Optimal Design of Three Phase Induction Motor Using Geometric Programming Optimisation Approach

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Abstract: This paper presents the use of Geometric Programming optimisation approach in the optimal design of a three phase induction motor. The objective function used in the optimisation is the efficiency of the induction motor. The result of the stator and rotor geometry variables obtained from the optimisation using geometric programming is compared with those obtained analytically. An optimal efficiency of 93.54 % was obtained from the Geometric Programming optimisation approach as compared with 88.38 % from the analytical approach.

Keywords: Geometric Programming, Optimisation, Efficiency, Induction Motor, Variables, Constraints, Minimisation, Objective functions, Design

1. Introduction

The wide applications of induction motors have led to the quest of improving its energy consumption and efficiency. The minimisation of the electrical energy consumption and maximisation of efficiency through an improved design is the major concern of induction motor designer, manufacturers and end users. The optimal design of an induction motor is mainly a proper sizing technique which is subject to a set of constraints. These constraints could be thermal, mechanical or users specifications [1]. Optimisation is a science of determining the best solution to certain mathematically defined problems, which are often models of physical reality. Optimisation involves the setting up of an objective function and making it to reach a maximum or minimum value while keeping all variables within an acceptable limit or range. The objective function of an induction motor could be the efficiency [2-3], losses [4], cost of material [5] or the torque [6]. Several optimisation techniques can be used in solving an objective function derived from an induction motor subject to its stated constraints. Most of the expressions used in the formulation of the objective functions for induction motors are nonlinear and this has prompted some authors to solve these objective functions using nonlinear optimisation techniques. Some optimisation techniques that have been applied in the design of induction motor are, the Genetic Algorithm [4,7-10], the Finite Element Method [11-12] and most recently the Particle Swarm [13]. All these aforementioned non linear optimisation techniques have been found to yield good results, but their results at times are not global optimum and in most cases the infeasibility of the problem may not be detected early.

In this paper, we propose the application of Geometric Programming to three phase induction motors. The proposed technique has been proved successful in the cost minimisation of transformers in [14] and synchronous motors [15]. The aim of this paper is to further enhance the contribution in the optimal design of induction motor using a technique that is little known in this area.

Interest in Geometric Programming as an optimisation tool is not new [16], and the real advantages of this optimisation technique are only starting to be appreciated now [17]. The Geometric Programming techniques are now extremely efficient and reliable. The geometric Programming optimisation technique uses the concepts of monomials and posynomials functions as the form to express the objective function and constraints.

The paper is organised as follows; section 2 outlines a basic review on Geometric programming, section 3 deals with the derivation of the objective function and the constraint using the concept of monomials and posynomials, section 4 indicates the optimisation and the validation of results with those found in literature and we conclude in section 5.

2. A Review on Geometric Programming

A geometric programme (GP) is an optimisation problem of the form [18-19],

minimise $f_0(x)$

subject to $f_i(x) \leq 1 \quad i = 1, \ldots., m,$

$$g_i(x) = 1, \quad i = 1, \ldots\ldots, p.$$

(1)

where f_i are posynomial functions, g_i are monomials, and x_i are the optimisation variables.

A monomial function is defined as,

$$g_i(x) = c_i x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n} \tag{2}$$

where c is a positive real constant called the monomial coefficient, and $a_{1,\ldots,}a_n$ are real and may be negative or fractional constants that are referred to as the exponents of the monomial.

The sum of monomial functions is named a posynomial function; that is,

$$f_i(x) = \sum_{k=1}^{K} c_k x_1^{a_{1k}} x_2^{a_{2k}} \cdots x_n^{a_{nk}} \tag{3}$$

The geometric programming problem (1) has nonlinear constraints but a transformation exists which causes considerable simplification. For further details, the reader is referred to [20].

3. Motor Design Problem formulation

The objective function used here in the design optimisation of three-phase induction motor is the efficiency. The efficiency of a machine is defined as the ratio of the output power to the input power and depends on various power losses such as copper loss, iron loss, frictional loss and windage loss. Only two of the losses will be considered. These are copper and iron losses.

The overall copper losses P_{cu} occurring in the stator and rotor slots of a three phase induction motor are as follows.

$$P_{cu} = \rho_{cu}\left(mNJ\left(2L + 2.3\pi\frac{D}{p} + 0.024\right)I + 4m(Nk_w I)^2 \frac{L_b}{s_s d_b w_b} + \frac{2D_e}{\pi p^2 a_e}\right) \tag{4}$$

The calculation of the iron losses P_{fe} is less exact because of the non-linear magnetic characteristics of iron. The losses are of two types; the hysteresis losses and the eddy current losses. Consequently, the iron losses in a three-phase induction motor can be expressed by the following equation.

$$P_{fe} = \rho_{fe}\left(s_s W_{st} L d_{ss} k_1 f^{k_2} B_{st}^{k_3} + \pi(D + 2d_{ss} + d_{sc}) d_{sc} L k_1 B_{sc}^{k_3} f^{k_2}\right) \quad (5)$$

When considering the two losses and the output powerP_{out}, the overall efficiency η of a three phase induction motor can be defined as;

$$\eta = \frac{P_{out}}{P_{out}+P_{cu}+P_{fe}} \quad (6)$$

The induction motor specification used in this paper is given in Table 1.

TABLE 1: SPECIFICATIONS

Nomenclature	Specification
ρ_{cu} Mass density of core material	$4800 kg/m^3$
m Number of phase	3
p Number of poles	-
I Prime current	-
k_w Winding factor	-
L_b Length of rotor bar	-
s_s Number of slots	-
k_1 material parameter	1.9×10^{-3}
k_2 material constant	1.24
k_3 Material constant	2
ρ_{fe} Electrical resistivity of iron	$1.72 \mu\Omega - cm$

The geometry of the stator and rotor are defined by several sizing parameters which are invariable and variable. The invariable sizing parameters are fixed or predefined at the inception of the design and are mainly made up of physical constraints. The variable sizing parameters do not have predefined optimum values. They may be mutually independent and without constraint, others may be dependent, either on some invariable sizing parameters or on mutual independent ones.

Eleven mutually independent variable parameters which define the geometry of the rotor and stator are identified from the objective function of (6) and will be subjected to optimisation analysis in this paper. These are, the stator tooth widthW_{st}, the rotor tooth width W_{rt}, the width of the rotor bar w_b , the depth of rotor bar d_b , the rotor diameter D_r , the stator bore diameter D ,stator core depth d_{sc} , mean diameter of end ring D_e , area of end ring a_e, the depth of stator slot d_{ss} , depth of rotor slot d_{rs}. The stator outer diameter D_0, the stator slot conductor current density J, the end ring current density J_e, the stator core flux density B_{sc}, the stator tooth flux density B_{st}, the air gap l_g and the ampere conductor per metre Q, are taken as constraining parameter.

The task here is to obtain optimal values of these mutually independent variables subject to certain constraints that will minimise losses (maximise efficiency).

4. Optimisation and Validation

Inverting (6) transforms it into the sum of monomials which is a required condition in geometric programming optimisation. Thus we minimise the objective function subject to twelve inequality and six equality constraints as stated below.

$$minimise\ \frac{1}{\eta} = 1 + \frac{\rho_{fe}}{P_{out}}\left(s_s W_{st} L d_{ss} k_1 f^{k_2} B_{st}^{k_3} + \pi(D + 2d_{ss} + d_{sc}) d_{sc} L k_1 B_{sc}^{k_3} f^{k_2}\right) +$$

$$+ \frac{\rho_{cu}}{P_{out}}\left(mNJ\left(2L + 2.3\pi\frac{D}{p} + 0.024\right)I + 4m(Nk_wI)^2\frac{L_b}{s_s d_b w_b} + \frac{2D_e}{\pi p^2 a_e}\right)$$

$$subject\ to.\quad D + 2d_{ss} + 2d_{sc} \le D_o$$

$$1.15 s_s \frac{W_{st}}{\pi p} \le d_{sc}$$

$$L + 0.02 \le L_b$$

$$0.0002 + 0.002\sqrt{DL} \le l_g$$

$$B_{st} \le 1.6$$

$$B_{sc} \le 1.2$$

$$J_e \le 7 \times 10^6$$

$$J \le 2 \times 10^6$$

$$l_g \le 0.005$$

$$Q\pi D/2mI \le N$$

$$d_b + 0.002125 \le d_{rs}$$

$$(w_b + 0.0025 + W_{rt}) + 2d_{rs} \le D_r$$

$$D_r + 2l_g \le D$$

$$0.95 W_{st}\frac{s_s}{s_r} \le W_{rt}$$

$$20000 \le Q$$

$$\frac{1.5\pi D}{p} = L$$

$$\frac{0.95 W_{st} s_s}{s_r} = W_{rt}$$

$$\frac{2B_{sc} d_{sc} p}{\pi B_g} = D$$

$$0.9D = D_e$$

$$\frac{B_{st}}{\pi} = B_g$$

$$\frac{D}{2p} = d_{ss}$$

(7)

A geometric programming toolbox gplab [21] written in Matlab [22] environment was used in solving the formulated problem. The optimisation result of a 7.5kW, 3-phase, 380 Volts, 4-pole, 50 Hz induction motor as compared with the analytical method is given in Table2. The most influential design parameter is the efficiency and as shown in the table the increase in efficiency is 5.16%. The result first of all shows that the geometrical programming optimisation method significantly improves the efficiency of the design and secondly the stator and rotor geometry parameter setting with minimum power losses was achieved.

TABLE 2: Optimisation Result

		Optimal Design	Analytical Design
Design variables	Stator bore diameter D (m)	0.1446	0.1260
	Rotor diameter D_r (m)	0.1346	-
	Stator tooth width W_{st} (m)	0.0031	0.0055
	Rotor tooth width W_{rt} (m)	0.0028	0.0040
	Depth of stator core d_{sc} (m)	0.0196	-
	Depth of stator slot d_{ss} (m)	0.0185	0.0175
	Depth of rotor slot d_{rs} (m)	0.0181	0.0178
	Length of rotor bar L_b (m)	0.1900	-
	Depth of rotor bar d_b (m)	0.0164	-
	Width of rotor bar w_b (m)	0.0028	-
	Mean diameter of end ring D_e (m)	0.1300	-
	Area of end ring a_e (m)	0.0013	-
	Axial length L (m)	0.1703	0.1600
Constraining Conditions	Stator conductor current density $J \leq 2 \times 10^6$ A/m	1.2×10^6 A/m	-
	End ring current density $J_e \leq 7 \times 10^6$ A/m	4.5×10^6 A/m	-
	Air gap flux density $B_g \leq 0.5T$	0.415 T	-
	Stator tooth flux density $B_{st} \leq 1.6T$	1.304 T	1.2000 T
	Stator core flux density $B_{sc} \leq 1.2T$	1.2 T	-
	Ampere conductor/metre $Q \leq 30000AT/m$	24250 AT/m	-
	Stator outer diameter $D_o \leq 0.22\ m$	0.22 m	0.2000 m
	Length of air gap $l_g \leq 0.005\ m$	0.005 m	0.0040 m
	Efficiency (η) %	93.54	88.38

5. Conclusion

The geometric programming optimisation process has been applied in the design of a three phase induction motor with the task of maximizing the efficiency of the motor. The optimisation leads to the determination of the stator and rotor geometry under certain constraining conditions. The result of the stator and rotor geometry variables obtained from the optimisation using geometric programming is compared with those obtained analytically. As shown in Table II, the geometric programming approach gives a better result as compared with the analytical approach.

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