SYNCHRONIZATION OF 4D RABINOVICH HYPERCHAOTIC SYSTEMS FOR SECURE COMMUNICATION

K. S. Ojo* and S.T. Ogunjo

Department of Physics, Federal University of Technology, P. M. B. 704, Akure, Nigeria
**Corresponding author: Email:kaystephe@yahoo.com*

Abstract

In this article, we investigate the synchronization of two identical four dimensional Rabinovich hyperchaotic systems evolving from different initial conditions. The active control technique is used to design control variables which enable the states variables of the two hyperchaotic systems to globally asymptotically achieve synchronization. The results show that the error state variables move hyperchaotically with time when the controllers are deactivated for $0 < t < 80$ while, the error converge to zero when the controllers are activated for $t \geq 80$. The system was successfully applied to secure communication using the additive encryption scheme to embed a signal in a component of the system.

Key words: synchronization, hyperchaotic system, secure communication, modulation

1. Introduction

In the last few years, synchronization of chaotic systems have received considerable interest among scientists in various fields. The results of chaos synchronization are utilized in biological systems [19,20], chemical reaction, secret communication and cryptography[24-26], nonlinear oscillator field, economic principles [21], finance [22,23] and some other fields. The first ideal of synchronizing two identical chaotic systems with different initial conditions was introduced by Pecora and Carroll [1] and the method is realized in electronic circuits. Synchronization technique has improved in recent years, and many different methods are applied theoretically and experimentally to synchronize the chaotic systems. Various active and nonlinear methods were used for chaos synchronization of two identical systems. Most of the works, however, considered low dimension chaotic systems with one positive Lyapunov exponent. Hyperchaotic systems possessing at least two positive Lyapunov exponents have more complex behaviour and abundant dynamics than chaotic systems and are more suitable for engineering applications such as secure communication. Hence, how to realize hyperchaotic systems synchronization is interesting and challenging work. Fortunately, some existing method of synchronizing low dimensional chaotic systems like adaptive control, active control, active backstepping control, sliding mode control methods can be generalized to synchronize hyperchaotic systems [16-18].

At first sight it is hard to understand what the practical use of an abstract theory like the synchronization theory can be. However, the theory has already been used in some communication devices. So synchronization can be used for encoding and communication. Another engineering application is the synchronization of two or more (industrial) robots. If necessary, they can be made to behave the same. Also the understanding of some biological phenomena (concerning pancreas, heart and neurons but also the behaviour of fireflies) is improved by the theory**.** Some researches has been done on the synchronization of spatio temporal systems Like fluid flows (also for other physical applications of synchronization). In quantum theory research is done on quantum clock synchronization**.** However, because of the very special dynamical behavior of quantum systems this kind of synchronization is difficult to compare with the 'standard' theory on synchronization and its study has a more information theoretical flavour.

Many methods have been proposed to achieve chaos control and synchronization, such as the passive control method Wang *et al* [13], backstepping design method Yassen [3], impulsive control method Wen *et al* [13], adaptive control Yassen [5], active control, Njah [7], sliding mode control Wang et al [6]. Bai and Lonnggren [8] proposed the method of identical chaos synchronization using active control. The technique was latter generalized to non-identical systems by Ho and Hung [9], Thus breaking the limit of it applicability beyond identical chaotic systems. Recently, the generalized active control (GAC) scheme was employed by Chen and Lee [10] to synchronize non-identical systems consisting of Lorenz, Chen and L$\ddot{u}$ systems with a new chaotic systems attributed to Chen and Lee [10]. Chaos synchronization using the active control has continue to receive wide application in variety of dynamical systems including geophysical model [11], spatiotemporal dynamical systems [12] etc. Despite the numerous advantage of active control method of synchronization, its application to synchronization of hyperchaotic systems with respect to secure communication has not been adequately explored.

The aim of this work is to synchronized two hyperchaotic 4D Rabinovich systems using active control method. We also aim to apply the successful synchronization to secure communication using the additive encryption scheme. The organization of the rest of this paper is as follows: Section 2 deals with system description. Section 3 deals with synchronization of the hyperchaotic systems. Section 4 is concern with application to secure communication, while section 5 concludes the paper.

2. Theoretical Consideration

In Liu et al., [14], reported a controlled 4D Rabinovich hyperchaotic system as follows

$$\begin{aligned}
\dot{x}_1 &= hx_2 - ax_1 + x_2 x_3 \\
\dot{x}_2 &= hx_1 - bx_2 - x_1 x_3 + x_4 \\
\dot{x}_3 &= -dx_3 + x_1 x_2 \\
\dot{x}_4 &= -kx_2
\end{aligned} \tag{1}$$

where a,b,d,h,k are positive constant parameters. When $(a,b,d,h,k) = (4,1,1,6.75,2)$, the 4D Rabinovich hyperchaotic system (1) has four Lyapunov exponents. $\lambda_{LE1} = 0.3066$, $\lambda_{LE2} = 0.0582$, $\lambda_{LE3} = -0.000$, $\lambda_{LE4} = -6.3642$.

And the Lyapunov dimension is $D_L = 3.0573$. Moreover, numerical simulations have verified that system (1) indeed has a hyperchaotic attractor when (a,b,d,h,k) = (4,1,1,6.75,2).[14]. The literature, Liu et al., (2010) also pointed out that the theoretical and numerical study indicates that chaos and hyperchaos are produced with the help of a Lienard-like oscillatory motion around a hypersaddle stationary point at the origin. The circuit implementation of the 4D Rabinovich hyperchaotic system has been carried out [15].

3. Complete synchronization using active control

Let (1) be the drive, then, the response system is

$$\begin{aligned}
\dot{y}_1 &= hy_2 - ay_1 + y_2 y_3 + u_1 \\
\dot{y}_2 &= hy_1 - by_2 - y_1 y_3 + y_4 + u_2 \\
\dot{y}_3 &= -dy_3 + y_1 y_2 + u_3 \\
\dot{y}_4 &= -ky_2 + u_4
\end{aligned} \tag{2}$$

where $u_i(t)$, $i = 1,2,3,4$ are control functions to be determined. Subtracting the drive equation (1) from the response (2), we obtain the error dynamics as

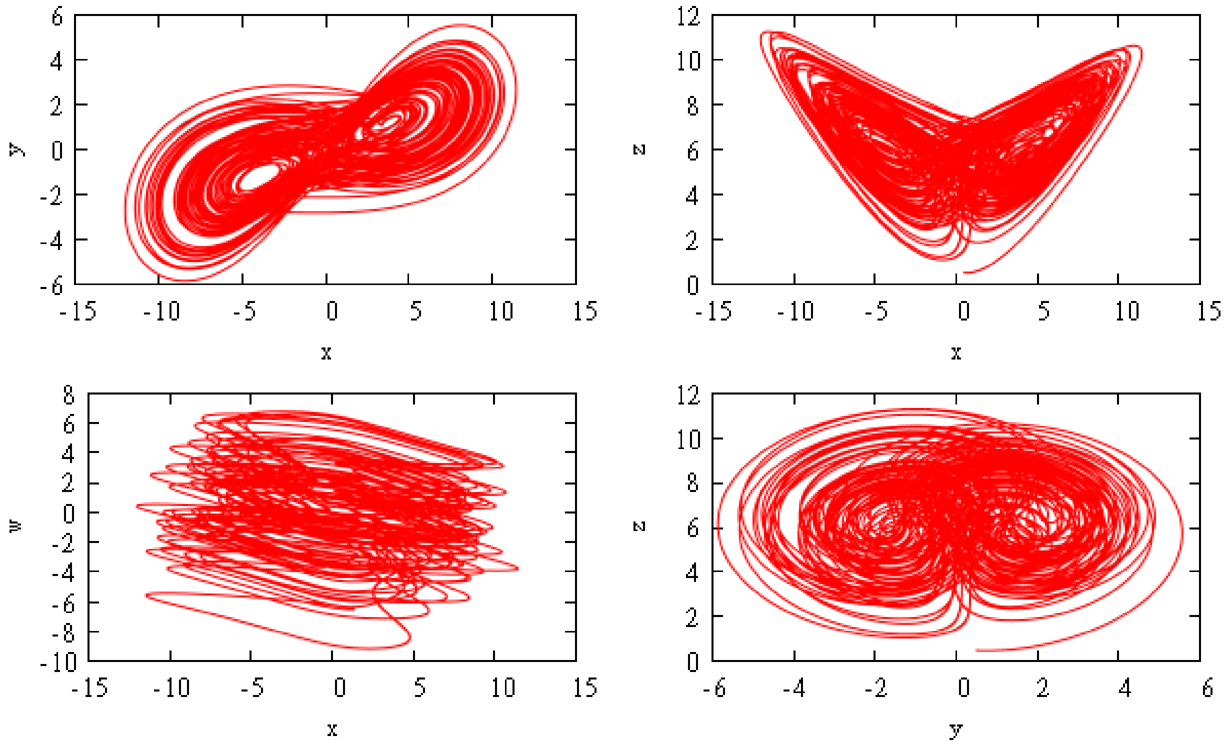

Fig.1. Phase portraits of hyperchaotic attractors of (1) with $a = 4, b = 1, c = 6.75,\ d = 1$ and $e = 2.0$

$$
\begin{aligned}
\dot{e}_1 &= he_2 - ae_1 + y_2 y_3 - x_2 x_3 + u_1(t) \\
\dot{e}_2 &= he_1 - be_2 - y_1 y_3 + x_1 x_3 + e_4 + u_2(t) \\
\dot{e}_3 &= -de_3 + y_1 y_2 - x_1 x_2 + u_3(t) \\
\dot{e}_4 &= -ke_2 + u_4(t)
\end{aligned}
\qquad (3)
$$

where $e_i = y_i - x_i,\ i = 1,2,3,4$. In the absence of the controls, the error dynamics system (3) would have an equilibrium at (0,0,0,0). If the controls are chosen such that the equilibrium (0,0,0,0) is unchanged, then the synchronization between the drive system(1) and the response system(2) reduces to that of finding the asymptotic stability of the error system(3) at equilibrium. To achieve this, the control functions are re-defined to eliminate terms in (3), which cannot be expressed as linear terms in e_1, e_2, e_3 *and* e_4 as follows:

$$
\begin{aligned}
u_1 &= -y_2 y_3 + x_2 x_3 + v_1(t) \\
u_2 &= y_1 y_3 - x_1 x_3 + v_2(t) \\
u_3 &= -y_1 y_2 + x_1 x_2 + v_3(t) \\
u_4 &= x_4 + v_4(t)
\end{aligned} \tag{4}
$$

Substituting (4) into (3) yeilds

$$
\begin{aligned}
\dot{e}_1 &= he_2 - ae_1 + v_1(t) \\
\dot{e}_2 &= he_1 - be_2 + e_4 + v_2(t) \\
\dot{e}_3 &= -de_3 + v_3(t) \\
\dot{e}_4 &= -ke_2 + v_4(t)
\end{aligned} \tag{5}
$$

Using the active control method, a constant matrix **A** is chosen which will control the error dynamics (3) such that the feedback matrix

$$
\begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \mathbf{A} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix} \tag{6}
$$

where

$$
\mathbf{A} = \begin{pmatrix} (k_1 + a) & -h & 0 & 0 \\ -h & (k_2 + h) & 0 & -1 \\ 0 & 0 & (k_3 + d) & 0 \\ 0 & k & 0 & k_4 \end{pmatrix} \tag{7}
$$

In (7), the four eigenvalues k_1, k_2, k_3, k_4 have been chosen as $-1, -1, -1, -1$ in order that a stable and synchronized state is achieved.

To numerically verify the effectiveness of the proposed synchronization scheme, we simulate the dynamics of the drive system (1) and the response system(2). In the simulation, the fourth order Runge-Kutta algorithm is empolyed with time step of 0.001 and fixing the parameter values $a = 4, b = 1, c = 6.75, d = 1$ and $e = 2$ to ensure hyperchaotic dynamics of the state variables. We then solved equations (1) and (2) with the control functions with initial conditions of the drive system and the response given as $(0.5, 0.5, 0.5, 0.5)$ and $(1,1,1,1)$ respectively. The results show that the error state variables move hyperchaotically with time when the controllers are deactivated for $0 < t < 80$ while, the error converge to zero when the controllers are activated for $t \geq 80$ as shown in Fig. 2. The results depicted in Fig. 2 show that complete synchronization of systems (1) and (2) have been achieved.

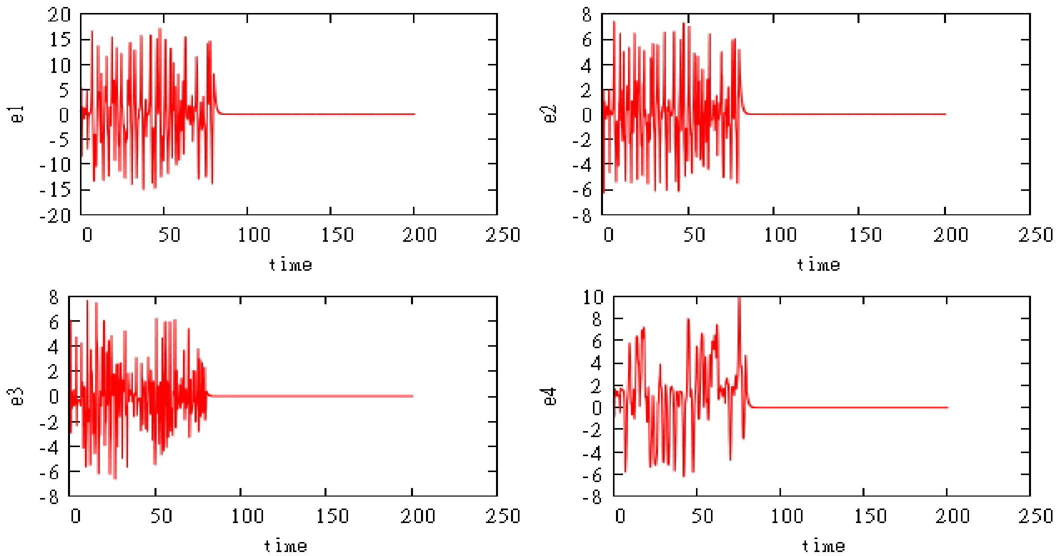

Fig. 2. Error dynamics between the two new hyperchaotic systems with the controller deactivated for $0 < t < 80$ and activated for $t \geq 80$

4. Secure information transmission via synchronization

The basic idea of chaotic secure communication is based on using chaotic nonlinear oscillator as a broadband signal generator. The signal is combined with message to produce unpredictable signal which is transmitted from the transmitter to the receiver. At the receiver the pseudo-random signal is generated through the inverse operation, original message is recovered. In order for this scheme to properly work, the receiver must synchronize robustly enough so as to admit the small perturbation in the drive signal due to the addition of the message. The power of the information signal must be much lower than that of the chaotic signal to effectively bury the information signal. The signal from the master serves two purposes: to control the slave system so as to synchronize it with the master and to carry the information signal just like any other communication scheme, the purpose of chaos secure communication is to hide message during transmission. The suitability of chaotic systems for application in secure communication is based on the feature of chaotic carriers such as: broadband or wide spectrum (which reduces the fading of the signal and increase the transmission capacity); orthogonality (which reduces signal distortion); sensitivity to slight changes in the initial conditions and system parameter as well complexity and noise-likeness dynamics which lead to unpredictability, thereby making extraction of hidden message difficult [13]. The secret keys are the set of value of the system parameters and since the system parameter are real numbers, the number of possible keys is infinite, thereby, enhancing confidentiality.

In this chaotic masking scheme, encryption is achieved by mixing information signal with the chaotic carrier signal using mixing algorithm which is simply a function of information and chaotic carrier signals. So far many mixing algorithm have been proposed to achieve chaotic masking: some of which are additive masking; multiplicative masking etc [13]. Here we demonstrate our secure communication scheme using the additive encryption masking scheme. The information signal is chosen to be a periodic function $sm = 0.1 + \sin 0.05\pi t$, with this choice the chaotic carrier x_1 remain chaotic. The encrypted information is given by the masking algorithm $em = x_1 + sm$. Consequently, the decrypted information rm is given by the inverse function $rm = em - y_1$. The chaotic signal x_1 of the master is transmitted to the slave via a coupling channel for synchronization between the master and the slave, the information signal sm is masked in the encrypted signal em and transmitted to the receiver. The decrypted information rm is extracted by inverse function. The schematic diagram for the communication scheme is shown in Fig. 3 while, the numerical simulation results for the communication scheme is shown in Fig. 4.

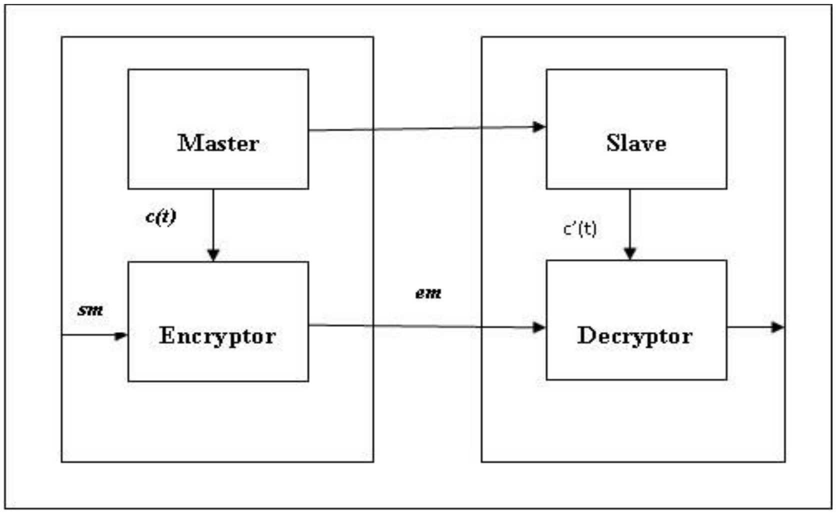

Fig. 3: Block diagram of diagram of a typical chaotic communication system.

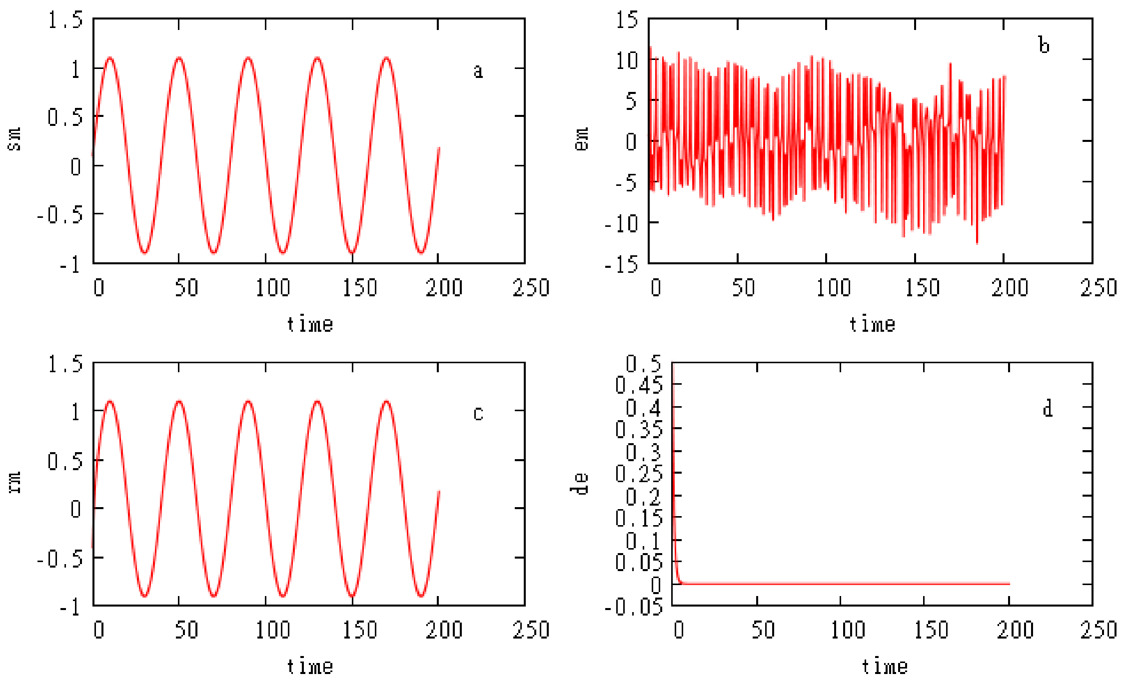

Fig.4: Chaotic masking of signals using the synchronized 4D Hyperchaotic Rabinovich system. (a.) original information (sm); (b). encrypted signal (em); (c.) decrypted signal (rm); (d.) decrypted error (de).

5. Conclusion

We have investigated the synchronization of two identical four dimensional Rabinovich hyper-chaotic systems evolving from different initial conditions. The active control technique is used to design control variables which enable the states variables of the two hyperchaotic systems to globally asymptotically achieve synchronization. The synchronization result was applied to secure communication. Numerical simulation results are presented to show the effectiveness of the proposed communication scheme.

Reference

[1] Pecora L. M. Carroll T. L. Synchronization of chaotic systems. Phys. Rev. Lett. 64, 821(1990).

[2] F. O. Wang, C. X. Liu, Synchronization of unified chaotic system based on passive control. Physica D 225 (2007) 55-60.

[3] M. T. Yessen, Chaos control of chaotic dynamic systems using backstepping designed, Chaos Solitons and Fractals 27 (2006) 537- 548.

[4] C. T. Wen, Y. Ji, Z. G. Li, Practical impulsive synchronization of chaotic systems with parametric uncertainty and mismatch. Physics Letters A 361 (2007) 108-114.

[5] M. T. Yassen, Adaptive chaos and synchronization for uncertain new chaotic dynamical systems. Physics Letters A 350 (2006) 36-43.

[6] H. Wang, Z. Z. Han, Q. W. Zhang, Y. Xie, Sliding mode control for chaotic systems based on LMI, Communication in Nonlinear Science and Numerical Simulations (2008).

[7] A. N. Njah, Synchronization via active control of identical and non-identical ϕ^6 chaotic oscillators with external excitations. Journal of Sound and Vibration 327 (2009) 322-332.

[8] E. W. Bai, K. e Lonngren, synchronization of two Lorenz chaotic systems using active control. Chaos Solitons and Fractals 319 (2009) 51-58.

[9] M. C. Ho, Y. C. Hung, synchronization of two chaotic systems using generalized active control. Physics Letters 302 (2002) 424-428.

[10] H. K. Chen, C. I. Li, Chaos Solitons and Fractals 21 (2004) 957

[11] U. E. Vincent, Phys Lette. A 343, (2005) 133

[12] S. Codereanu, Chaos Solitons and Fractals 15 (2003) 507

[13] H-K. Chen, T-N. Lin, J-H. Chen, Chinese Journal of Applied Physics 42 (2003) 7603.

[14] Liu. Y., Yang. Q., Pang. G.,: A hyperchaotic system from the Rabinovich system. J. Comput. Appl. Math. 234. 101-113 (2010)

[15] Liu. Y., Circuit Implementation and finite-time synchronization of the 4D Rabinovich hyperchaotic system. Engineering Nonlinear Dynamics, Vol 67, No: 1 (2012), 89 - 96

[16] K. S. Sudheer and M. Sabir, Adaptive Modified function projective synchronization between hyperchaotic Lorenz system and hyperchaotic Lu system with uncertain parameters. Physics Letters A 373 (2009) 3774-3748

[17] K. S. Sudheer and M. Sabir, Adaptive function projective synchronization of two-cell quantum-CNN chaotic oscillators with uncertain parameters. Physics Letters A 373 (2009) 1847-1851

[18] K. S. Sudheer and M. Sabir, Adaptive modified function projective synchronization of multiple time-delayed chaotic Rossler system. Physics Letters A 373 (2011) 1176-1179

[19] Wang H., Wang Q., Lu Q., Bursting oscillations, bifurcation and synchronization neuronal systems. Chaos, Solitons and Fractals 44(2011) 667 – 675.

[20] Njah A.N., and Ojo K.S., Backstepping Control and Synchronisation of parametrically and externally excited ϕ^6 with application to secure communication. International Journal of Modern Physics B, Vol 24, No 23(2010) 4581 – 4593.

[21] Kyrtsou C., Labys W. (2006) Evidence for chaotic dependence between US inflation and commodity prices. Journal of Macroeconomics 28(1):256-266.

[22] Yao H and Cui J, Stability of a Time-Delay and Impulsive Financial Model. International Journal of Nonlinear Science Vol 9 (2010) N0 2, pp 229 – 232.

[23] Son W and Park Y. Delayed feedback on the dynamical model of a financial system. Chaos, Solitons and Fractals 44 (2011) 208 - 217

[24] Teh-Lu L. and Shin-Hwa T. Adaptive synchronization of chaotic systems and its application to secure communications, Chaos, Solitons and Fractals 11 (2000) 1387-1396

[25] Wang X, Zhang N., Ren X., and Zhang Y., Synchronization of spatiotemporal chaotic systems and application to secure communication of digital image *Chinese Phys. B* 20 020507

[26] K. Li ., M. Zhao. and X., Fu Projective Synchronization of Driving–Response Systems and Its Application to Secure Communication. Circuits and Systems I: Regular Papers, IEEE Transactions. Volume: 56 Issue: 10 (2009), 2280 - 2291