COMPARABILITY OF CTE AND VAR ON NORMAL,TWO AND THREE PARAMETER WEIBULL DISTRIBUTION ON A PORTFOLIO MARKET CLOSE.

Osu, Bright O and OgwoObiageri E.

Department of mathematics

Abia State University

Uturu, Nigeria

(e-mail: megabrait@yahoo.com)

ABSTRACT

The basic Question for value at risk (V_aR) is; how much can we lose on our trading portfolio by tomorrows market close or on a bad day? Or how risky is the position? This is a pure question of risk measure. There are several approaches in answering this question which one is probabilistic (or statistical) approach .This paper examines the use of other risk measures in addressing this question. A special attention is given to the use of two and three parameter Weibull in answering this question by implementing scenarios analysis to perform stress testing of Weibull conditional tail expectations (WBCTE) based risk measurement systems. We then compare behaviours of V_aR and CTE based on normal distribution and Weibull two and three parameter CTE in answering the question on a continuous and discrete distributions. Results show that WBCTE performs better than the V_aR and the CTE on normal distribution.

KEYWORDS : VAR ,CTE, Two and Three parameter WB CTE.

Mathematical Subject Classification: 91B30, 91G10, 91B80.

INTRODUCTION

One dollar today is worth more than one dollar tomorrow or in a years' time! The simple reason behind it is that money can be invested and thus 'produce' more money (for example, the investor can receives more money from returns if he decides to invest more money at a higher risk rate [1]. However, investing money involves different levels of risks depending on the choice of the investment and a higher value of money at risk brings about higher returns. In this paper, we addressed the question of risk measurement by summarizing other well known risk measurements and through some meaningful counter examples, we evaluate and compared the results through implementing platform analysis of Weibull CTE based risk measurement systems both in discrete and continuous distributions. These risk measurements are, Value at risk(V_aR), Conditional tail expectation (CTE) and Weibull based CTE. The parameter α is chosen to be 95%, 90% or 99% confidence level.

FORMULATION

Let $x > 0$ be a loss random variable with distribution function $F(x)$ the V_aR at confidence level α is defined as

$$V_aR_\alpha = \min(x: F(x) \geq \alpha) \Rightarrow \min(Q: p(L \leq Q) \geq \alpha)\ for\ 0 \leq \alpha \leq 1, \quad (1)$$

in other words V_aR is the 100th percentile of the distribution $F(x)$.

It is a loss in value of a risky asset over a defined period ,for a given confidence level α.it is the minimum amount of money one is expected to loss on a portfolio with maximum probability.

It is the standard risk used by the banking industry .it can be used by any entity to measure risk exposure, it is used most often by commercial and investment banks to capture the potential loss in value of their traded portfolios from adverse market movements over a specified period, this can then be compared to their available capital and cash reserves to ensure that losses can be covered without putting the firm at risk.V_aR is clearly on downside risk and can be calculated on either dollar or a percentage basis. Example if 1$ is invested in a bank at a discrete confidence level of Q_α= 10% $V_\alpha R$ per annum. Then , at the end of the second year the $V_\alpha R$ will be

$$1 \times (1 + Q_\alpha) \times (1 + Q_\alpha) = (1 + Q_\alpha)^2 \quad (2)$$

It is easy to say that after n years, the V_α R becomes $(1 + Q_\alpha)^m$dollar (discrete compounded). If we assume that the investor losses m amount during a year and the effective rate for each of the sub-periods of equal length is $\frac{Q_\alpha}{m}$,then , it is expected that at the end of the year the value at risk has grown to

$$(1 + \frac{Q_\alpha}{m})^m. \quad (3)$$

Taking it step further by assuming that the value at risk take place at increasingly frequent intervals (continuous compounded) expression (2) becomes

$$\lim_{m\to\infty}(1 + \frac{Q_\alpha}{m})^m = e^{Q_\alpha}. \quad (4)$$

The focus in V_aR is clearly on downside risk and potential loss [2]. It is used in banks to reflectstheir fear of liquidity crisis, where a low probability catastrophic occurrence creates a loss that wipes out the capital and creates a clients exodus.

To estimate the probability of loss with a confidence level, we need to define the probability distribution of the individual risk, the correlation across this risk and the effects of such risk on values. While the V_aR at investment banks is specified in terms of market volatility and economic growth, there is no reason why the risk cannot be defined more broadly or narrowly in specified contexts[3].

If the capital is set at V_aR_α within the interval (1, 0), the probability of ruin will be no greater than $1 - \alpha$ for a discrete distribution. It is possible that

$$p(X > V_aR_\alpha) < 1 - \alpha. \quad (5)$$

In its most general form, V_aR measures the potential loss in value of a risky asset or portfolio over a defined period, for a given confidence level.Thus the V_aR of an asset for 1 million at 95% confidence for 1 year means that there is 5% chance that the value of the asset will be equal or drop more than 1 million over a given time and 95% chance that tomorrows portfolio value will exceed that of today between the time interval. Elements of V_aR are Specific level of loss in value, a fixed time interval over which risk is assessed and a confidence level.

If the capital is set at V_aR_α within the time interval (0,1) , the probability of ruin will not be greater than $1-\alpha$implies (5). Given some confidence level $\alpha \in (0.1)$, the V_aR of a portfolio at the confidence level α is give by a smallest number ε :

$$V_aRQ_\alpha = \inf(\varepsilon \in R : p(loss)) < 1-\alpha. \qquad (6)$$

Where $1-\alpha$ is the shortfall or worst part of the distribution [4]. It is the worst expected loss or the insolvency capital. The shortfall for the portfolio X and solvency capital P(X) is defined by

$$\max\big(0, X-P(X)\big) = (X-P(X)). \qquad (7)$$

V_aR model produces an estimate of the maximum amount of money that the bank expected to lose on a particular portfolio over a given holding period with a given degree of statistical confidence level (the minimum daily amount of money one is expected to lose with maximum probability).

V_aR is a risk measure that only concerns about the frequency of the default, but not the size of the default. Which means that V_aR accesses the worst case ($1-\alpha$)event, but does not take into consideration what the loss will be if the worst case event actually occurs [5]. For instance, doubling the largest loss may not impact the V_aR at all. Although being a useful risk measure, V_aR is short of being consistent when used for comparing risk portfolios because of its non-coherence nature. It is an in appropriate risk measure for allocating capital charges, interpreted as trading limits among organization units of a bank[6]. It is inconsistent with diversification and can thus lead to suboptimal risk management if used in the context of portfolio optimization or hedging.

It is also inappropriate for the measurement of capital adequacy as it controls only the probability of the default, but not the average loss in the case of default.

The conditional tail expectation is the conditional value at risk (CV_aR) at confidence level, defined as

$$CTE_\alpha = E(L/L > Q_\alpha) = \frac{1}{1-\alpha}\int_{Q_\alpha}^{\infty} xf(x)dx \qquad (8)$$

where $Q_\alpha = V_aR_\alpha$, F(x) is the probability distribution function of a random variable X and

$E(L/L > Q_\alpha)$is the aggregate expected loss given that the aggregate exceeds some threshold Q_α (V_aR).

By [3], the limited expected value function is

$$E[L \wedge Q_\alpha] = E[\min(L, Q_\alpha)]$$

$$= \int_0^{Q_\alpha} xf(x)dx + Q_\alpha\big(1-F(Q_\alpha)\big)$$

$= \int_0^{Q_\alpha} xf(x)dx + Q_\alpha(1-\alpha).$ (9)

So we can re-write the CTE for the continuous case as;

$$CTE_\alpha = \frac{1}{1-\alpha}\{E[L] - E[L \wedge Q_\alpha] - Q_\alpha(1-\alpha)\}$$

$= Q_\alpha \frac{1}{1-\alpha}\{E[L] - E[L \wedge Q_\alpha]\}.$ (10)

This is the conditional expected loss above the V_aR or V_aR outbreak (a loss which exceeds the VAR threshold). CTE addresses some of the problem with VAR; it is the expected loss given that the loss falls in the worst part of the distribution $(1-\alpha)$ [6].It is when the probability distribution of a random loss X is continuous.CV_aR also called expected shortfall is the term commonly used in finance.The demonstration about V_aR not being a coherent risk measure, as an alternative, CTE was advocated which also has the following expression; prescribe α as a security level, then

CTE $= V_aR_\alpha + \frac{p(X > V_aR_\alpha)}{1-\alpha}.E(X - \frac{V_aR_\alpha}{X} > V_aR_\alpha).$ (11)

It is well known that CTE reflects not only the frequency of the shortfall, but also the expected value of the shortfall, hence it is coherent which makes it a superior risk measure than V_aR[7].

However, CTE although being coherent, reflects only losses exceeding the V_aR and consequently lacks incentives for mitigating losses below theV_aR and does not properly adjust for extreme low frequency and high severity losses since it only accounts for the expected shortfall [8].

Overbeck[9] also discussedV_aR and CTE as risk measures; he argued that V_aR VAR is an 'all or nothing' risk measure on any capital requirement. In that if the extreme event occurs,there will be no capital to cushion losses, since the extreme event is the one that uses up all the capital. He also argued that CTE provides a definition of "bad times" which are those where losses exceeds some threshold, not using all the available capital.

In this sequel we are motivated to suggest a risk measure which takes into consideration a measure of right-tail when one is concerned with variability along the right (or left) tail of the loss distribution.

The Weibull Distribution

In probability theory and statistics, Weibull Distribution is a continuous probability distribution. It was named after Waloddi Weibull who described it in detail in 1951[10] .The formula for the probability density function for the general Weibull distribution is;

$$f(x) = \frac{\lambda}{\alpha}\left(\frac{x-\mu}{\alpha}\right)^{\lambda-1} exp\left(-\left(\frac{x-\mu}{\alpha}\right)^{\lambda}\right),\ x \geq \mu;\ \ \lambda, \alpha > 0\ , \quad (12)$$

where λ is the shape parameter, μ is the location parameter and α is the scale parameter. The case where $\mu = 0$ and α=1 is called the standard Weibull distribution. When $\mu = 0$, it is called two – parameter Weibull distribution. Thus the equation for the three-parameter Weibull Distribution reduces to the two- parameter;

$$f(x) = \frac{\lambda}{\alpha}\left(\frac{x}{\alpha}\right)^{\lambda-1} exp\left(-\left(\frac{x}{\alpha}\right)^{\lambda}\right),\ x \geq 0;\ \ \alpha, \lambda > 0, \quad (13)$$

and that of the standard Weibull distribution becomes.

$$f(x) = \lambda x^{\lambda-1} exp\left(-x^{\lambda}\right),\ x \geq \mu;\ \ \lambda > 0. \quad (14)$$

The reliability function of the Weibull distribution in (8) is given by; $exp\left(-\left(\frac{x-\mu}{\alpha}\right)^{\lambda}\right)$,while the failure rate function $\frac{\lambda}{\alpha}\left(\frac{x-\mu}{\alpha}\right)^{\lambda-1}$

Equation (8) can be applied in any kind of distribution.

THE WEIBULL CTE

Assuming that the value at risk takes place at increasingly frequent intervals the risk flow in future can be related to the present value at risk by

$$\lim_{n\to\infty}\left(1-\frac{Q_\alpha}{m}\right)^m = e^{-Q_\alpha(T-t)}, \qquad (15)$$

where T is the future time and t is the present time.

Lemma 1: Given the probability density function (p.d.f) of the standard normal distribution, two and three parameter Weibull distribution then, their CTE are given respectively as[11];

$$CTE_\alpha = \mu + \frac{\sigma}{1-\beta}\emptyset\left(\frac{Q_{\alpha-\mu}}{\sigma}\right), \qquad (16)$$

$$CTE_\alpha = \frac{e^{-Q_\alpha}}{\alpha(1-\alpha)} \qquad (17)$$

and

$$CTE_\alpha = \frac{e^{-Q_\alpha}}{(1-\alpha)}\,. \qquad (18)$$

Proof

With a little calculation using $f_x\ (X) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}}(\frac{x-\mu}{\sigma}),-\infty < x < \infty$, the pdf of the normal distribution, equations (12) and (13) with equation (8) the CTE associated with the normal distribution, the two and three-parameter Weibull are derived. In what follows, we shall derive the tail conditional variance ($TCV_\alpha(x)$) for the Weibull distribution.

Theorem 1: Given equations (8) and (12) above, we have the tail conditional variance for a Weibull distribution as;

$$TCV_\alpha(x) = \begin{cases} \frac{\lambda^2}{1-\alpha}\Gamma\left(\frac{2}{r}+1\right),\ ifZ_\alpha = 0 \\ \frac{\lambda^2 r}{1-\alpha}\left\{\Gamma\left(\frac{2}{r}+1\right)-\Gamma\left((Z_\alpha)^2;\ \frac{2}{r}+1\right)\right\}, ifZ_\alpha \neq 0 \end{cases}. \qquad (19)$$

Proof.

Equation (8) can be written as

$$CTE_\alpha = \frac{1}{1-\alpha}\left\{\int_0^\infty xf(x)dx - \int_0^{Q_\alpha} xf(x)dx\right\},$$

and by (12) we have;

$TCV_\alpha(x) = \frac{r}{\lambda(1-\alpha)} \int_{Z_\alpha}^{\infty} (x-\mu)^2 \frac{(x-\mu)^{r-1}}{\lambda} e^{-\left(\frac{x-\mu}{\lambda}\right)^r} dx,$ (20) with $x \geq \mu,\ 0 \leq \alpha < 1,\ r, \lambda > 0$. Put $z = \frac{x-\mu}{\lambda}$ in (20) to get

$TCV_\alpha = \frac{\lambda^2}{1-\alpha} \int_{Z_\alpha}^{\infty} z^{r+1} e^{-z^r} dz.$ (21)Let $y = z^r$, it is easy to see (with a little calculation) that

$$TCV_\alpha = \frac{\lambda^2}{1-\alpha} \int_{Z_\alpha}^{\infty} y^{\frac{2}{r}} e^{-y} dz$$

$= \frac{\lambda^2}{1-\alpha} \Gamma\left(\frac{2}{r}+1\right)$, for $Z_\alpha = 0$.

To evolve a generalTCV_α formula for a Weibull Distribution, (i.e. for $Z_\alpha \neq 0$), we follow the steps in[12]and show that the general TCV formula for a Weibull Distribution where$Z_\alpha \neq 0$ is given as:

$$TCV_\alpha = \frac{\lambda^2}{1-\alpha} \int_{Z_\alpha}^{\infty} z^{r+1} e^{-z^r} dz, \quad r > o, \qquad \lambda > 0$$

$= \frac{\lambda^2 r}{1-\alpha} \left\{ \Gamma\left(\frac{2}{r}+1\right) - \Gamma\left((Z_\alpha)^2;\ \frac{2}{r}+1\right) \right\}.$

Where $\Gamma\left((Z_\alpha)^2;\ \frac{2}{r}+1\right) = \int_0^{Z_\alpha} y^{\frac{2}{r}} e^{-x} dx$ denotes the incomplete gamma function.

COMPARATIVE ANALYSIS OF THE V_aR, CTE_α AND WBCTE.

FOR DISCRETE DISTRIBUTION

Suppose that the daily returns are normally distributed with $\mu = 0$ and a 100 basis point per day.

The question is how much we can lose on this portfolio by tomorrow's market close on a $V_aR_{5\%}$and$V_aR_{1\%}$. From (1) we have $V_aR_\alpha = \min(Q\!: p(L \leq Q_\alpha)) \geq \alpha$for $0 \leq \alpha \leq 1$. And for $\alpha = 95\%$ and 99% gives$V_aR_{5\%} = 16450 \Rightarrow$ 983550.

This means that there is a 5% chance that the daily losses on 1million portfolio is equal or exceed only 16450 and 95% chance of being worth 983550 or more tomorrow.

$V_aR_{1\%} = 23260 \Rightarrow 976740.$

This means that there is 1% chance that the daily loss on 1m portfolio is equal or exceed only 23260 and 99% chance of being worth 976740 or more tomorrow.

USING 2 AND 3 PARAMETER WB CTE IN WEIGHING THE RISK

For α =99 %, $V_aR_{Q_\alpha} = 0.00232$ and for α =95%$V_aR_{Q_\alpha} = 0.00164$. From (17) WBCT$E_{5\%}$= 21018 ⇒ 978982, and from (18), we have WBCT$E_{5\%} = 19967 \Rightarrow 980033$.This shows that 21018 and19967 are the minimum daily loss of the portfolio that will exceed only 5% of the time. which implies that there is a 5% chance that the daily losses on 1m portfolio equals or exceeds 19967 and 21018 and a 95% chance of being worth 980033 and 978982 or more tomorrow.Again from (17)WBCT$E_{1\%} = 100778 \Rightarrow 899222$and (18) gives the WBCTE_(1%)=99770⇒900230.This shows that 100778 and 99770 are the minimum daily loss of the portfolio that will exceed only 1% of the time which implies that there is a 1% chance that the daily losses on 1m portfolio is equal or exceed 100778 and 99770 and a 99% chance of being worth 899222 and 900230 or more tomorrow. A higher value of money in weighing risk corresponds to higher security on market portfolio and self financing portfolio. Higher values of risk involve higher reward or returns.

CALCULATING V_aR OVER A PERIOD OF TIME

A week or one year = $\sqrt{days}$×dailyV_aR.Thus, 5 business days for $V_aR_{5\%} = 36283 \Rightarrow 963717$.

Calculating for 1 year (using 250 days as a number of the trading days in a year), we have

$V_aR_{5\%} = 260097 \Rightarrow 739903$. This means there is 5% chance of losing 36283 and 260097 or more and 95% chance of being worth 963717and739903 by the end of 5days and the year. It implies that

$p(L \leq Q_\alpha) = \alpha$ (22)

and the distribution function of a loss random variable is

$P(L \leq Q_\alpha) = F_L Q_\alpha = \alpha \Rightarrow Q_\alpha = F^{-1}(\alpha).$ (23) Taking

$V_aR = min(Q\colon p(L \leq Q_\alpha) \geq \alpha$ (24)

The reason for the *min* is that we may not have a value that exactly matches equation (22) for continuous distribution.

Suppose we have random losses on a given portfolio andthen table 1.

$$L = \begin{cases} 100 & P = 0.05 \\ 70 & P = 0.045 \\ 10 & P = 0.10 \\ 0 & P = 0.85 \end{cases}$$

CDF	$p(L \leq Q_\alpha)$
100	1.00
70	0.99
10	0.95
0	0.85

Table 1: Probability of loss portfolio.

Then calculating for V_aR of 99%, we notice that there is no value of Q_α for which $p(L \leq Q_\alpha) = 0.99$. So we choose the smallest value for the loss that gives at least a 99% probability which is V_aR of 70.$V_aR$of 70 is the smallest number that gives at least 99% probability. Therefore 70= $\min(Q: p(L < Q_\alpha) \geq 0.99)$.For 95%, 90% and 80% V_aR measure for the loss distribution is 10, 10, and 0 respectively.

Using 2 and 3 parameter WBCTE for $V_aR_{Q_\alpha}$ of 99% we have (17)and (18) lying above the distribution that is, it is more than 100 and also for 95% we have (17) and (18) which also lie above the distribution , hence WB CTE adjust for extremely high severity losses.

For example given (μ=3,σ=109) loss, since the loss random variable is continuous then 95% = $Q_{0.95}$ so that;

$\phi(\frac{V_aR-\mu}{\sigma}) = V_aR_{Q_\alpha},$ (25)

and$p(\text{L} \leq Q_{0.95}) = 0.95 \Rightarrow 212.29$.

For VAR 99% we have $p(L \leq Q_{0.99}) = 0.99 \Rightarrow 286{,}57$ where ϕ is the standard normal cumulative distribution. $V_aR$accesses the worst case $(1-\alpha)$ but does not take into consideration what the los will be if eventually the worst case occurs, (Wang 2001).

CTE addresses some of the problem with V_aR. It is the expected loss given that the loss falls in the worst part of the distribution.

The worst part of the distribution is the part above the $V_aR_{Q_\alpha}$, if Q_α falls in the continuous part of the loss distribution (that is not in the probability mass).

The CTE at confidence level α given $V_aR_{Q_\alpha}$ is given by

$\text{CTE}_\alpha = E(L/L > Q_\alpha)$ (26)

This formula does not work if Q_α falls in a probability mass, that is if there exist some ε>0 such that

$Q_{\alpha+\varepsilon} = Q_\alpha$ (27)

In this case, if we consider only losses strictly greater than Q_α, we are using less than the worst case$(1-\alpha)$of the distribution .

If losses greater than or equal to Q_α is considered, we may be using more than the worst case $(1-\alpha)$ of the distribution. That is define$B^1 = \max(B: Q_\alpha = Q_B)$ then

$CTE_\alpha = \frac{(B^1-\alpha)Q_\alpha + (1-B^1) - E(L/L > Q_\alpha)}{1-\alpha}$ (28)

Comparing the CTE and the WBCTE on a Discrete Distribution

Suppose X is a loss random variable with probability function

$$X = \begin{cases} 0 & \text{with} \quad p = 0.9 \\ 100 & \text{with} \quad p = 0.06 \\ 1000 & \text{with} \quad p = 0.04 \end{cases} .$$

Consider first 90% CTE, the 90%$V_aR_{Q_\alpha}$ is $Q_{0.90} = 0$

Since CTE is the mean loss above the V_aR at α level, then for any ε>0 $Q_{0.90+\varepsilon} > Q_{0.90}$

CTEbecomes (using (28));

$\text{CTE}_{0.90} = E(X/X > 0) = 460.$

This implies that 460 is the mean loss given that the loss lies in the upper 10% of the distribution.

Consider $\text{CTE}_{0.95}$from (27) with ε>0 we have $Q_{0.96} = Q_{0.95}$ the 95% $V_aR_{Q_{0.95}} = 100.$

To get the mean loss in the upper 5% of the distribution with $B^1 = 0.96$, (28) gives $\text{CTE}_{0.95} = 820$, which is the mean loss. (17) gives WB $\text{CTE}_{0.90} = 1111$ and WB $\text{CTE}_{0.95} = 7915$, while by (18) we have WB $\text{CTE}_{0.90} = 1000$ and WB $\text{CTE}_{0.95} = 1860$.

Comparing the Normal CTE and the Weibull CTE for a continuous Loss Distribution

Given (16) as the CTE formula for the normal distribution $\text{N}(\mu, \sigma^2)$with mean $\mu$and varianceσ^2, we have $\text{CTE}_{0.95} = 257.83$ and$\text{CTE}_{0,99} = 323.52$.But by(14) we have $\text{WBCTE}_{0.95} = 408$ and $\text{WBCTE}_{0.99}$=1012.7, while (15)gives $\text{WBCTE}_{0.95} = 387$ and gives $\text{WBCTE}_{0.99} = 1002.5$ respectively.

CONCLUSION

We have shown through meaningful counter examples herein theperformances of the WB CTE, the Normal CTE and the $V_aR$in weighing risk. The performance of WBCTE shows that if the extreme events occurs, there will be enough capital to cushion losses, which are those where losses exceed some threshold not using up all available capital. WB CTE provides the expected loss over the threshold Q_α (VAR) and CTE on normal, hence provides authentic and reliable definition for bad times. It also brings about more returns on portfolios because higher values of money is involved at risk compared to other risk measures like VAR and CTE based on normal distribution.

Suppose a risk manager is weighing the cost of risk management against the benefit of capital relief.CTE does not promote risk management but WB distortion risk measure does because the distortion function (risk adjusted function) acts as hedging referring to a strategy intended to reduce or minimize risk by making the outcome more certain which we will discuss in further study. In CTE, there is a capital penalty instead of capital relief for either removing or reducing the initial loss amount. However, the WB distortion offers a capital relief.

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