Stochastic Modeling and Performance Analysis of a Repairable Series-Parallel System with Independent Failures

Ibrahim Yusuf
Department of Mathematical Sciences, Faculty of Science, Bayero University, Kano, Nigeria
ibrahimyusif@yahoo.com
08065281616

Abstract: The paper deals with modeling and performance evaluation of a series-parallel with independent failures using Markov Birth-Death process and probabilistic approach. The system consists of five subsystems arranged in series and parallel configurations with three possible states, working, reduced capacity and failed. Through the transition diagram, systems of differential equations are developed and solved recursively via probabilistic approach. Failure and repair rates for all the subsystems are assumed constant. Availability matrices for each subsystem have been developed to provide various performance values for different combinations of failure and repair rates of all subsystems. Performance of each subsystem of series-parallel system is evaluated. The results of this paper will enhance the system performance and useful for timely execution of proper maintenance improvement, decision, planning and optimization.

Keywords: Performance optimization, availability, availability matrices

1. **Introduction**

Due to the importance of series-parallel systems in various industries, determination of their availability has become an increasingly important issue. System availability represents the percentage of time the system is available to users. Failure is an unavoidable phenomenon which can be dangerous and costly and bring about less production and profit. Proper maintenance planning plays a role in achieving high system reliability, availability and production output. It is therefore important to keep the equipments/systems always available and to lay emphasis on system availability at the highest order.

A large volume of literature exists on the issue of predicting performance evaluation of various systems. Kumar et al [1] discussed the reliability analysis of the Feeding system in the paper industry, Kumar el al.[2] discussed the availability analysis of the washing system in the paper industry, Kumar el al. [3] deal with reliability, availability and operational behavior analysis for different systems in paper plant. Kumar el al. [4] discussed the behavior analysis of Urea decomposition in the fertilizer industry under the general repair policy. Kumar et al.[5] studied the design and cost analysis of a refining system in a Sugar industry. Srinath [6] has explained a Markov model to determine the availability expression for a simple system consisting of only one component. Gupta el al. [7] has evaluated the reliability parameters of butter manufacturing system in a diary plant considering exponentially distributed failure rates of various components. Gupta et al. [8] studied the behavior of Cement manufacturing plant. Arora and Kumar [9] studied the availability analysis of the cool handling system in paper plant by dividing it into three subsystems. Singh and Garg [10] perform the availability analysis of the core veneer manufacturing system in a plywood manufacturing system under the assumption of constant failure and repair rates.

In the present paper, we study a series-parallel system consisting of five different subsystems arranged in series. Through the transition diagram obtained in this study, systems of differential equations are developed and solved recursively via probabilistic approach. Availability matrices for each subsystem have been developed to provide various performance values for different combinations of failure and repair rates of all subsystems. Performance of each subsystem of series-parallel system is evaluated.

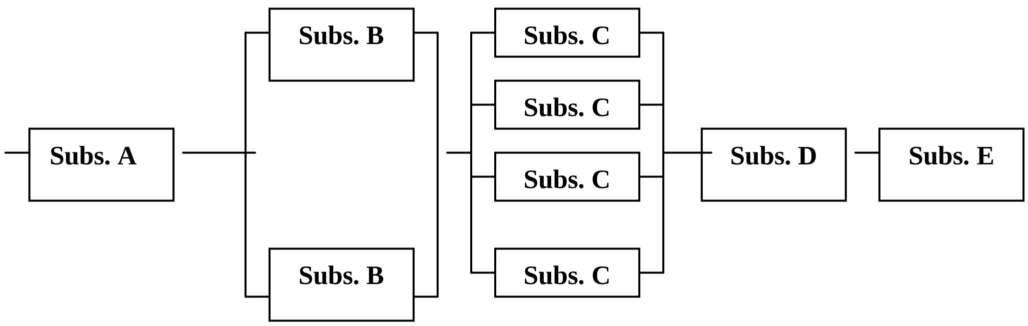


Fig.1 reliability block diagram of the system

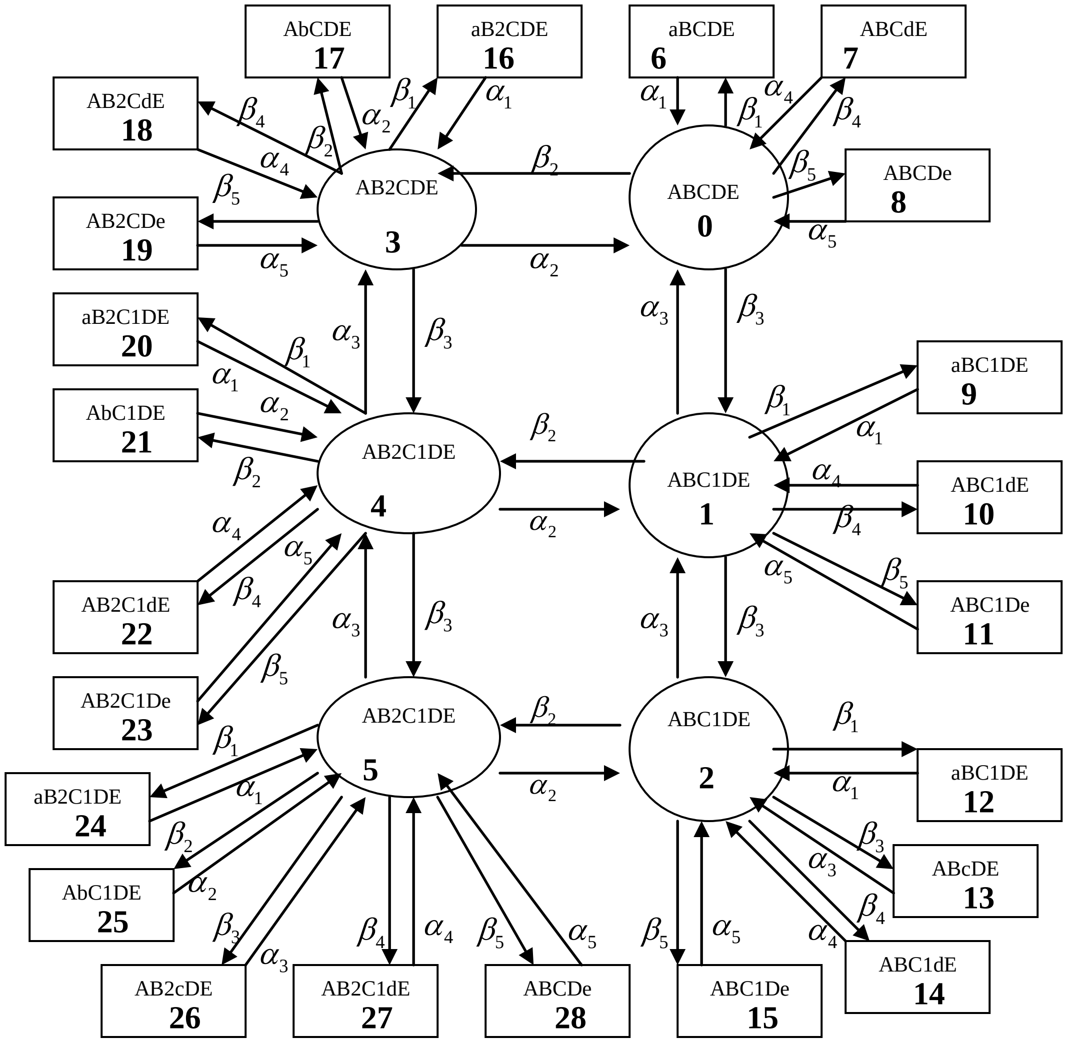


Fig.2 transition diagram of the system

2. System Descriptions

2.1 System structure

The System consists of five dissimilar subsystems which are:

1. Subsystem A: A single unit in series whose failure cause complete failure of the entire system.
2. Subsystem B: Consists of two active parallel units. Failure of one unit, the system to work in reduced capacity. Complete failure occurs when both units failed.

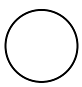
3. Subsystem C: consisting of four units in which two are in operation while the remaining two on standby. Failure of the system occurs when all the four units have failed.
4. Subsystem D: A single unit in series whose failure cause complete failure of the entire system.
5. Subsystem E: A single unit in series whose failure cause complete failure of the entire system.

2.2 Assumptions

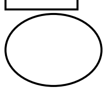
The assumptions used in model development are as follows:

1. Failure and repair rates are constant over time and are statistically independent (Kumar et al. 2007)[11]
2. At any given time, the system is either in operating state, reduced capacity or in failed state.(Kumar et al 2009)[12]
3. System failure/repair follows exponential distribution
4. System work in a reduced capacity
5. Repair is as good as new and standby units in subsystems C are of the same nature
6. Subsystems do not fail simultaneously

2.3 Notations

 Indicate the system is in full working state

 Indicate the system is in failed state

 Indicate the system in reduced capacity state

A, B, C,D,E represent full working state of subsystem
B2 denote that the subsystem B is working in reduced capacity
C1 denote subsystem is working on standby unit
a, b, c,d,e represent failed state of subsystem
$\beta_1, \beta_2, \beta_3, \beta_4, \beta_5$ represent failure rates of subsystems A, B,C
$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$: represent repair rates of subsystems A,B,C
$P_0(t)$, $P_1(t)$, $P_2(t)$: Probability of the system working with full capacity at time t
$P_3(t)$, $P_4(t)$, $P_5(t)$: Probability of the system in reduced capacity state
$P_6(t)$ to $P_{28}(t)$: Probability of the system in failed state
$P_i'(t), i = 0,1,2,...,28$: represents the derivatives with respect to time t
Av: Steady state availability of the system

3. Performance Modeling of the System

The following system linear differential equations associated with the transition diagram (Fig. 2) are derived:

$$\left(\frac{d}{dt} + \sum_{i=1}^{5} \beta_i\right) P_0(t) = \alpha_1 P_6(t) + \alpha_2 P_3(t) + \alpha_3 P_1(t) + \alpha_4 P_7(t) + \alpha_5 P_8(t) \qquad (1)$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{5}\beta_i\right)P_1(t)+\alpha_3 P_1(t)=\alpha_1 P_9(t)+\alpha_2 P_4(t)+\alpha_3 P_2(t)+\alpha_4 P_{10}(t)+\alpha_5 P_{11}(t)+\beta_3 P_0(t) \quad (2)$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{5}\beta_i\right)P_2(t)+\alpha_3 P_2(t)=\alpha_1 P_{12}(t)+\alpha_2 P_5(t)+\alpha_3 P_{13}(t)+\alpha_4 P_{14}(t)+\alpha_5 P_{15}(t)+\beta_3 P_1(t) \quad (3)$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{5}\beta_i\right)P_3(t)+\alpha_2 P_3(t)=\alpha_1 P_{16}(t)+\alpha_2 P_{17}(t)+\alpha_3 P_4(t)+\alpha_4 P_{18}(t)+\alpha_5 P_{19}(t)+\beta_2 P_0(t) \quad (4)$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{5}\beta_i\right)P_4(t)+\alpha_2 P_4(t)=\alpha_1 P_{20}(t)+\alpha_2 P_{21}(t)+\alpha_3 P_5(t)+\alpha_4 P_{22}(t)+\alpha_5 P_{23}(t)+\beta_2 P_1(t) \quad (5)$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{5}\beta_i\right)P_5(t)+\alpha_3 P_5(t)=\alpha_1 P_{24}(t)+\alpha_2 P_{25}(t)+\alpha_3 P_{26}(t)+\alpha_4 P_{27}(t)+\alpha_5 P_{28}(t)+\beta_3 P_4(t) \quad (6)$$

$$\left(\frac{d}{dt}+\alpha_m\right)P_i(t)=\beta_m P_j(t) \quad (7)$$

$m=1:\ i=6, j=0$; $m=4:i=7, j=0$; $m=5:i=8, j=0$

$m=1:i=9, j=1$; $m=4:i=10, j=1$; $m=5:i=11, j=1$;

$m=1:i=12, j=2$; $m=3:i=13, j=2$; $m=4:i=14, j=2$; $m=5:i=15, j=2$;

$m=1:i=16, j=3$; $m=2:i=17, j=3$; $m=4:i=18, j=3$; $m=5:i=19, j=3$;

$m=1:i=20, j=4$; $m=2:i=21, j=4$; $m=4:i=22, j=4$; $m=5:i=23, j=4$;

$m=1:i=24, j=5$; $m=2:i=25, j=5$; $m=3:i=26, j=5$; $m=4:i=27, j=5$;

$m=5:i=28, j=5$.

With initial condition $P_i(t)=\begin{cases}1, & i=0\\ 0, & i\neq 0\end{cases}$

4. Steady State Availability of the System

Setting $\frac{d}{dt}=0$ as $t\to\infty$ in equations $(1-7)$ and solving them recursively we obtained the steady state probabilities given below:

$$P_1 = X_3 P_0 \quad (8)$$

$$P_2 = X_3^2 P_0 \quad (9)$$
$$P_3 = X_2 P_0 \quad (10)$$
$$P_4 = X_2 X_3 P_0 \quad (11)$$
$$P_5 = X_2 X_3^2 P_0 \quad (12)$$
$$P_6 = X_1 P_0 \quad (13)$$
$$P_7 = X_4 P_0 \quad (14)$$
$$P_8 = X_5 P_0 \quad (15)$$
$$P_9 = X_1 X_3 P_0 \quad (16)$$
$$P_{10} = X_3 X_4 P_0 \quad (17)$$
$$P_{11} = X_3 X_5 P_0 \quad (18)$$
$$P_{12} = X_1 X_3^2 P_0 \quad (19)$$
$$P_{13} = X_3^3 P_0 \quad (20)$$
$$P_{14} = X_4 X_3^2 P_0 \quad (21)$$
$$P_{15} = X_5 X_3^2 P_0 \quad (22)$$
$$P_{16} = X_1 X_2 P_0 \quad (23)$$
$$P_{17} = X_2^2 P_0 \quad (24)$$
$$P_{18} = X_2 X_4 P_0 \quad (25)$$
$$P_{19} = X_2 X_5 P_0 \quad (26)$$
$$P_{20} = X_1 X_2 X_3 P_0 \quad (27)$$
$$P_{21} = X_2^2 X_3 P_0 \quad (28)$$
$$P_{22} = X_2 X_3 X_4 P_0 \quad (29)$$
$$P_{23} = X_2 X_3 X_5 P_0 \quad (30)$$
$$P_{24} = X_1 X_2 X_3^2 P_0 \quad (31)$$
$$P_{25} = X_2^2 X_3^2 P_0 \quad (32)$$
$$P_{26} = X_2 X_3^3 P_0 \quad (33)$$
$$P_{27} = X_2 X_3^2 X_4 P_0 \quad (34)$$
$$P_{28} = X_2 X_3^2 X_5 P_0 \quad (35)$$

Where $X_1 = \frac{\beta_1}{\alpha_1}, X_2 = \frac{\beta_2}{\alpha_2}, \; X_3 = \frac{\beta_3}{\alpha_3}, X_4 = \frac{\beta_4}{\alpha_4}, \; X_5 = \frac{\beta_5}{\alpha_5}$

P_0 (the probability of full working state) is determine using the condition (normalizing):

$$\sum_{i=0}^{28} P_i = 1 \quad (36)$$

Thus

$$P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} + \ldots + P_{28} = 1$$

$$P_0\left(1+X_3+X_3^2+X_2+X_2X_3+X_2X_3^2+X_1+X_4+X_5+...+X_2X_3^2X_5\right)=1 \quad (37)$$

$$P_0=\frac{1}{D} \quad (38)$$

where

$$D=\left(1+X_3+X_3^2+X_2+X_2X_3+X_2X_3^2+X_1+X_4+X_5+...+X_2X_3^2X_5\right) \quad (39)$$

The steady state availability A_v is summation of all working and reduced capacity states probabilities. Thus

$$A_v=P_0+P_1+P_2+P_3+P_4+P_5= \quad (40)$$

$$P_0(1+X_3+X_3^2+X_2+X_2X_3+X_2X_3^2)=\frac{1+X_3+X_3^2+X_2+X_2X_3+X_2X_3^2}{D}$$

5. Results and Discussion

Table 1 Availability matrix of the subsystem A of series-parallel system

β_1 \ α_1	0.1	0.185	0.27	0.355	0.44	$\alpha_2 = 0.399$ $\beta_2 = 0.109$ $\alpha_3 = 0.252$ $\beta_3 = 0.105$ $\alpha_4 = 0.425$ $\beta_4 = 0.099$ $\alpha_5 = 0.39$ $\beta_5 = 0.076$
0.05	0.5068	0.5736	0.6030	0.6195	0.6302	
0.068	0.4644	0.5432	0.5797	0.6007	0.6143	
0.086	0.4286	0.5160	0.5581	0.5829	0.5993	
0.104	0.3979	0.4913	0.5381	0.5662	0.5849	
0.122	0.3713	0.4689	0.5195	0.5504	0.5713	

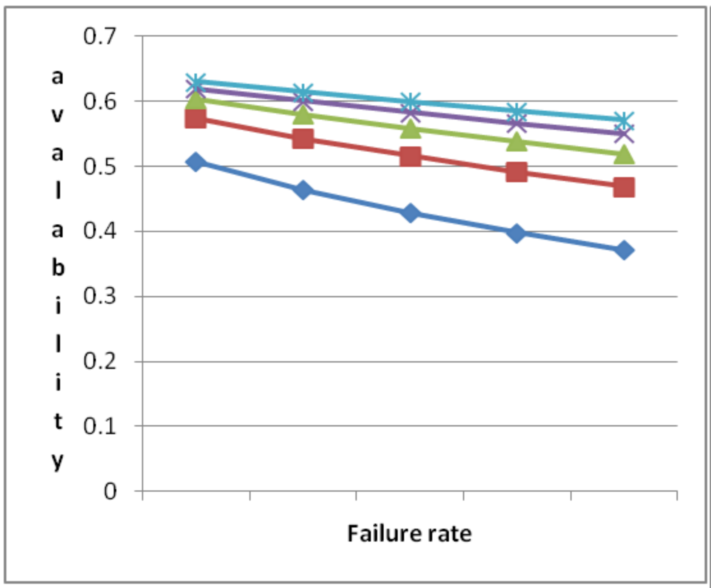


Fig.3 effect of β_1 on availability

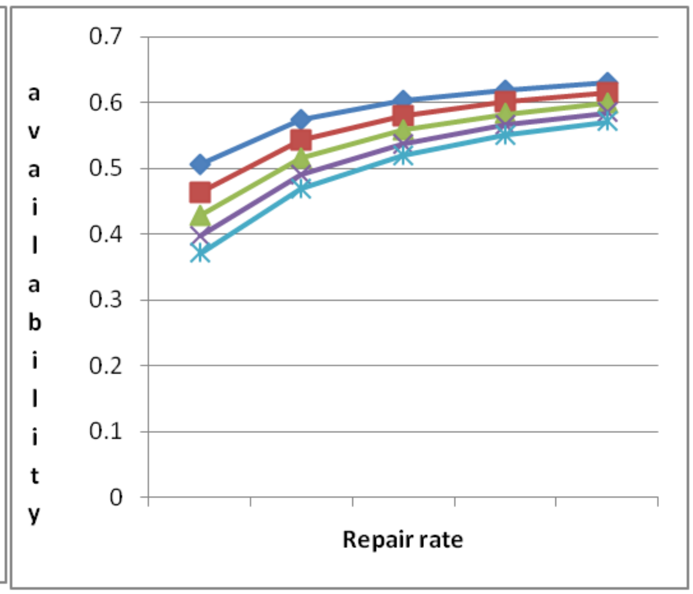


Fig.4 effect of α_1 on availability

Table 2 Availability matrix of the subsystem B of series-parallel system

α_2 / β_2	0.1	0.2	0.3	0.4	0.5	$\alpha_1 = 0.439$ $\beta_1 = 0.051$ $\alpha_3 = 0.399$ $\beta_3 = 0.105$ $\alpha_4 = 0.425$ $\beta_4 = 0.067$ $\alpha_5 = 0.25$ $\beta_5 = 0.099$
0.075	0.6261	0.8065	0.8825	0.9250	0.9650	
0.100	0.5530	0.7408	0.8326	0.8825	0.9162	
0.125	0.4891	0.6840	0.7835	0.8431	0.8825	
0.150	0.4380	0.6345	0.7408	0.8065	0.8508	
0.175	0.3962	0.5912	0.7021	0.7725	0.8208	

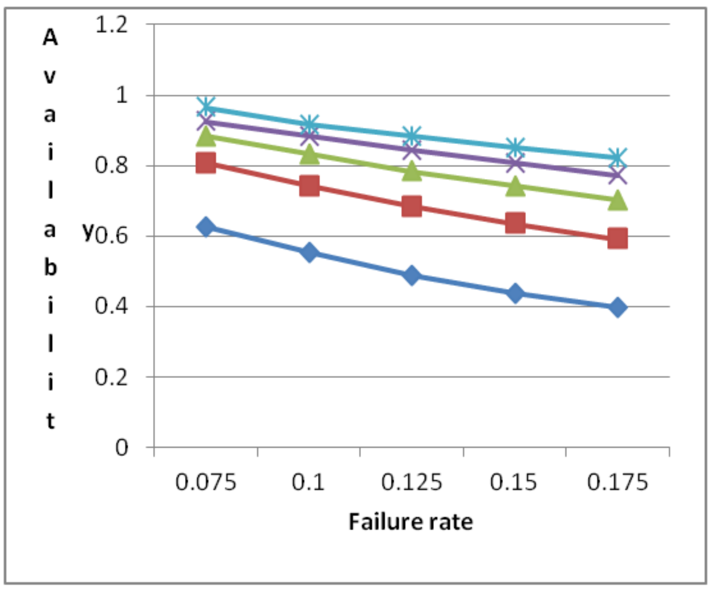


Fig. 5 effect of β_2 on availability

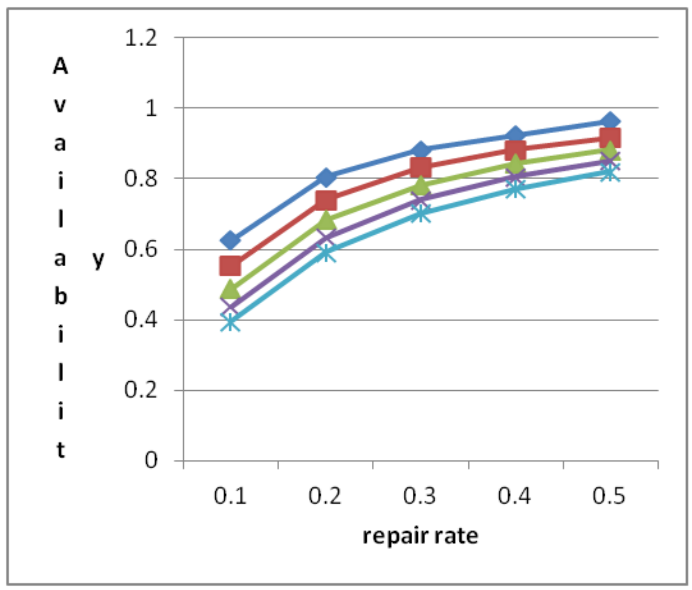


fig. 6 effect of α_2 on availability

Table 3 Availability matrix of the subsystem C of series-parallel system

β_3 \ α_3	0.125	0.2	0.275	0.35	0.425	$\alpha_1 = 0.44$ $\beta_1 = 0.05$ $\alpha_2 = 0.169$ $\beta_2 = 0.077$ $\alpha_4 = 0.44$ $\beta_4 = 0.105$ $\alpha_5 = 0.25$ $\beta_5 = 0.099$
0.04	0.7012	0.7474	0.7693	0.7824	0.7913	
0.0575	0.6522	0.7132	0.7435	0.7615	0.7735	
0.075	0.6076	0.6814	0.7188	0.7414	0.7565	
0.0925	0.5672	0.6513	0.6952	0.7220	0.7401	
0.11	0.5430	0.5583	0.6726	0.7033	0.7241	

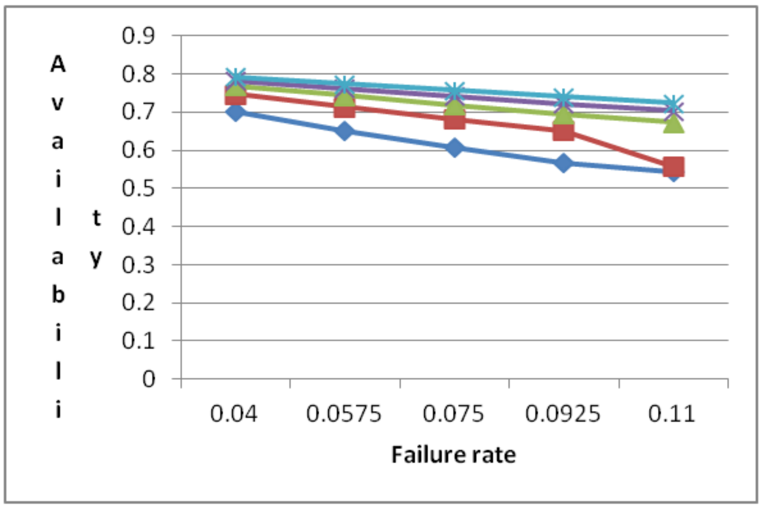


Fig. 7 effect of β_3 on availability

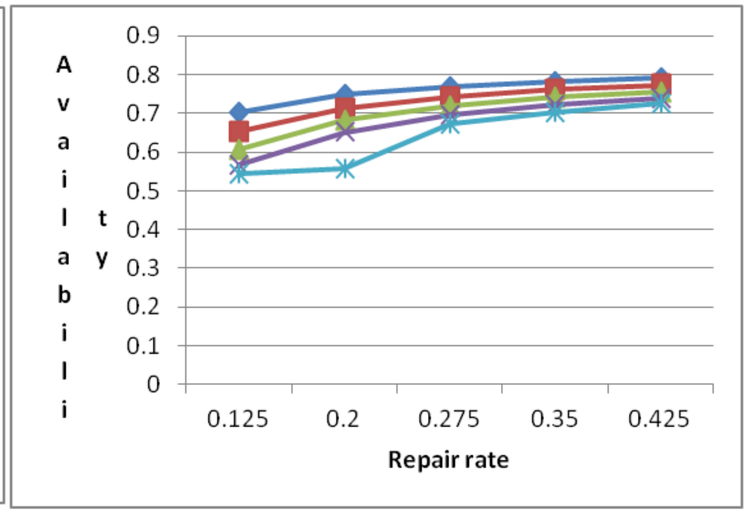
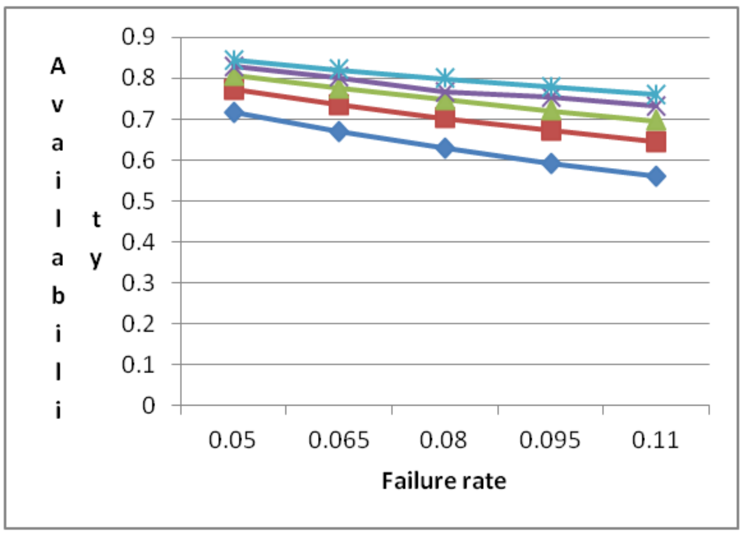


Fig. 8 Effect of α_3 on availability

Table 4 Availability matrix of the subsystem D of series-parallel system

β_4 \ α_4	0.155	0.23	0.305	0.38	0.455
0.05	0.7154	0.7736	0.8070	0.8286	0.8438
0.065	0.6691	0.7365	0.7762	0.8024	0.8209
0.08	0.6284	0.7027	0.7476	0.7670	0.7993
0.095	0.5924	0.6719	0.7211	0.7546	0.7788
0.11	0.5602	0.6437	0.6964	0.7328	0.7593

$\alpha_1 = 0.43$
$\beta_1 = 0.051$
$\alpha_2 = 0.49$
$\beta_2 = 0.075$
$\alpha_3 = 0.4$
$\beta_3 = 0.067$
$\alpha_5 = 0.12$
$\beta_5 = 0.08$



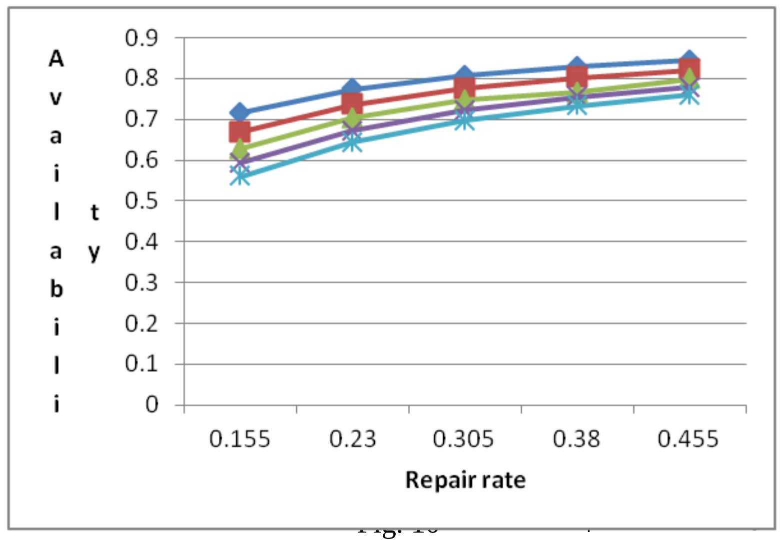


Table 5 Availability matrix of the subsystem D of series-parallel system

β_5 \ α_5	0.1	0.195	0.29	0.385	0.48	$\alpha_1 = 0.44$ $\beta_1 = 0.059$ $\alpha_2 = 0.499$ $\beta_2 = 0.0775$ $\alpha_3 = 0.425$ $\beta_3 = 0.075$ $\alpha_4 = 0.16$ $\beta_4 = 0.096$
0.055	0.6278	0.7536	0.8100	0.8419	0.8624	
0.066	0.5865	0.7229	0.7858	0.8221	0.8457	
0.077	0.5510	0.6945	0.7631	0.8033	0.8296	
0.088	0.5195	0.6684	0.7416	0.7852	0.8142	
0.099	0.4914	0.6441	0.7213	0.7680	0.7992	

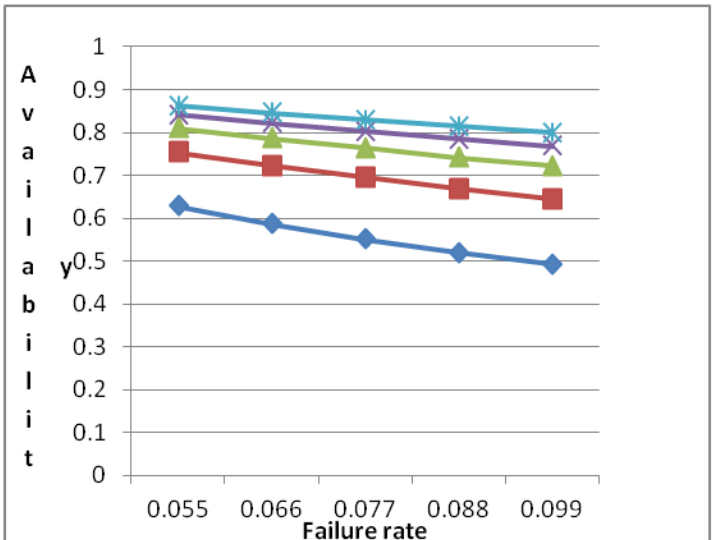


Fig. 11 effect of β_5 on availability

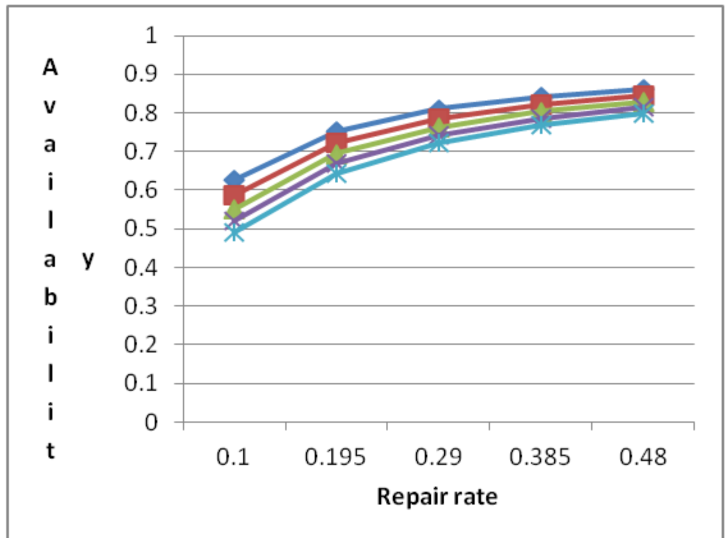


Fig. 12 effect of α_5 on availability

Table 6 Optimum values of Failure/Repair rates of Subsystems of Series-Parallel system

S/N	Subsystem	Failure rate β_i	Repair rate α_i	Maximun

				Availability Level
1	A	0.05	0.44	63%
2	B	0.075	0.5	96%
3	C	0.04	0.425	79%
4	D	0.05	0.455	84%
5	E	0.055	0.48	86%

Table 1 and Fig. 3-4 depict the effect of failure and repair rate of subsystem A on the availability of series-parallel system. It is observed that as the failure rate of subsystem A increases from 0.05 to 0.122, the unit availability decreases by 13.50%. Similarly as the repair rate of subsystem A increases from 0.1 to 0.44, the unit availability increases by 12.30%.

Table 2 and Fig. 5-6 depict the effect of failure and repair rate of subsystem B on the availability of series-parallel system. It is observed that as the failure rate of subsystem B increases from 0.075 to 0.175, the unit availability decreases by 22.99%. Similarly as the repair rate of subsystem A increases from 0.1 to 0.5, the unit availability increases by 33.89%.

Table 3 and Fig. 7-8 depict the effect of failure and repair rate of subsystem C on the availability of series-parallel system. It is observed that as the failure rate of subsystem C increases from 0.04 to 0.11, the unit availability decreases by 15.82%. Similarly as the repair rate of subsystem C increases from 0.125 to 0.425, the unit availability increases by 9.01%.

Table 4 and Fig. 9-10 depict the effect of failure and repair rate of subsystem D on the availability of series-parallel system. It is observed that as the failure rate of subsystem D increases from 0.05 to 0.11, the unit availability decreases by 15.52%. Similarly as the repair rate of subsystem D increases from 0.155 to 0.455, the unit availability increases by 12.84%.

Table 5 and Fig. 11-12 depict the effect of failure and repair rate of subsystem E on the availability of series-parallel system. It is observed that as the failure rate of subsystem E increases from 0.055 to 0.099, the unit availability decreases by 13.64%. Similarly as the repair rate of subsystem E increases from 0.1 to 0.48, the unit availability increases by 23.46%.

Table 6 helps in determining the subsystem with maximum availability. It is observed that subsystem B is having maximum availability (96%). Shown in the Table 6 are the optimum values of failure and repair rates for maximum availability for each subsystem. From Table 6, it is observed that the most critical subsystem as far as maintenance is concerned and required immediate attention is subsystem B, as the effect of failure rates on system availability is higher (22.99%) than that of subsystems A,C,D and E.

6. Conclusion

Explicit expression for the availability model is developed and used for the evaluation of performance of different subsystems of the series-parallel system in this study. Using the model, tables 1-6 are constructed to show the relationship between failure and repair rates on system availability. The availability decreases as the failure rate increases. Similarly as availability increases so also the repair rates. The model will assist maintenance engineers and managers for proper maintenance utilization. The results of this study will be beneficial to the plant management for the availability analysis of series-parallel system.

References

[1] kumar, D., Singh Jai and Pandey PC. Reliability analysis of the feeding system in the paper industry. Microelectron Reliability, vol. 28, no. 2, 1988. pp 213-215.
[2] Kumar, D., Singh Jai and Pandey PC. Availability analysis of the washing system in the paper industry. Microelectron Reliability, vol.29, 1989, pp 775-778.
[3] Kumar, D., Singh Jai and Pandey PC. Operational behavior and profit function for a bleaching and screening in the paper industry. Microelectron Reliability, vol. 33, 1993, pp 1101-1105.
[4] Kumar, D., Singh Jai and Pandey PC. Behavior analysis of urea decomposition in the fertilizer industry under general repair policy. Microelectron Reliability, vol. 31, no. 5, 1991, pp 851-854.
[5] Kumar, D., Singh Jai and Pandey PC. Design and cost analysis of a refining system in a Sugar industry. Microelectron Reliability, vol. 30, no. 6, 1990, pp 1025-1028.
[6] Srinath, L.S. Reliability Engineering 3rd edition, East west press Pvt Ltd. New Delhi, India, 1994.
[7] P. Gupta, A. Lal, R. Sharma and J. Singh. Numerical analysis of reliability and availability of series processes in butter oil processing plant. International Journal of Quality and Reliability Management, vol. 22, no. 3 2005, pp 303-316.
[8] P. Gupta, A. Lal, R. Sharma and J. Singh. Behavioral study of the Cement Manufacturing plant. A numerical approach. Journal of Mathematics and systems sciences. Vol.1, no. 1, 2005, pp 50-69.
[9] Arora, N., and Kumar, D. System analysis and maintenance management for coal handling system in a paper plant. International Journal of Management and Systems, 2000.
[10] Singh, J., and Garg, S. Availability analysis of core veneer manufacturing system in plywood industry. International Conference on Reliability and safety engineering, India institute of Technology, Kharagpur, 2005,pp 497-508.
[11] Kumar, D., Tewari, P.C. and Sharma, R. Simulated availability of CO_2 cooling system in a fertilizer plant. Industrial engineering Journal. Vol. 36, no. 10, 2007, pp 19-23
[12] Gupta, S., Tewari, P.C. and Sharma, A.K. Reliability and availability analysis of handling unit of a steam thermal plant. International Journal of Engineering Research and Industrial Application, vol.1, no.5, 2008, pp 53-62.
[13] Kumar, S., Tewari, P.C. and Kumar, S. Performance evaluation and availability analysis of ammonia synthesis unit in a fertilizer plant.