Simulation Modeling for the Stochastic Analysis and Performance Evaluation of a Repairable Series-Parallel System

Ibrahim Yusuf [*1] and Nafiu Hussaini [2]
[1,2] Department of Mathematical Sciences, Bayero University, Kano, Nigeria
ibrahimyuisf@yahoo.com Nafiu_hussaini@yahoo.com

Abstract
The paper deals with modeling and performance evaluation of a series-parallel using Markov Birth-Death process and probabilistic approach. The system consists of three subsystems arranged in series and parallel configurations with two possible states, working and failed. Through the transition diagram, systems of differential equations are developed and solved recursively via probabilistic approach. Failure and repair rates for all the subsystems are assumed constant. Availability matrices for each subsystem have been developed to provide various performance values for different combinations of failure and repair rates of all subsystems. Performance of each subsystem of series-parallel system is evaluated. The results of this paper will enhance the system performance and useful for timely execution of proper maintenance improvement, decision, planning and optimization.

Keywords: Performance optimization, availability, availability matrices

1. **Introduction**

Due to the importance of series-parallel systems in various industries, determination of their availability has become an increasingly important issue. System availability represents the percentage of time the system is available to users. Failure is an unavoidable phenomenon which can be dangerous and costly and bring about less production and profit. Proper maintenance planning plays a role in achieving high system reliability, availability and production output. It is therefore important to keep the equipments/systems always available and to lay emphasis on system availability at the highest order.

A large volume of literature exists on the issue of predicting performance evaluation of various systems. Kumar et al [1] discussed the reliability analysis of the Feeding system in the paper industry, Kumar el al. [2] discussed the availability analysis of the washing system in the paper industry, Kumar el al. [3] deal with reliability, availability and operational behavior analysis for different systems in paper plant. Kumar el al.[4] discussed the behavior analysis of Urea decomposition in the fertilizer industry under the general repair policy. Kumar et al. [5] studied the design and cost analysis of a refining system in a Sugar industry. Srinath [6] has explained a Markov model to determine the availability expression for a simple system consisting of only one component. Gupta el al.[7] has evaluated the reliability parameters of butter manufacturing system in a diary plant considering exponentially distributed failure rates of various components. Gupta et al. [8]studied the behavior of Cement manufacturing plant. Arora and Kumar [9] studied the availability analysis of the cool handling system in paper plant by dividing it into three subsystems. Singh and Garg [10]perform the availability analysis of the core veneer manufacturing system in a plywood manufacturing system under the assumption of constant failure and repair rates.

In the present paper, we study a series-parallel system consisting of three different subsystems arranged in series. Through the transition diagram obtained in the study, systems of differential equations are developed and solved recursively via probabilistic approach. Availability matrices for each subsystem have been developed to provide various performance values for different combinations of failure and repair rates of all subsystems. Performance of each subsystem of series-parallel system is evaluated.

2. System Description

The series-parallel system in this study consists of three subsystems, namely:

Subsystem A is a single unit in series whose failure brings about system failure.

Subsystem B consists of three units in a cold standby. Failure of the system occurs when all the three units have failed.

Subsystem C is a single unit in series whose failure brings about system failure.

2.1 Assumptions

The assumptions used in model development are as follows:

1. Failure and repair rates are constant over time and are statistically independent
2. At any given time, the system is either in operating state or in failed state.
3. System failure/repair follows exponential distribution
4. System do not work in a reduced capacity
5. Repair is as good as new and standby units in subsystems B are of the same nature
6. Units do not fail simultaneously.

2.2 Notations

Indicate the system is in full working state

Indicate the system is in failed state

A, B, C represent full working state of subsystem

B_1 denote that the subsystem B is working on standby unit

a, b, c represent failed state of subsystem

$\beta_1, \beta_2, \beta_3$ represent failure rates of subsystems A, B,C

$\alpha_1, \alpha_2, \alpha_3$: represent repair rates of subsystems A,B,C

$P_0(t)$: Probability of the system working with full capacity at time t

$P_1(t)$, $P_2(t)$: Probability of the system in cold standby state

$P_3(t)$ to $P_9(t)$: Probability of the system in failed state

$P_i'(t), i = 1, 2, ..., 9$: represents the derivatives with respect to time t

Av : Steady state availability of the system

2.3 Reliability block diagram o and transition diagram of the system

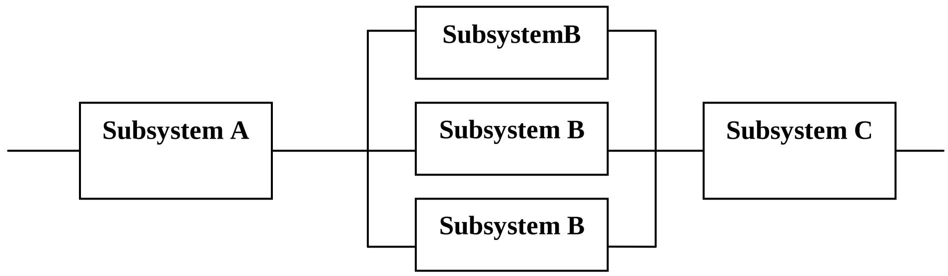

Fig. 1 Reliability block diagram of the system

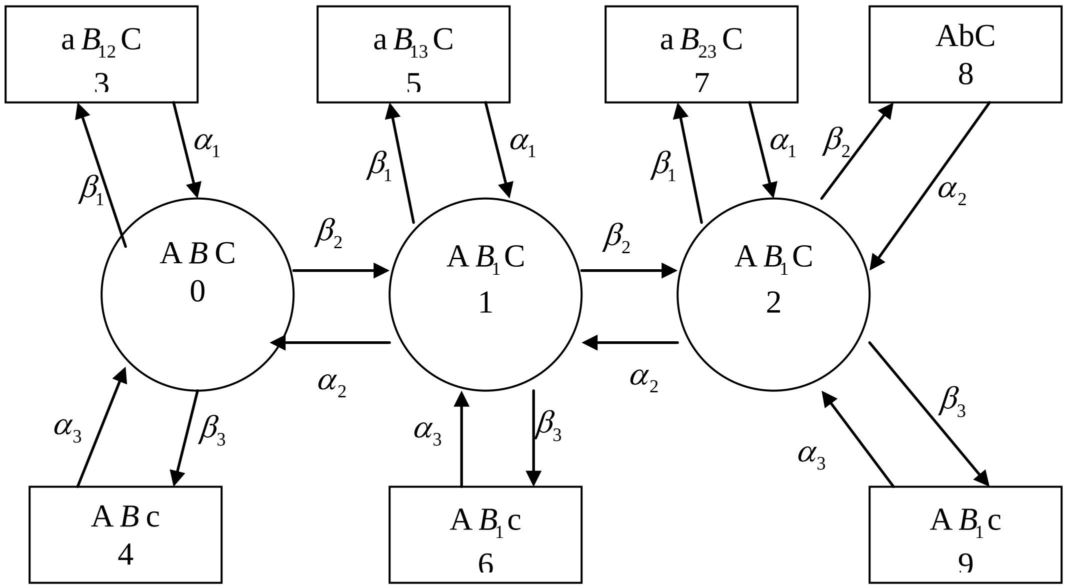

Fig. 2 Transition diagram of the System (series-parallel system)

3. Simulation Modeling

$$P_0'(t) + \sum_{i=1}^{3} \beta_i P_0(t) = \alpha_1 P_3(t) + \alpha_2 P_1(t) + \alpha_3 P_4(t) \tag{1}$$

$$P_1'(t) + \sum_{i=1}^{3} \beta_i P_1(t) + \alpha_2 P_1(t) = \alpha_1 P_5(t) + \alpha_2 P_2(t) + \alpha_3 P_6(t) + \beta_2 P_0(t) \quad (2)$$

$$P_2'(t) + \sum_{i=1}^{3} \beta_i P_2(t) + \alpha_2 P_2(t) = \alpha_1 P_7(t) + \alpha_2 P_8(t) + \alpha_3 P_9(t) + \beta_2 P_1(t) \quad (3)$$

$$P_3'(t) + \alpha_1 P_3(t) = \beta_1 P_0(t) \quad (4)$$

$$P_4'(t) + \alpha_3 P_4(t) = \beta_3 P_0(t) \quad (5)$$

$$P_5'(t) + \alpha_1 P_5(t) = \beta_1 P_1(t) \quad (6)$$

$$P_6'(t) + \alpha_3 P_3(t) = \beta_3 P_1(t) \quad (7)$$

$$P_7'(t) + \alpha_1 P_7(t) = \beta_1 P_2(t) \quad (8)$$

$$P_8'(t) + \alpha_2 P_8(t) = \beta_2 P_2(t) \quad (9)$$

$$P_9'(t) + \alpha_3 P_9(t) = \beta_3 P_2(t) \quad (10)$$

With initial condition at time $t = 0$

$$P_i(t) = \begin{cases} 1 & for\ i = 0 \\ 0 & for\ i \neq 0 \end{cases}$$

Solving equation (1) – (10) recursively we have

$$P_1(t) = \left(\frac{\beta_2}{\alpha_2}\right) P_0(t) \qquad P_4(t) = \left(\frac{\beta_3}{\alpha_3}\right) P_0(t) \qquad P_7(t) = \left(\frac{\beta_1}{\alpha_1}\right)\left(\frac{\beta_2}{\alpha_2}\right)^2 P_0(t)$$

$$P_2(t) = \left(\frac{\beta_2}{\alpha_2}\right)^2 P_0(t) \qquad P_5(t) = \left(\frac{\beta_1}{\alpha_1}\right)\left(\frac{\beta_2}{\alpha_2}\right) P_0(t) \qquad P_8(t) = \left(\frac{\beta_2}{\alpha_2}\right)^3 P_0(t)$$

$$P_3(t) = \left(\frac{\beta_1}{\alpha_1}\right) P_0(t) \qquad P_6(t) = \left(\frac{\beta_3}{\alpha_3}\right)\left(\frac{\beta_2}{\alpha_2}\right) P_0(t) \qquad P_9(t) = \left(\frac{\beta_3}{\alpha_3}\right)\left(\frac{\beta_2}{\alpha_2}\right)^2 P_0(t)$$

4. Steady state availability

Using normalizing condition:

$$P_0(t) + P_1(t) + P_2(t) + P_3(t) + P_4(t) + P_5(t) + P_6(t) + P_7(t) + P_8(t) + P_9(t) = 1 \quad (11)$$

$$P_0(t) = \frac{1}{\left(1 + \frac{\beta_2}{\alpha_2} + \left(\frac{\beta_2}{\alpha_2}\right)^2\right)\left(1 + \frac{\beta_1}{\alpha_1} + \frac{\beta_3}{\alpha_3}\right) + \left(\frac{\beta_2}{\alpha_2}\right)^3} \quad (12)$$

$$Av = P_0(t) + P_1(t) + P_2(t) = \frac{1 + \frac{\beta_2}{\alpha_2} + \left(\frac{\beta_2}{\alpha_2}\right)^2}{\left(1 + \frac{\beta_2}{\alpha_2} + \left(\frac{\beta_2}{\alpha_2}\right)^2\right)\left(1 + \frac{\beta_1}{\alpha_1} + \frac{\beta_3}{\alpha_3}\right) + \left(\frac{\beta_2}{\alpha_2}\right)^3} \tag{13}$$

5. Performance evaluation

Performance and availability of engineering/industrial systems are affected by the failure and repair rates of the subsystems that make up the system. The model in (13) is used to predict the availability and performance of series-parallel system in this study through the known values of both failure and repair rates of the subsystems. Table 1-3 are the availability matrices generated using the model in equation (13) for subsystems A, B, and C of the series-parallel system in this study. Different combinations of failure and repair rates are used in the construction of the matrices. The plots of the matrices are given in figure 3-8. In table 4 we present the optimum value of the availability level with the corresponding failure and repair rates for maintenance decision, planning and optimization.

6. Results and Discussion

The performance of each subsystem is studied and analyzed through the model obtained. The availability values are given in the Table 1-3 and are plotted in figure 3-8, which reveal the effect of both failure and repair rates of subsystem on availability of series-parallel system.

Table 1 Availability matrix of the subsystem A of series-parallel system

β_1 \ α_1	0.1	0.2	0.3	0.4	0.5
0.05	0.3780	0.4174	0.4325	0.4404	0.4453
0.06	0.3642	0.4089	0.4263	0.4356	0.4414
0.07	0.3514	0.4007	0.4204	0.4309	0.4375
0.08	0.3395	0.3928	0.4146	0.4263	0.4337
0.09	0.3283	0.3853	0.4089	0.4218	0.4300

$\alpha_2 = 0.5$
$\beta_2 = 0.9$
$\alpha_3 = 0.5$
$\beta_3 = 0.09$

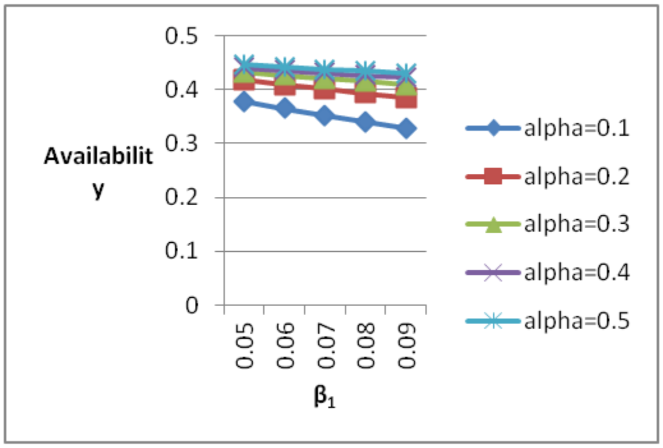

Fig. 3 effect of β_1 on availability

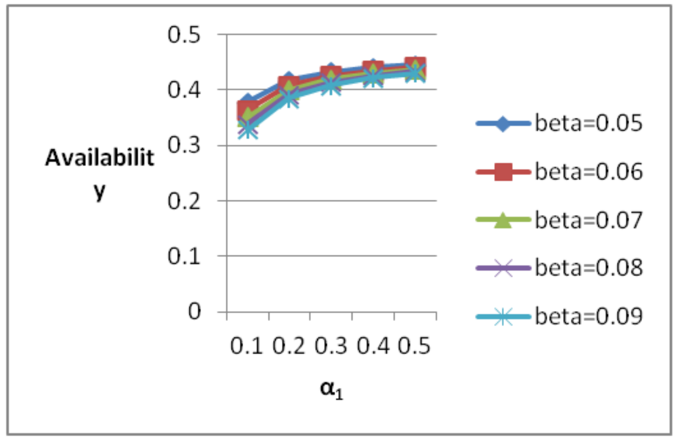

Fig. 4 effect of α_1 on availability

Table 1 and figure 2 and figure 3 shows the effect of failure rate and repair rate of subsystem A on the system availability of the series-parallel system. From constant values of both failure and repair rates of other subsystems, as the failure rate of subsystem A increases from 0.05 to 0.09, the unit availability decreases by 3.85%. Similarly, as the repair rate of subsystem A increases from 0.1 to 0.5, the subsystem availability increases by 5.37%.

Table 2 Availability matrix of the subsystem B of series-parallel system

β_2 \ α_2	0.1	0.2	0.3	0.4	0.5
0.5	0.1766	0.3094	0.4032	0.4668	0.5093
0.6	0.1503	0.2698	0.3606	0.4273	0.4751
0.7	0.1308	0.2387	0.3250	0.3918	0.4426
0.8	0.1157	0.2138	0.2950	0.3606	0.4126
0.9	0.1037	0.1935	0.2698	0.3333	0.3853

$\alpha_1 = 0.2$
$\beta_1 = 0.09$
$\alpha_3 = 0.5$
$\beta_3 = 0.09$

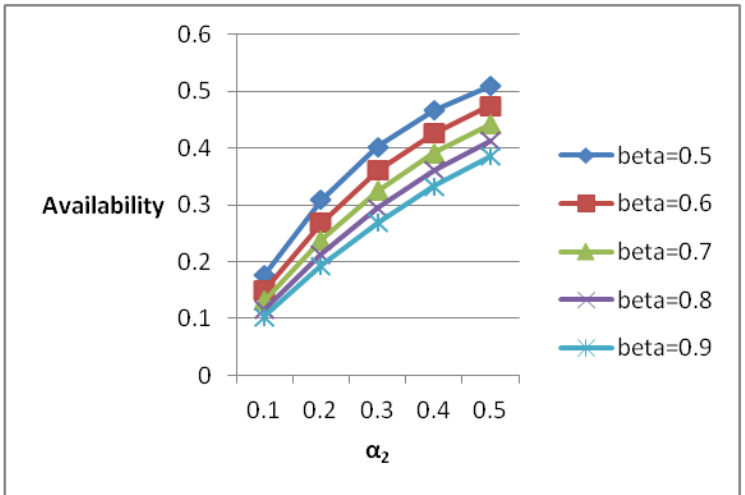

Fig. 5 effect of α_2 on availability

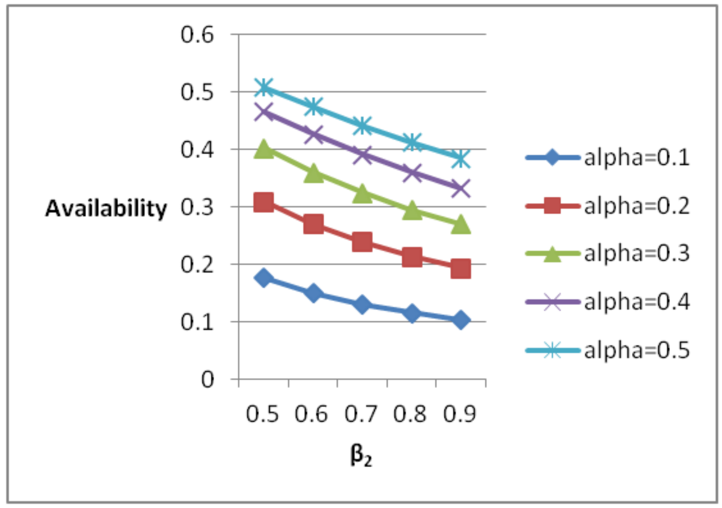

Fig. 6 effect of β_2 on availability

Table 2 and figure 5 and figure 6 shows the effect of failure and repair rates of subsystem B on the availability of series-parallel system. Keeping the values of both failure and repair rates of other subsystems fixed, as the failure rate of subsystem B increases from 0.5 to 0.9, the subsystem availability decreases by 7.29% . Similarly as the repair rate of subsystem B increases from 0.1 to 0.5, the subsystem availability increases by 3.32%

Table 3 Availability matrix of the subsystem C of series-parallel system

β_3 \ α_3	0.1	0.2	0.3	0.4	0.5
0.05	0.3430	0.3752	0.3873	0.3936	0.3975
0.06	0.3316	0.3682	0.3823	0.3898	0.3944
0.07	0.3210	0.3616	0.3775	0.3860	0.3913
0.08	0.3110	0.3552	0.3728	0.3828	0.3883
0.09	0.3016	0.3490	0.3682	0.3787	0.3853

$\alpha_1 = 0.2$
$\beta_1 = 0.09$
$\alpha_2 = 0.2$
$\beta_2 = 0.9$

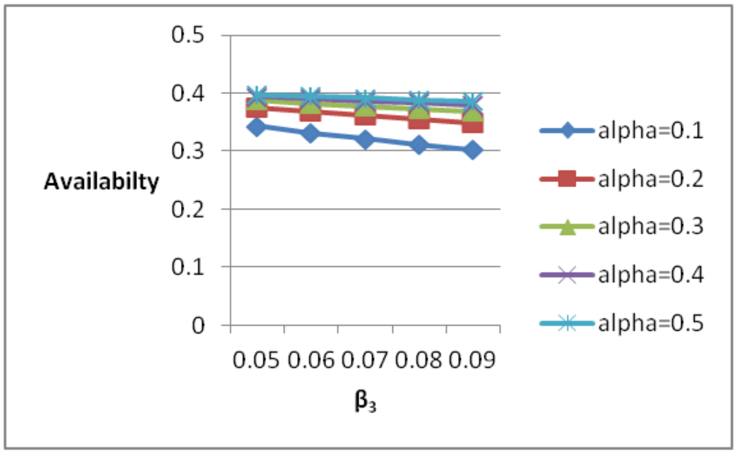

Fig. 7 effect of β_3 on availability

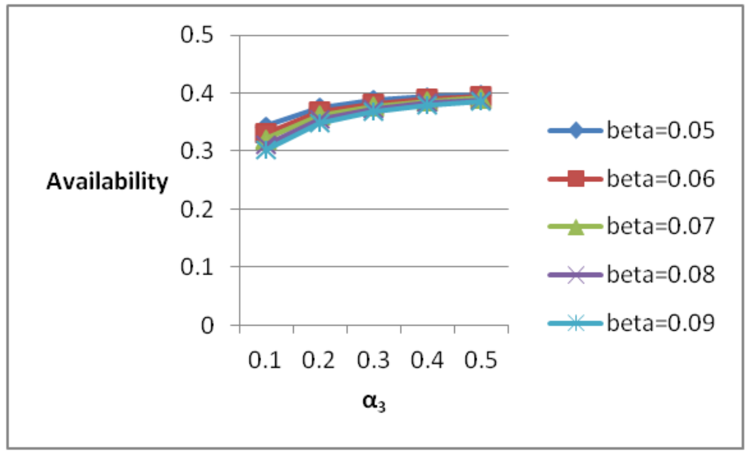

Fig. 8 effect of α_3 on availability

Table 3 and figure 7 and figure 8 shows the effect of failure and repair rates of subsystem C on the availability of series-parallel system. Keeping the values of both failure and repair rates of other subsystems fixed, as the failure rate of subsystem B increases from 0.05 to 0.09, the subsystem availability decreases by 4.14% . Similarly as the repair rate of subsystem C increases from 0.1 to 0.5, the subsystem availability increases by 5.45%.

Table 4 Optimum values of Failure/Repair rates of Subsystems of Series-Parallel system

S/N	Subsystem	Failure rate β_i	Repair rate α_i	Maximun Availability Level
1	A	0.05	0.5	44%
2	B	0.5	0.5	50%
3	C	0.05	0.5	39%

Table 4 reveals the subsystems with their maximum availability. From the table, it can be seen that subsystem B has the highest availability (50%). Also the optimum values of both failure and repair rates for maximum availability for each subsystem are displayed in the table.

7. Conclusion

The model in this study is used for evaluation of performance of subsystems A,B, and C of series-parallel system. The figures 3-8 and table 1-3 have shown availability decreases with increase in failure rates and increases with increase in repair rates. Equation 13 is the availability simulation model for the analysis and performance evaluation of subsystems A, B, and C of series-parallel system under study. From the analysis of the results, it can seen that from table 1-4 and fig. 3-8, subsystem C is the most critical in terms of maintenance and therefore much attention is needed. Less attention is needed to subsystems A and B when compared to subsystem C. This result is useful to the plant management for maintenance improvement, decision, planning and optimization.

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