter H_a on horizontal velocity and temperature distribution of the fluid and particles. It is noticed that increase in magnetic parameter $H_a(0, 0.5, 1.0)$ increases sharply the velocity distribution of both the fluid and particles while the temperature distribution of the fluid decreases. This is due to the fact the rate of transport is considerably increase with increase of H_a since the fluid is electrically conductive and flow in the direction of magnetic field line.

Figure 4-6 reveal the effect of porosity parameter M on velocity and temperature distribution of the fluid and particles. Increase in porosity parameter $M(1.0, 1.5, 2.0)$ increases the velocity distribution of both the fluid and particle while the temperature distribution of the fluid decreases. This occur as a results of pumping the fluid from colder lower half region to the center of the plate

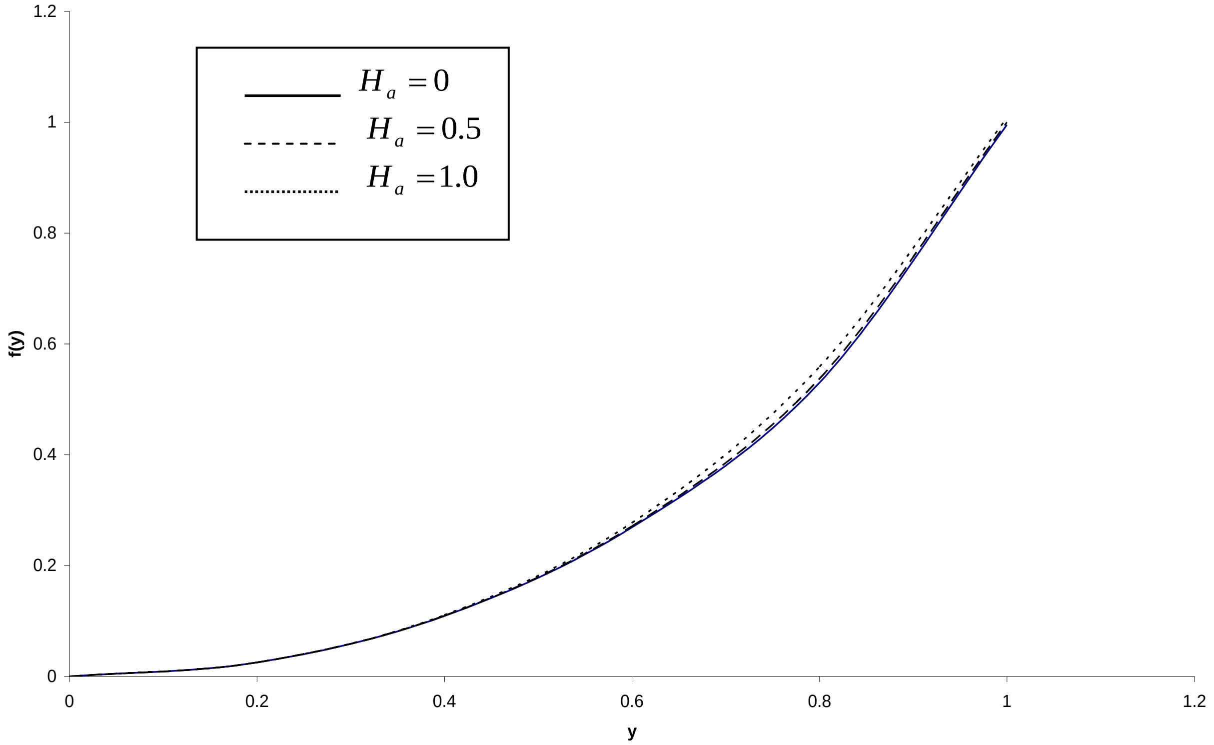

Figure 1: Velocity profile of fluid for different values of H_a

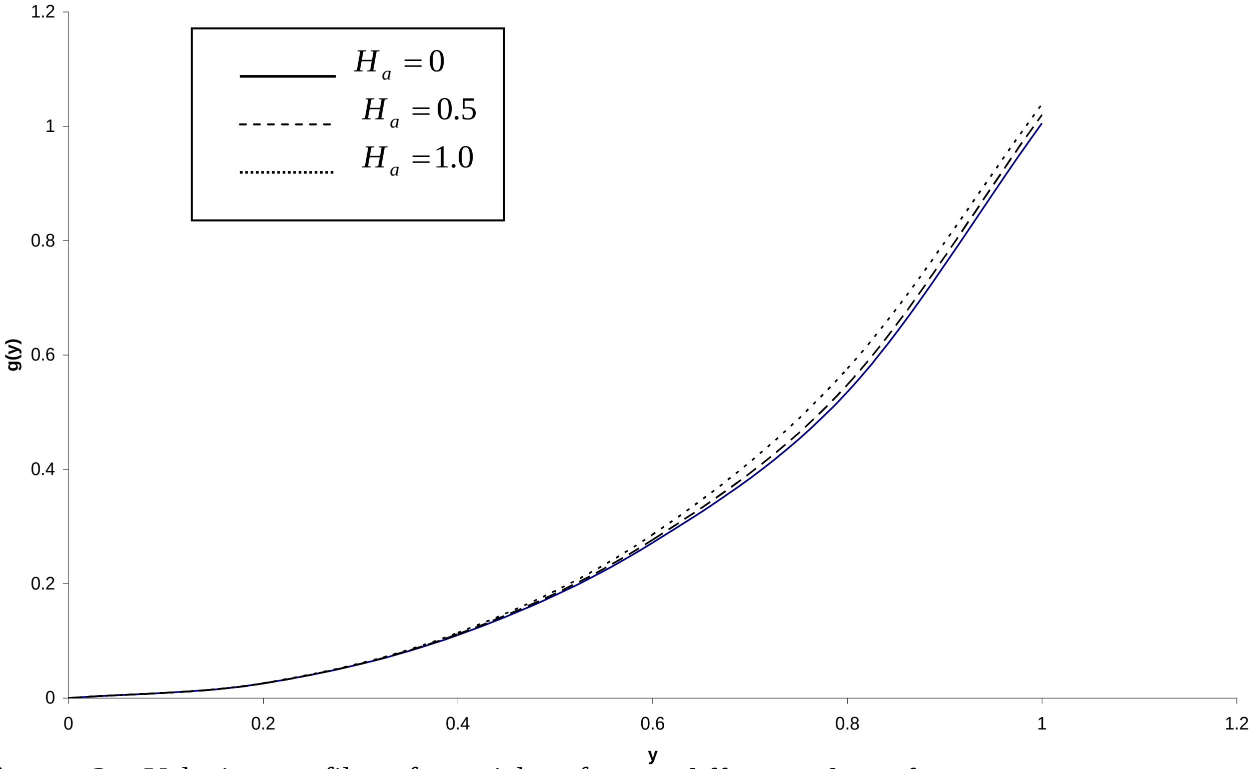

Figure 2: Velocity profile of particles for different values of H_a

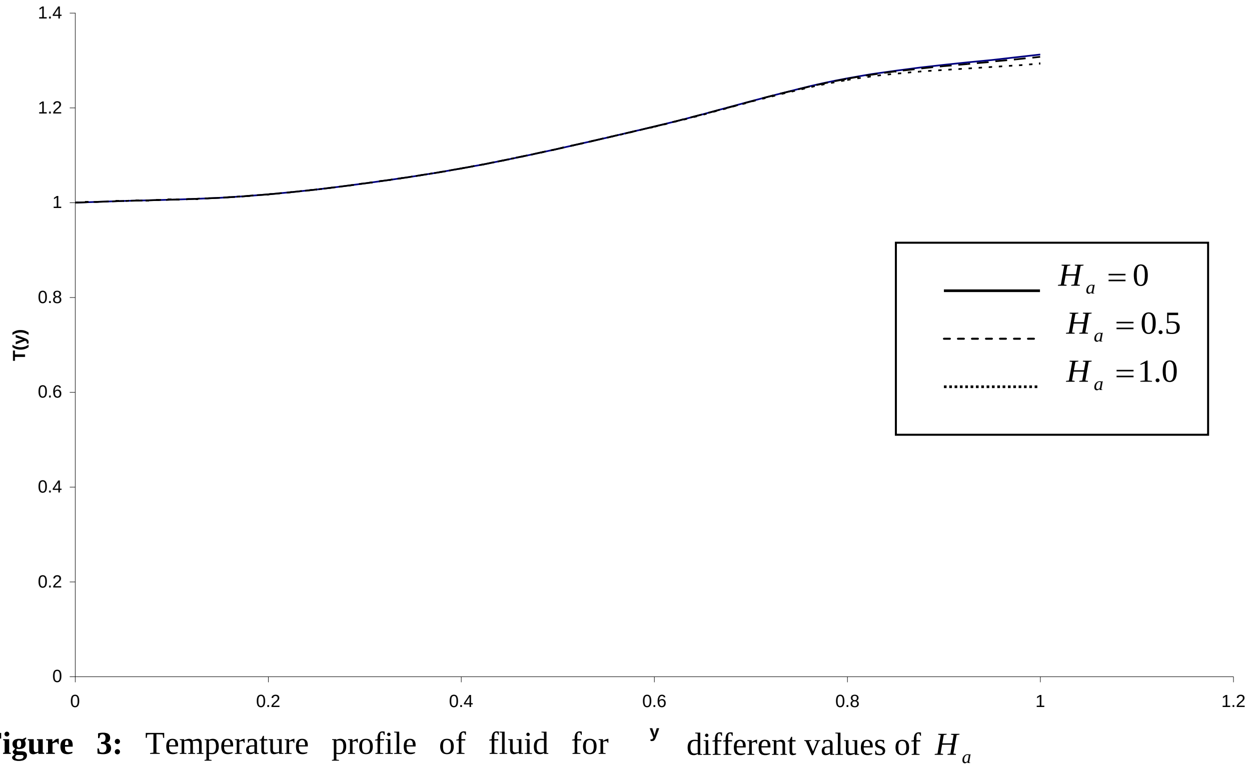

Figure 3: Temperature profile of fluid for different values of H_a

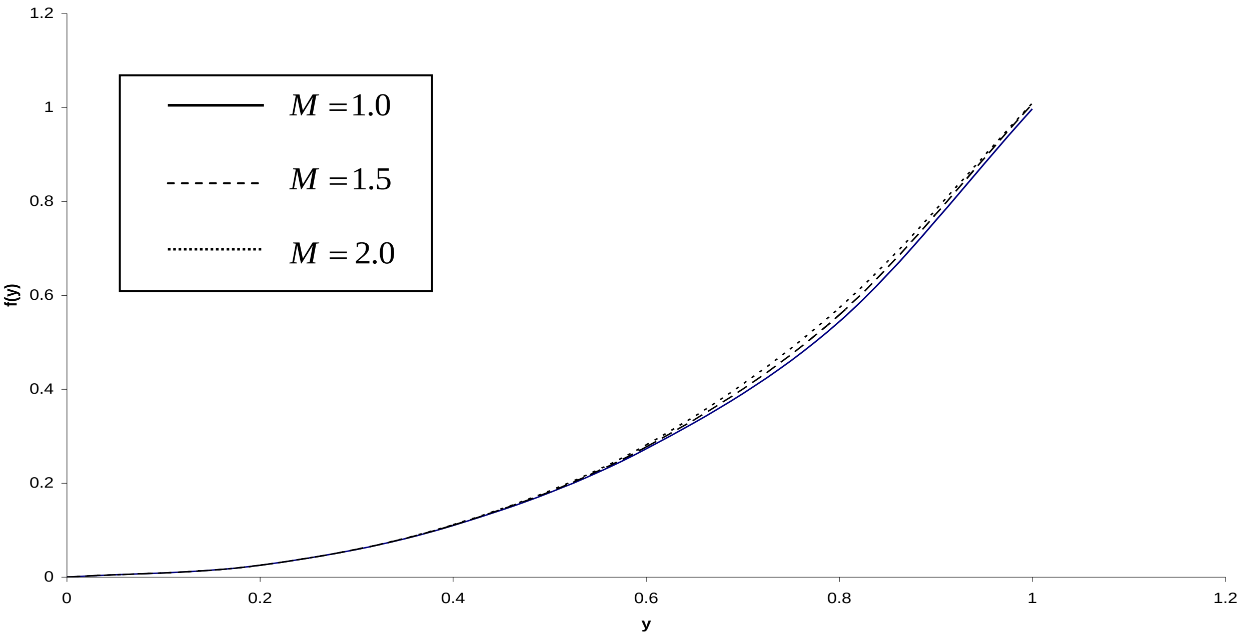

Figure 4: Velocity profile of fluid for different values of M

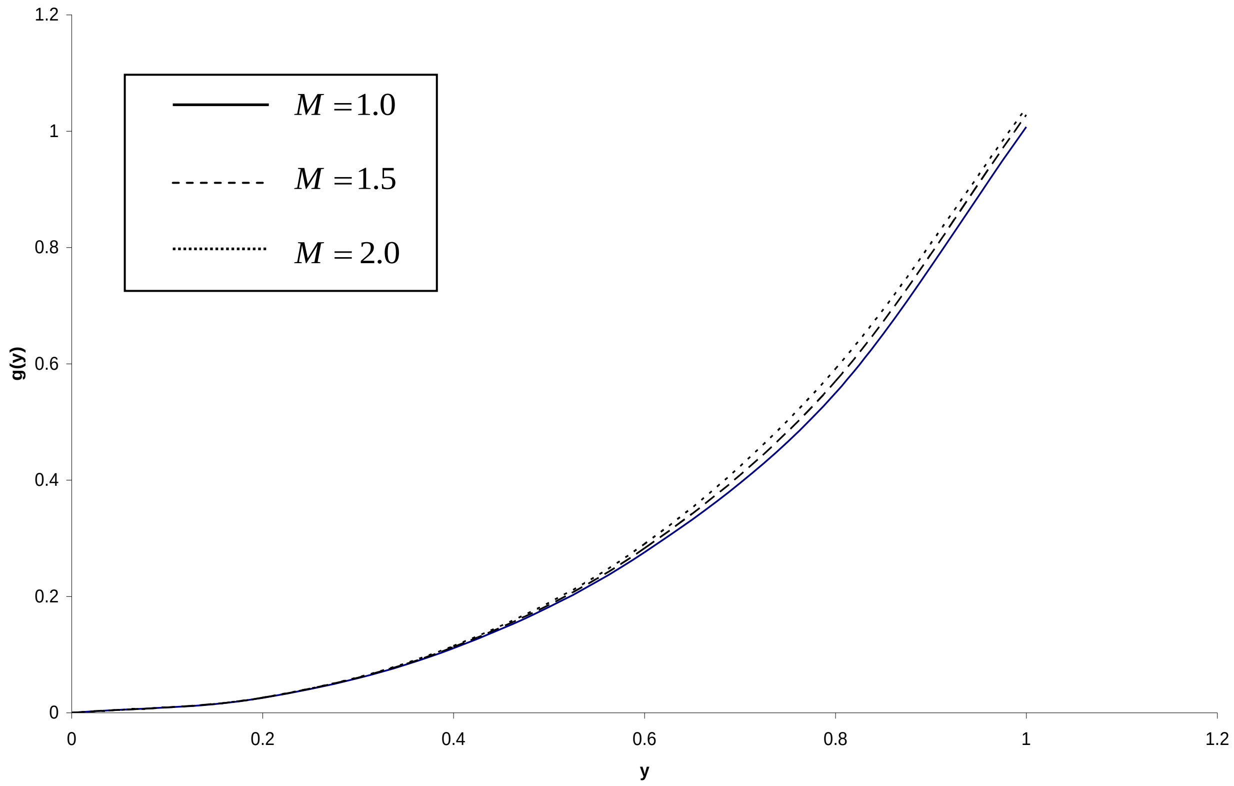

Figure 5: Velocity profile of particle for different values of M

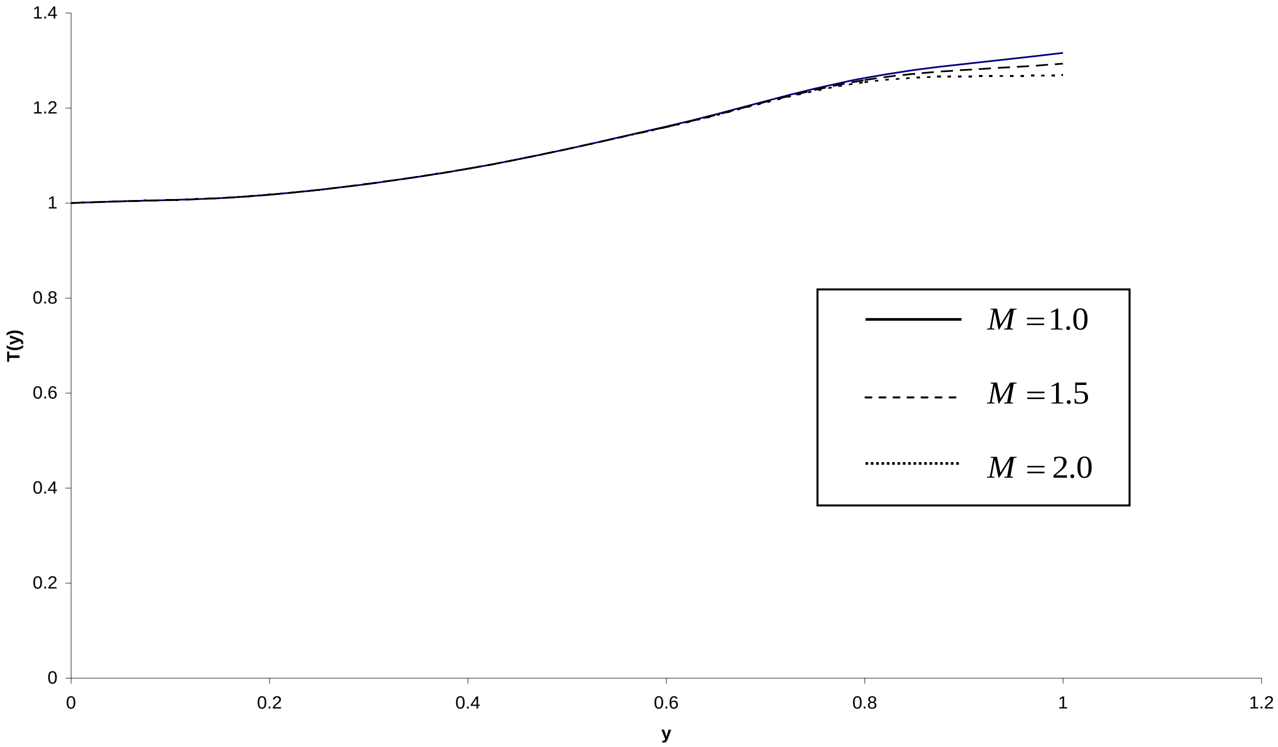

Figure 6: Temperature profile of fluid for different values of M

Conclusions

In the present work, the flow of electrically conducting viscoelastic dusty fluid has been investigated. Numerical computations are carried out using *Adams-Bashford-Moulton* classical method.

Effects of the magnetic and porosity parameter on the flow and heat transfer characteristic have been examined. From this investigation, we can draw the following conclusions:

* The effect of transverse magnetic field is to increase the velocity field which in turn causes the enhancement of the temperature distribution across the plates.
* The velocity distribution of the fluid and parameter increase as results of increase in porosity parameter which made the temperature profile lower throughout the flow region.

Therefore, we can conclude that to predict more accurate results, the effects of magnetic field and porosity parameter have to be considered in the analysis of magnetohydrodynamics flow of viscoelastic dusty fluid and heat transfer.

References

[1] Sujit, K. K. and Emmanuel, S., "Viscoelastic boundary layer flow and heat transfer over and exponential stretching sheet ", International Journal of Heat and Mass Transfer ", Vol. 48, pp. 1534-1542 (2005).

[2]. Shyamanta, C., "MHD flow and heat transfer of a dusty viscoelastic stratified fluid down an inclined channel", Theoretical and Applied Mechanics, Vol.26, pp.1-14 (2001).

[3] Attia, H. A., "Unsteady Hartman flow of a viscoelastic fluid considering the Hall effects", Canadian Journal of Physics, Vol. 82, pp. 127-135 (2004).

[4]. Yamaguchi, H., Engineering fluid Mechanics, Springer, Netherlands (2008).

[5] Kannani K. and Ramamurthy, V., "Induced dusty flow due to normal oscillations of wavy wall ", Int. J. Math, Vol. 26, No. 4, pp. 199-223 (2001)

[6] Ames, W. F., Numerical Solution of Partial Differential Equations, 2nd ed., Academic Press, New York (1977).

[7] Dieter, B., Non-Linear Magnetohydrodynamics, Cambridge University Press, United Kingdom (2004).