Quantum phase transition in the Heisenberg model: A case study of two-spin system.

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ABSTRACT

This study presents a detailed and quantitative study of the magnetic properties of Heisenberg two-spin system. At the critical longitudinal magnetic fields of h_{zc}=J and transverse field of h_{xc}=J, the system undergoes a quantum phase transition (QPT) to the ferromagnetic state. It is found that the combined effect of longitudinal and transverse fields (mixed fields) hastens up the quantum transition from antiferromagnetism to ferromagnetism. For this mixed fields, a critical field of h_{xzc}=0.7071J is observed. The experimental evidence of this QPT has been observed at optimum doping in the cuprate $Bi_2Sr_{2-x}La_xCuO_{6+\delta}$ (Balakirev et al 2003).

The quantum magnetization observed at zero temperature for the Heisenberg spin-two systems was completely wiped out at finite temperatures due to thermal fluctuation. A temperature increase is expected to disrupt the ferromagnetic alignment of spins thereby favouring antiferromagnetism.

I. INTRODUCTION

Mechanical systems composed of many interacting parts are known to undergo Phase transition when an external parameter such as temperature, pressure or magnetic field is smoothly varied. In physics, a quantum phase transition (QPT) is a phase transition between different quantum phases of matter at absolute zero temperature. Contrary to classical phase transitions, quantum phase transitions can only be accessed by varying a physical parameter such as magnetic field or pressure at absolute zero temperature (Ando, 2004). The transition describes an abrupt change in the ground state of a many-body system due to its quantum fluctuations. At the critical point where the quantum phase transition (QPT) occurs, the ground state of the system undergoes a qualitative change in some of its properties (Sachdev, 1999). Osterloh et al (2002), showed that in a class of one-dimensional magnetic systems, the QPT is associated with a change of entanglement, and that the entanglement shows scaling behaviour in the vicinity of the transition point. This behaviour was discussed in detail for the Heisenberg model (Osborne and Nielsen, 2002; Giamarchi, 2004).

On the other hand, classical phase transitions are driven by a competition between the energy of a system and the entropy_of its thermal fluctuations. A classical system does not have entropy at zero temperature and therefore no phase transition can occur. Their order is determined by the first discontinuous derivative of a thermodynamic potential. A phase transition from water to ice, for example, involves latent heat (a discontinuity of the heat capacity) and is of first order. The aim of this work is to study the magnetic properties and quantum phase transition in the Heisenberg spin-two system.

The remainder of this paper is organized as follows. In section 2, we give a brief description of the Heisenberg spin-two system. Section 3 investigates the effect of longitudinal field on the spin-two system. Section 4 investigates the effect of transverse field on the spin-two system. Section 5 investigates the effect of mixed field on the spin-two system. Section 6 investigates the response of this system to finite temperatures. We present and discuss results in section 7. We conclude in section 8.

2. TWO-SPIN HEISENBERG SYSTEM

This section Present the simplest possible Heisenberg cluster, i.e. the spin-1/2 two–site dimer. Periodic boundary conditions (PBC) is imposed on the spins so that $S_{N+1}^{z} = S_{1}^{z}$. Thus, the topology of the spin space is that of a circle as shown in Fig.1.

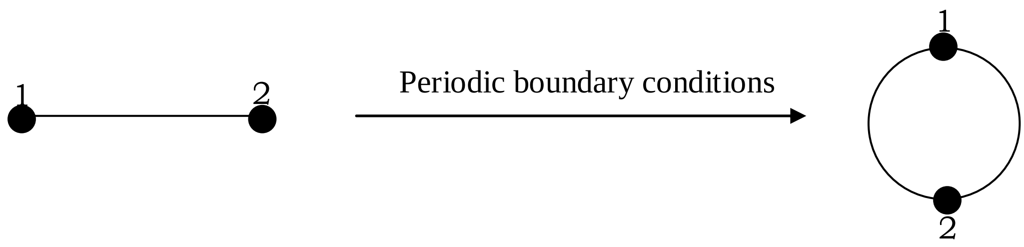

Fig. 1. A two site chain. A two-site open chain whose topology is that of a circle on the application of periodic boundary conditions.

The Heisenberg model in one dimension is given by

$$H = J\sum_i \left[S_i^z S_{i+1}^z + \frac{1}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+\right)\right] - h_Z \sum_i S_i^Z - h_Z \sum_i S_i^x \qquad (1)$$

where J is the superexchange coupling parameter between spins on site $\boldsymbol{i}$ and j which decays rapidly with the distance between these sites. S_i^Z and S_i^x are the spin operator in the z- and x-direction. The spin raising (S_i^+) and lowering (S_i^-) operators help to preserve the antiferromagnetic ground state. These operators act in the reduced Hilbert space of no doubly occupied sites. h_Z and h_x are the longitudinal and transverse field respectively.

Since a spin have two configurations in space (i.e. spin up or spin down) and the exclusion of doubly occupied site is understood, the size of the Hilbert space of a Heisenberg cluster with N spins is 2^N. Hence, for two-spin system, the size of the Hilbert space is 4. The basis states are:

$$|1\rangle = |1\uparrow, 2\uparrow\rangle, |2\rangle = |1\downarrow, 2\downarrow\rangle, |3\rangle = |1\uparrow, 2\downarrow\rangle, |4\rangle = |1\downarrow\, 2\uparrow\rangle\} \qquad (2)$$

3. EFFECT OF EXTERNAL LONGITUDINAL MAGNETIC FIELD.

The Hamiltonian for longitudinal field for the two-spin system expressed in the form of creation and annihilation operators is given by

$$H_{h_z} = -\vec{h}_z\left(S_1^z + S_2^z\right) = -h_z / 2\left(C_{1\uparrow}^\dagger C_{1\uparrow} - C_{1\downarrow}^\dagger C_{1\downarrow} + C_{2\uparrow}^\dagger C_{2\uparrow} - C_{2\downarrow}^\dagger C_{2\downarrow}\right) \qquad (3)$$

If the transverse field is switched off, the action of H in Eq. (1) on the Hilbert space of the two-spin system gives

$$H|1\uparrow,2\uparrow\rangle = \frac{J}{4}|1\uparrow,2\uparrow\rangle - \frac{h_z}{2}\left[|1\uparrow,2\uparrow\rangle + |1\uparrow,2\uparrow\rangle\right] = \frac{J}{4}|1\rangle - h_z|1\rangle \tag{4}$$

$$H|1\downarrow,2\downarrow\rangle = \frac{J}{4}|1\downarrow,2\downarrow\rangle - \frac{h_z}{2}\left[-|1\downarrow,2\downarrow\rangle - |1\downarrow,2\downarrow\rangle\right] = \frac{J}{4}|2\rangle + h_z|2\rangle \tag{5}$$

$$H|1\uparrow,2\downarrow\rangle = \frac{J}{2}|1\downarrow,2\uparrow\rangle - \frac{J}{4}|1\uparrow,2\downarrow\rangle - \frac{h_z}{2}\left[|1\uparrow,2\downarrow\rangle - |1\uparrow,2\downarrow\rangle\right] = \frac{J}{2}|4\rangle - \frac{J}{4}|3\rangle \tag{6}$$

$$H|1\downarrow,2\uparrow\rangle = \frac{J}{4}|1\uparrow,2\downarrow\rangle - \frac{J}{2}|1\downarrow,2\uparrow\rangle - \frac{h_z}{2}\left[|1\downarrow,2\uparrow\rangle - |1\downarrow,2\uparrow\rangle\right] = \frac{J}{2}|3\rangle - \frac{J}{4}|4\rangle \tag{7}$$

Observe that the external longitudinal field does not have effect on antiferromagnetic states. Therefore, the Hamiltonian matrix in the presence of external longitudinal magnetic field gives

$$H = \begin{bmatrix} \frac{J}{4} - h_Z & 0 & 0 & 0 \\ 0 & \frac{J}{4} + h_Z & 0 & 0 \\ 0 & 0 & -\frac{J}{4} & \frac{J}{2} \\ 0 & 0 & \frac{J}{2} & -\frac{J}{4} \end{bmatrix} \tag{8}$$

The energy levels arising from Eqn. 8 are given by.

$$\left.E_1/J = 1/4 - h_Z/J,\;\; E_2/J = 1l4 + h_Z/J,\;\; E_3/J = 1/4,\;\; E_4/J = -3/4\right\} \tag{9}$$

4. EFFECT OF EXTERNAL TRANVERSE MAGNETIC FIELD

The Hamiltonian for transverse field expressed in the form of creation and annihilation operators for two-spin system is given by Eq. (14).

$$H_{h_x} = -\vec{h}_x\left(S_1^x + S_2^x\right) = -h_x/2\left(c_{1\uparrow}^{\dagger}c_{1\downarrow} + c_{1\downarrow}^{\dagger}c_{1\uparrow} + c_{2\uparrow}^{\dagger}c_{2\downarrow} + c_{2\downarrow}^{\dagger}c_{2\uparrow}\right) \tag{10}$$

The actions of the transverse field on each of the basis states are

$$H_{h_x}|1\uparrow,2\uparrow\rangle = -\frac{h_x}{2}\left[|1\uparrow,2\downarrow\rangle + |1\downarrow,2\uparrow\rangle\right] = -\frac{h_x}{2}\left[|3\rangle + |4\rangle\right] \quad (11)$$

$$H_{h_x}|1\downarrow,2\downarrow\rangle = -\frac{h_x}{2}\left[|1\downarrow,2\uparrow\rangle + |1\uparrow,2\downarrow\rangle\right] = -\frac{h_x}{2}\left[|4\rangle + |3\rangle\right] \quad (12)$$

$$H_{h_x}|1\uparrow,2\downarrow\rangle = -\frac{h_x}{2}\left[|1\uparrow,2\uparrow\rangle + |1\downarrow,2\downarrow\rangle\right] = -\frac{h_x}{2}\left[|1\rangle + |2\rangle\right] \quad (13)$$

$$H_{h_x}|1\downarrow,2\uparrow\rangle = -\frac{h_x}{2}\left[|1\downarrow,2\downarrow\rangle + |1\uparrow,2\uparrow\rangle\right] = -\frac{h_x}{2}\left[|2\rangle + |1\rangle\right] \quad (14)$$

If the longitudinal field is switched off, the Hamiltonian matrix arising from the action of H in Eq. (1) on the Hilbert space of the two-spin system is given by

$$H = \begin{bmatrix} \frac{J}{4} & 0 & -\frac{h_x}{2} & -\frac{h_x}{2} \\ 0 & \frac{J}{4} & -\frac{h_x}{2} & -\frac{h_x}{2} \\ -\frac{h_x}{2} & -\frac{h_x}{2} & -\frac{J}{4} & \frac{J}{2} \\ -\frac{h_x}{2} & -\frac{h_x}{2} & \frac{J}{2} & -\frac{J}{4} \end{bmatrix} \quad (15)$$

The energy levels arising from the matrix in Eq. (15) are the same as those in Eq. (9). This invariably suggests that both the longitudinal and transverse fields have the same effect on the two- spin system. We shall further confirm this observation in section 7.

5. MIXED FIELDS

The Hamiltonian for mixed field (i.e. involving both longitudinal and transverse fields) is given by

$$H_h = -\sum_i \left(h_z S_z^i + h_x S_x^i\right) \quad (16)$$

In terms of creation and annihilation operators, Eq. (16) is expanded for two-spin system to give

$$H_h = -h_z/2\left(C_{1\uparrow}^{\dagger}C_{1\uparrow} - C_{1\downarrow}^{\dagger}C_{1\downarrow} + C_{2\uparrow}^{\dagger}C_{2\uparrow} - C_{2\downarrow}^{\dagger}C_{2\downarrow}\right) - h_x/2\left(c_{1\uparrow}^{\dagger}c_{1\downarrow} + c_{1\downarrow}^{\dagger}c_{1\uparrow} + c_{2\uparrow}^{\dagger}c_{2\downarrow} + c_{2\downarrow}^{\dagger}c_{2\uparrow}\right) \quad (17)$$

The interaction of this mixed field with the two-spin system gives

$$H_h\left|1\uparrow,2\uparrow\right\rangle = -\frac{h_x}{2}\left[\left|1\uparrow,2\downarrow\right\rangle + \left|1\downarrow,2\uparrow\right\rangle\right] - \frac{h_z}{2}\left|1\uparrow,2\uparrow\right\rangle = -\frac{h_x}{2}\left[\left|3\right\rangle + \left|4\right\rangle\right] - h_z\left|1\right\rangle \quad (18)$$

$$H_{h_x}\left|1\downarrow,2\downarrow\right\rangle = -\frac{h_x}{2}\left[\left|1\downarrow,2\uparrow\right\rangle + \left|1\uparrow,2\downarrow\right\rangle\right] + h_z\left|1\downarrow,2\downarrow\right\rangle = -\frac{h_x}{2}\left[\left|4\right\rangle + \left|3\right\rangle\right] + h_z\left|2\right\rangle \quad (19)$$

$$H_h\left|1\uparrow,2\downarrow\right\rangle = -\frac{h_x}{2}\left[\left|1\uparrow,2\uparrow\right\rangle + \left|1\downarrow,2\downarrow\right\rangle\right] - \frac{h_z}{2}\left[\left|1\uparrow,2\downarrow\right\rangle - \left|1\uparrow,2\downarrow\right\rangle\right] = -\frac{h_x}{2}\left[\left|1\right\rangle + \left|2\right\rangle\right] \quad (20)$$

$$H_{h_x}\left|1\downarrow,2\uparrow\right\rangle = -\frac{h_x}{2}\left[\left|1\downarrow,2\downarrow\right\rangle + \left|1\uparrow,2\uparrow\right\rangle\right] - \frac{h_z}{2}\left[\left|1\downarrow,2\uparrow\right\rangle - \left|1\downarrow,2\uparrow\right\rangle\right] = -\frac{h_x}{2}\left[\left|2\right\rangle + \left|1\right\rangle\right] \quad (21)$$

The Hamiltonian matrix arising from the interaction of this mixed field with two-spin system gives

$$H = \begin{bmatrix} \frac{J}{4} - h_z & 0 & -\frac{h_x}{2} & -\frac{h_x}{2} \\ 0 & \frac{J}{4} + h_z & -\frac{h_x}{2} & -\frac{h_x}{2} \\ -\frac{h_x}{2} & -\frac{h_x}{2} & -\frac{J}{4} & \frac{J}{2} \\ -\frac{h_x}{2} & -\frac{h_x}{2} & \frac{J}{2} & -\frac{J}{4} \end{bmatrix} \quad (22)$$

6. EFFECT OF FINITE TEMPERATURES ON TWO-SPIN SYSTEM

At finite temperatures, T enters the Hamiltonian through the uniform magnetization given by

$$m(h,T) = \frac{\sum_i S_i^z \exp(-\beta E_i)}{\sum_i \exp(-\beta E_i)} \quad (23)$$

For two-spin Heisenberg system, this is simplified to give

$$m(h,T)=\frac{2Sin(\beta h)}{2Cos(\beta h)+1+\exp(\beta J)} \tag{24}$$

In the unit of the Boltzmann constant k=1, the magnetization takes the form

$$m(h,T)=\frac{2Sin(h/T)}{2Cos\,h/T+1+\exp(J/T)} \tag{25}$$

Magnetic field dependence of the uniform magnetization has been calculated for fixed T=0.05, 0.1 and 0.2, and the temperature dependence of the uniform magnetization has also been calculated for fixed h=60J, 70J and 80J. These results as well as the figures illustrating the dependence for both magnetic field and temperature will be presented in section 7.

7. RESULTS AND DISCUSSION

This section presents and discusses the results obtained for two-spin Heisenberg system in the presence of external longitudinal, transverse and mixed magnetic field. The eigenvalues arising from the transverse field can easily be shown to be same with those of the longitudinal field. Therefore, our discussion will be restricted to longitudinal and mixed field.

I. LONGITUDINAL FIELD

The results presented in Tables 1 and 2 for longitudinal field have been obtained from the exact diagonalization of the Hamiltonian matrix in Eq. (8) and the eigenvalues in Eq.(9) respectively. As shown in Table 1 above, the threefold degenerate triplet excited states observed at zero magnetic field split into three distinct states at finite fields. At h_{zc}=J, S^z_{tot} =1 becomes the ground state energy, hence magnetization jumps from 0 to 1 (i.e. a jump from antiferromagnetic to ferromagnetic ground state). The four energy levels of this system for various values of h_z/J are shown in Table 2. Again, at h_z/J =1, the ground state shift from E_4/J to E_1/J, indicating a quantum phase transition from antiferromagnetism to ferromagnetism. This variation of the energy levels with the external magnetic field is shown in Fig.2, while the quantum of magnetization from 0 to 1 is captured by Fig. 3. The ground state energy E_4/J and the first excited state E_3/J are constant irrespective of the value of the field. This is because pure antiferromagnetic states are not affected by external longitudinal field. The energy level E_2/J exhibits a linear behaviour with h_z, while E_1/J exhibits an inverse behaviour with h_z.

Table1. Eigenvalues at $h_z=0$ and $h_z\neq 0$ for two-spin Heisenberg system

Eigenvalues (h=0)	Eigenvectors	Eigenvalues ($h\neq 0$)	S^z_{tot}
J/4	$\|1\uparrow,2\uparrow\rangle$	J/4-h	1
J/4	$\|1\downarrow,2\downarrow\rangle$	J/4+h	-1
-3J/4	$1/\sqrt{2}\|1\uparrow 2\downarrow - 1\downarrow 2\uparrow\rangle$	-3J/4	0
J/4	$1/\sqrt{2}\|1\uparrow 2\downarrow + 1\downarrow 2\uparrow\rangle$	J/4	0

Table 2. The energy levels of two-spin system as a function of h_Z

h_z/J	E_1/J	E_2/J	E_3/J	E_4/J
0	0.25	0.25	0.25	-0.75
0.5	-0.25	0.75	0.25	-0.75
1	-0.75	1.25	0.25	-0.75
1.5	-1.25	1.75	0.25	-0.75
2	-1.75	2.25	0.25	-0.75

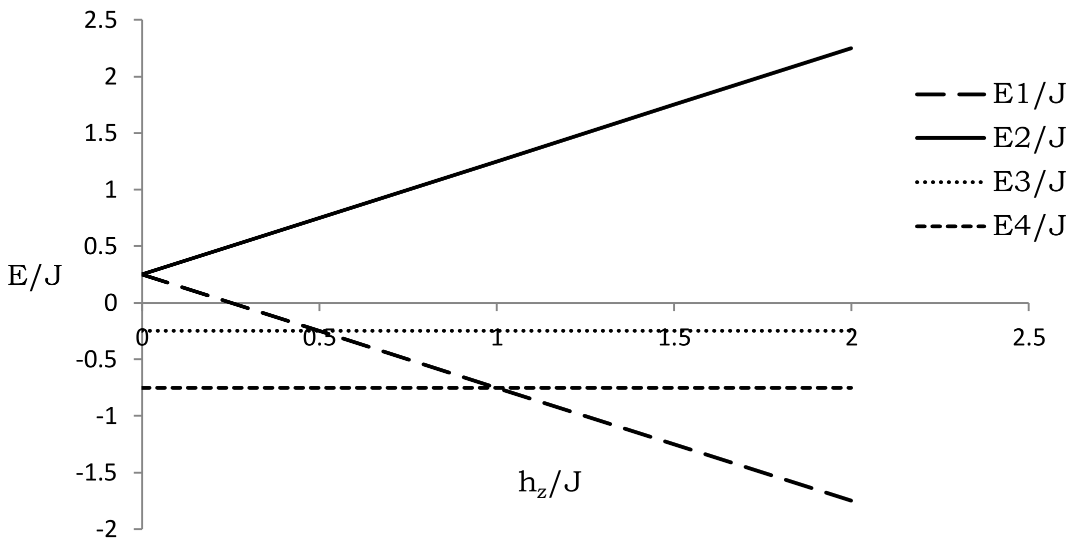

Fig.2.

Energy levels of two-spin Heisenberg system as a function of an external longitudinal field

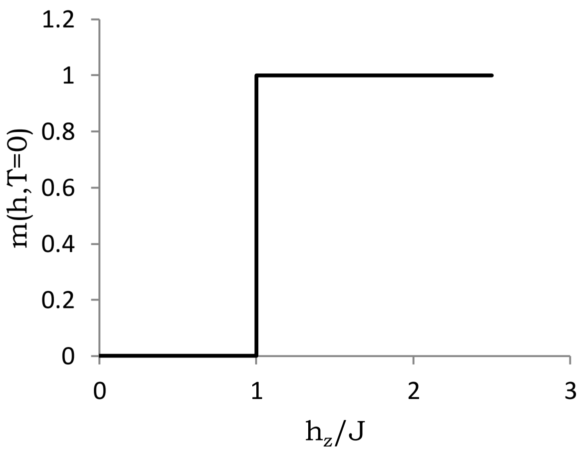

Fig. 3. Zero-temperature magnetization of two-spin Heisenberg system

II MIXED FIELD

The result for the response of the two-spin Heisenberg system to mixed field, which is the combined effect of longitudinal and transverse fields, is shown in Table 3. The response of the two-spin system to variations in h_x (keeping h_z constant) and its response to variations in h_z (keeping h_x constant), as shown in Table 3 is the same. Hence, for two-spin Heisenberg system, the longitudinal field as well as the transverse field has equal effect. However, ferromagnetic alignment of two spins is favoured if h_x and h_z are simultaneously varied as shown in the last column of Table 3. This study therefore reveals that the two-spin system responds faster to h_x and h_z when varied simultaneously than when varied separately.

The gradual emergence of ferromagnetic ground state as the fields h_z and h_x are gradually varied independently and simultaneously is shown in Table 4. The antiferromagnetic state (ATFS) is unaffected by external magnetic field, and maintains the value of -0.75 for all values of h_x or h_z. For zero and weak fields, this value is sufficient to sustain the system in an antiferromagnetic ground state. As the external field is switched on, the ferromagnetic state (E_1/J) becomes active and begins to compete with the antiferromagnetic ground state (E_4/J). At sufficiently large field, antiferromagnetic ground state is suppressed and ferromagnetic ground state emerges. As shown in Table 4, the emergence of ferromagnetic ground state is more rapid when h_z and h_x are simultaneously increased than when they are increased independently.

It is possible to determine the critical mixed field (h_{zxc}) at which the two-spin system undergoes a quantum phase transition from an antiferromagnetic ground state to a ferromagnetic ground state. As shown in Table 5, the system is observed to undergo a quantum phase transition at a critical field of h_{zxc}=0.7071 at J=1. This is lower than h_{zc}=h_{xc}=1 for the separate field of longitudinal and transverse respectively. This shows that quantum phase transition can be hastened up if both longitudinal and transverse are combined together (i.e. if they are mixed).

Table. 3. Effect of mixed field on two-spin Heisenberg system at J=1.

h_z	h_x	G.S (fixed h_z)	h_x	h_z	G.S (fixed h_x)	h_x	h_x	G.S (h_x=h_z)
0.1	0.5	-0.7500	0.1	0.5	-0.7500	0.20	0.20	-0.75000
0.1	1.0	-0.7500	0.1	1.0	-0.7500	0.40	0.40	-0.75000
0.1	1.5	-1.2533	0.1	1.5	-1.2533	0.60	0.60	-0.75000
0.1	2.0	-1.7525	0.1	2.0	-1.7525	0.80	0.80	-0.88137
0.1	2.5	-2.5200	0.1	2.5	-2.5200	1.00	1.00	-1.16421

Table 4. Emergence of ferromagnetic ground state

h_z	h_x	h_x=h_z	ATFS	Emerging FGS At fixed h_z	Emerging FGS At h_x=h_z	h_z
0.1	0.2	0.2	-0.7500	0.250000	-0.032843	0.1
0.1	0.4	0.4	-0.7500	-0.162311	-0.315685	0.1
0.1	0.6	0.6	-0.7500	-0.358276	-0.598530	0.1
0.1	0.8	0.8	-0.7500	-0.556226	-0.881371	0.1
0.1	0.9	0.9	0.7500	-0.655539	-1.022790	0.1
0.1	1.0	1.0	-0.7500	-0.754988	-1.164210	0.1
0.1	1.5	1.5	-0.7500	-1.255330	-1.871320	0.1

Table 5. The critical field for mixed field

h_z=h_x	AFGS	Emerging FGS
0.6000	-0.7500	-0.59853
0.6500	-0.7500	-0.66924
0.7000	-0.7500	-0.73995
0.7070	0.7500	-0.74985
0.7071	0.7500	-0.74999
0.7100	-0.7500	-0.75409
0.7200	-0.7500	-0.76823
0.7400	-0.7500	-0.79652
0.8000	-0.7500	-0.88137
1.0000	-0.7500	-1.16421

III EXPERIMENTAL REALIZATION OF QUANTUM PHASE TRANSITION IN CUPRATE SUPERCONDUCTOR

In a pulsed magnetic-field experiment, Balakirev et al (2003), found strong evidence at very low temperatures that there is indeed a quantum phase transition (QPT) at optimum doping in a cuprate superconductor $Bi_2Sr_{2-x}La_xCuO_{6+\delta}$ (BSLCO). In their work, as shown in Fig.4, the doping dependence of the normal-state Hall coefficient measured under 58T magnetic field was found to show a sharp break at optimum doping. This is an indication of a phase transition. Notably, the break in the dependence of Hall resistivity on magnetic field became sharper and sharper with lowering temperature, suggesting that the observed feature is truly a result of a zero-temperature transition. In subsequent experiment with the same material by Ando et al (2004), two QPTs in the superconducting doping regime were observed. From these similar behaviours across a wide range of different materials, one can argue that the occurrence of high temperature superconductivity is fundamentally related to quantum fluctuations associated with a zero-temperature phase transition. QPT in a transverse magnetic field has also been realized experimentally for the first time in 1D Ising ferromagnet $CoNb_2O_6$ (Coldea et al, 2010). Ultracold atoms in optical lattices has also provided a versatile tool with which to investigate fundamental properties of quantum many-body systems such as quantum phase transition and quantum spin dynamics (Weitenberg et al, 2011).

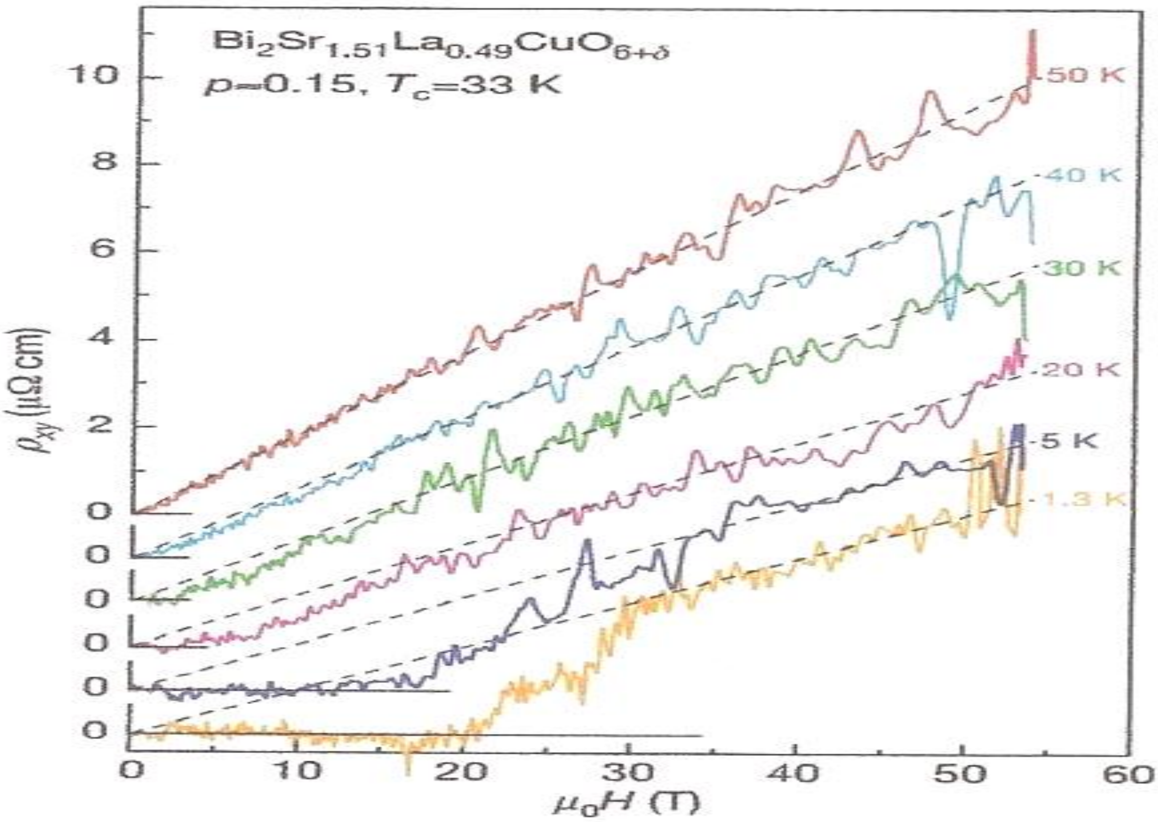

Fig.4. Experimental evidence for QPT in Cuprates.

(Source: Balakirev et al, 2003)

IV. UNIFORM MAGNETIZATION AT FINITE TEMPERATURES

At finite temperatures, the quantum jump observed in Fig.3. is smoothened out. This is because they are wiped out by thermal fluctuations. The magnetic field and temperature dependence of the uniform magnetization are calculated and presented in Tables 6 and 7 respectively. The temperature dependence of the uniform magnetization depends on the strength of the applied magnetic field. If the applied field is lesser than the critical field strength, the zero-temperature magnetization vanishes, and $m(h,T)$ is thermally activated as $m(h,T) \propto \exp(\beta h) + \exp(-\beta h)$. On the other hand, if the applied field exceeds the critical field strength at zero temperature, uniform magnetization is activated as $m(h,T) \propto 1 - \exp(-\beta h)$. In both cases, at sufficiently large temperatures, $m(h,T)$ exhibit Curie-like decay just like the Ising systems. This behaviour of $m(h,T)$ at finite temperatures is illustrated in Figs.5 and 6. An increase in h favours ferromagnetic alignment of the spins, while an increase in temperature favours antiferromagnetic alignment of the spins.

Table 6. Variation of $m(h_Z,T)$ with h_Z for fixed values of T for Heisenberg two- spin system.

h/J	$m(h_Z,T)$ T=0.05J	$m(h_Z,T)$ T=0.1J	$m(h_Z,T)$ T=0.2J
30	0.00007	0.00846	0.08355
40	0.00238	0.04659	0.17985
50	0.07265	0.21868	0.34441
60	0.71999	0.61584	0.55704
70	0.98828	0.90179	0.75062
80	0.99964	0.98134	0.87810
90	0.99998	0.99669	0.94518
100	0.99999	0.99942	0.97634

Table 7. Variation of $m(h_Z,T)$ with T for fixed values of h_Z for Heisenberg two- spin system.

T/J	m(h,T) h=60J	m(h,T) h=70J	m(h,T) h=80J
0.01	0.99116	1.00000	1.00000
0.04	0.76494	0.99610	0.99995
0.06	0.68711	0.97576	0.99865
0.10	0.61584	0.90179	0.98134
0.60	0.45530	0.53463	0.61000
1.00	0.36115	0.41835	0.47372
1.60	0.26439	0.30655	0.34783
2.00	0.22229	0.25809	0.29333

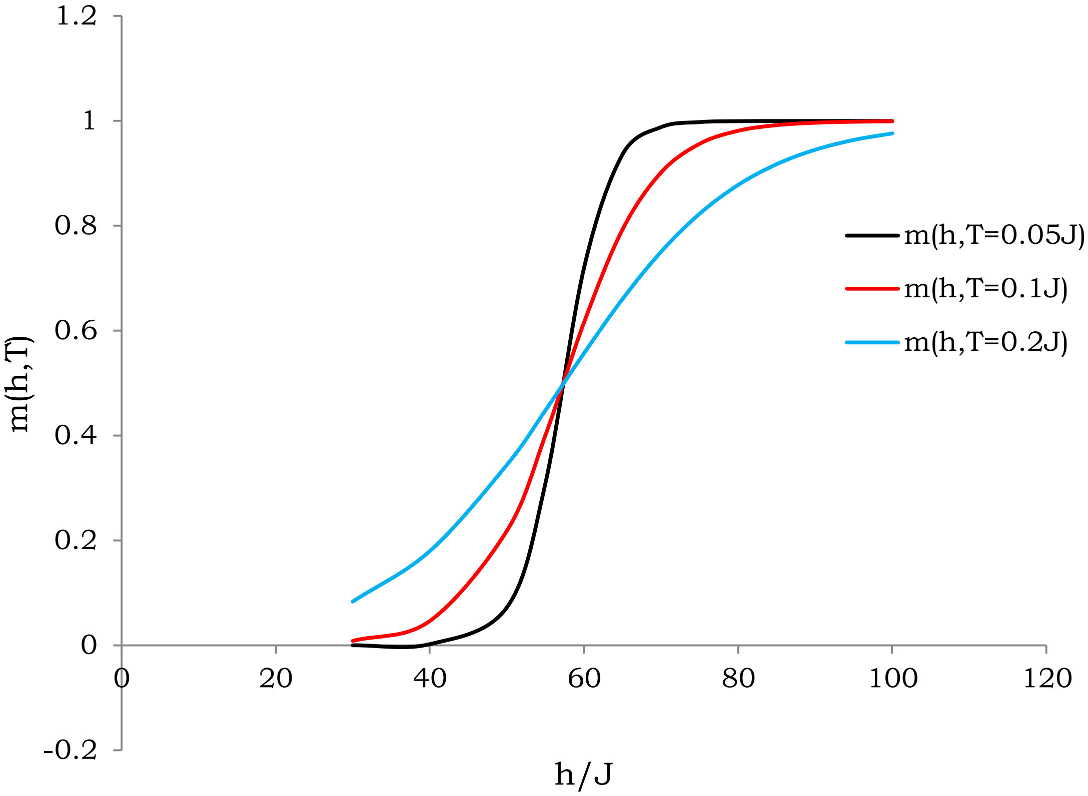

Fig 5. Magnetic field dependence of the uniform magnetization of a 2-site Heisenberg system for given values of T.

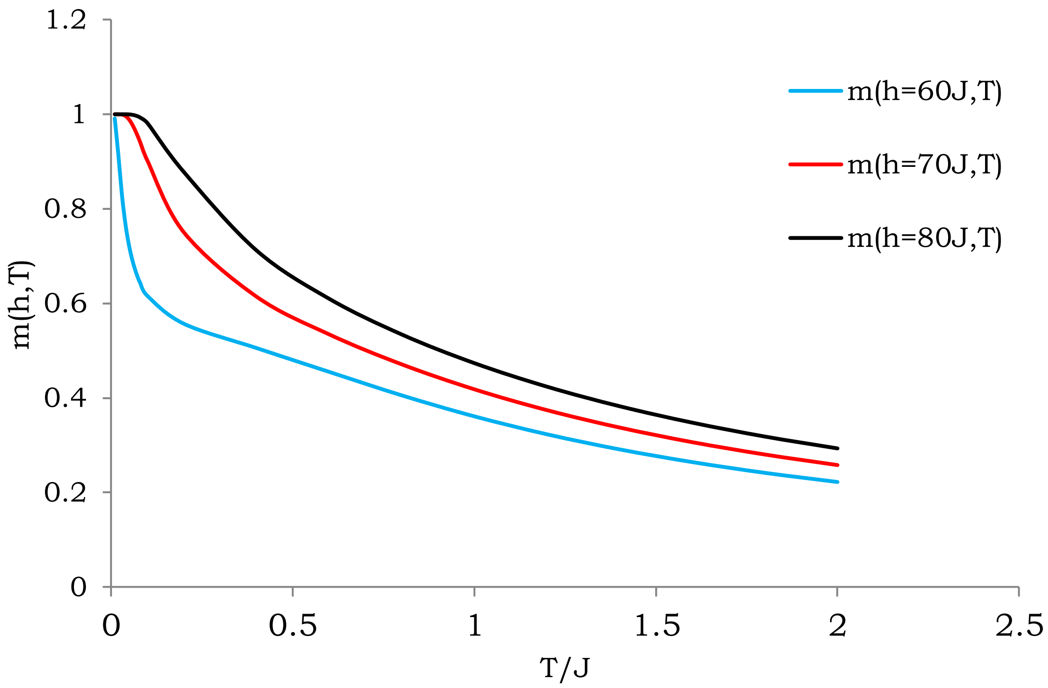

Fig 6. Temperature dependence of the uniform magnetization of a 2-site Heisenberg system for given values of h.

8. CONCLUSION

The magnetic properties of two-spin system have been studied with the Heisenberg quantum antiferromagnetic model. At the critical longitudinal magnetic fields of h_{zc}=J and transverse field of h_{xc}=J, the system undergoes a quantum phase transition (QPT) to the ferromagnetic state. The two-spin system exhibits the same behaviour to both transverse and longitudinal field. However, it is found that the combined effect of longitudinal and transverse fields (mixed fields) hastens up the quantum transition from antiferromagnet to ferromagnet. For this mixed field, a critical field of h_{xzc}=0.7071 is observed. At this critical field, the system is expected to undergo a quantum phase transition from antiferromagnetic ground to ferromagnetic ground state. The experimental evidence of this QPT has been observed at optimum doping in the cuprate $Bi_2Sr_{2-x}La_xCuO_{6+\delta}$ and 1D Ising ferromagnet $CoNb_2O_6$ (Balakirev et al 2003; Coldea et al, 2010).

The quantum magnetization observed at zero temperature for the Heisenberg spin-two systems was completely wiped out at finite temperatures due to thermal fluctuation. A temperature increase is expected to disrupt the ferromagnetic alignment of spins thereby favouring antiferromagnetism. On a macroscopic view, this accounts for the disappearance of magnetic field when a magnet is subjected to thermal heating in accordance with curie’s law.

REFERENCES

Ando Y., Ono S., Sun X. F., Takeya J., Balakirev F. F., Betts J. B. and Boebinger G. S. (2004). Quantum Phase Transitions in the Cuprate Superconductor $Bi_2Sr_{2-x}La_xCuO_{6+\delta}$. Phys. Rev. Lett. 92, pp247004-247007.

Balakirev F. F., Betts J.B., Migliori A., Ono S., Ando Y. and Boebinger G.S. (2003). Signature of optimal doping in Hall-effect measurements on a high-temperature superconductor. Nature, 424, pp912-915.

Coldea R.,Tennant D.A., Wheeler E.M., Wawrzynska E., Prabhakaran D., Telling M., Habicht K., Smeibidl P. and Kiefer K.(2010). Quantum criticality in an Ising chain: Experimental evidence for the emergent E8 symmetry. Science, 327, pp177-180.

Giamarchi T. (2004) Quantum physics in one dimension J.Phys. A. Math .Gen. 37, pp5275-5276.

Osborne T. J. and Nielsen M. A. (2002). Entanglement in a simple quantum phase transition. Phys. Rev. A 66, pp032110-032123

Osterloh A., Amico L., Falci G. and Rosario Fazio R. 2002.Scaling of entanglement close to a quantum phase transition. Nature 416, pp608-610

Sachdev S. (1999). Quantum Phase Transitions, Cambridge University Press, Cambridge U.K.

Weitenberg C., Endres M., Sherson J. F, Cheneau M., Schauß P., Fukuhara T., Bloch I., and Kuhr S. (2011). Single-spin addressing in an atomic Mott insulator. Nature 471, pp319-324 .