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Analysis of Direct Extrusion Operation Using The Bubnov-Galerkin Finite Element Model

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Abstract

This paper reports the analysis of direct extrusion operation using the Bubnov-Galerkin finite element model to get the pressure distributions along the cross-section of a blank. Four Lagrange quadratic elements were assembled to represent the blank. The governing equation is a one dimensional differential equation describing the pressure on the die-blank interface. The weighted residual form was obtained from the differential equation, the finite element model was obtained in a matrix form from the weighted residual, boundary condition were now applied to obtain the pressure distribution across the cross-section of the blank. Finite element results were obtained for a particular values of coefficient of friction and blank diameter and compared with the exact solution on a graph.

Keywords: Direct extrusion operation, Bubnov-Galerkin residual scheme, finite element model, Lagrange quadratic element.

1.0 Introduction

Extrusion is one of the major metal forming process in which a block of metal is passed through a die by means of a tensile force applied of the exit of the die in order to reduce the cross-section of the metal. Hence, the need therefore arises to analyze the drawing operation in order to predict the various stresses and pressure fields set up at a particular cross-section of a given blank material. The estimated pressures and stresses can thus be compared with the strength of the material and this aids the determination of the smallest pressure needed to cause the bulk plastic flow of the material. Being an important and versatile metal forming process, a large number of researchers into metal forming process exist in literature. Akpobi and Edobor [1] developed a model for analyzing forging process. Alfozan and Gunsdrkrts [2] proposed an upper bound element technique approach forging by forward and backward simulation. Navarrete et al [3] used a dimensional analysis approach to propose five dimensionless groups from the process variables in an attempt to simplify the forging stress determination. Oviawe and Omorodion [4] determined stresses in hot bar forging, Ovawe and Oviawe [5] analyzed axisymmetric forging operation while Oviawe and Asikhia [6] determined stresses in wire-drawing operation.

2.0 Materials and Method

In this research, the weighted residual finite element method was employed as a numerical tool, used in obtaining the extruding stresses and pressure distribution on a material during direct extrusion process. Due to symmetry, analysis was carried out on half the blank. The blank was represented by a mesh finite element and Bubnov-Galerkin weighted residual scheme was applied to get the value of the pressure at nodal points. Four quadratic elements were used to ensure an accurate solution. A numerical analysis was done to compare the finite element results with the exact solution.

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3.0 Results and Discussion

Formulation Of Governing Equation

A number of assumptions that were made in the formulation of the differential equation are:

- The billet to be extruded is rigid
- Redundant work is allowed for by an efficiency factor
- The billet was subjected to hydrostatic pressure, p.

Considering the equilibrium of a thin slice of billet of width dx, being extruded by direct method, as shown in Fig 1, we get

$$\frac{\pi}{4}D^2 dp = \pi D \mu P dx \qquad (1)$$

$$\pi D^2 dp = 4\pi\mu P dx \qquad (2)$$

Dividing through by πD, we obtain

$$D dp = 4\mu p dx \qquad (3)$$

$$\frac{dP}{dx} - \frac{4\mu P}{D} = 0 \qquad (4)$$

Equation (4) gives us the governing equation,
where, D is the billet diameter,
μ is the coefficient of friction between billet and container wall, and
P is the horizontal pressure on the slice.

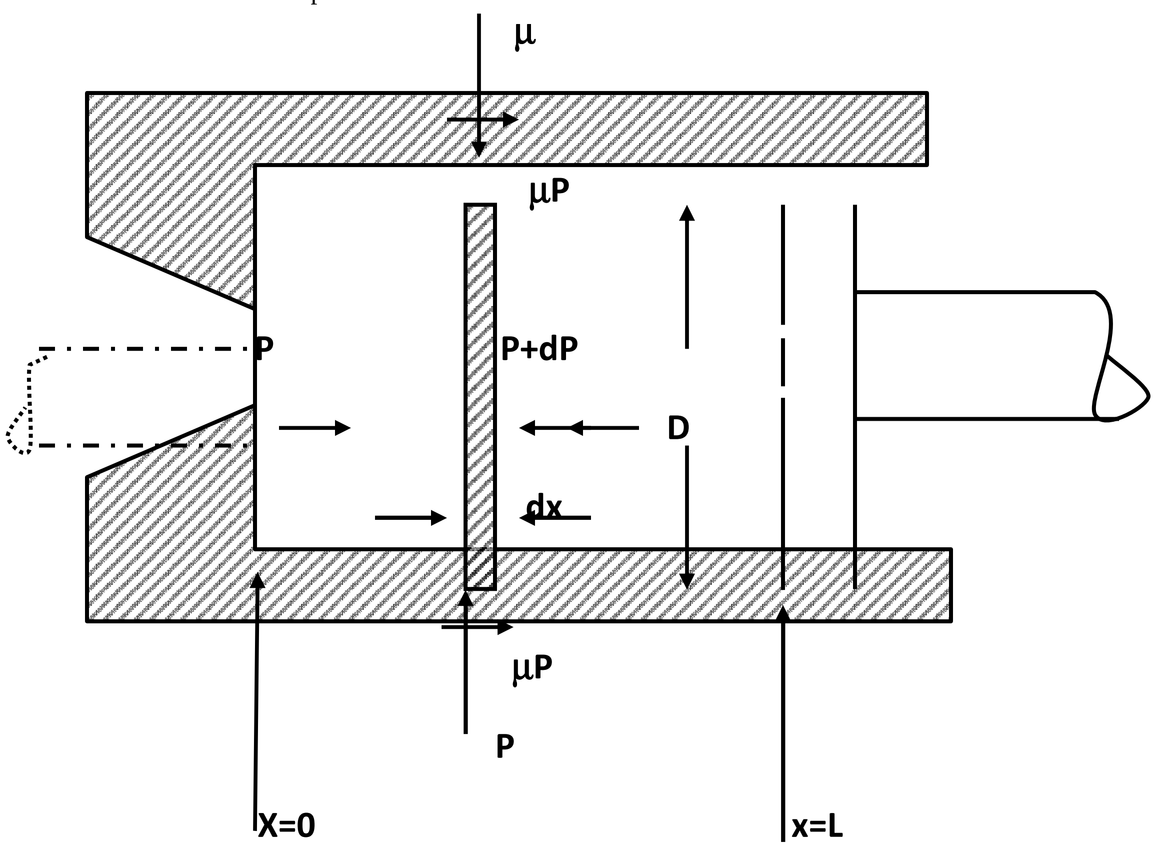

Fig 1: Free body diagram of direct extrusion process

Weighted Integral Formulation

The weighted integral form of equation (4) is obtained by multiplying it by the weight function, V and integrating with respect to x over the domain enclosing an element, to get

$$\int_o^L V\left(\frac{dP}{dx}-\frac{4\mu P}{D}\right)dx=0 \tag{5}$$

or

$$\int_o^e V\frac{dp}{dx}dx-\int_o^c V\frac{4\mu p}{D}dx=0 \tag{6}$$

As examination of equation (6) reveals the solution and hence once differentiable with respect to x. Thus, the Lagrange family of interpolation functions can be used satisfactorily.
Let us assume that the solution P is approximated as follows:

$$P\approx P^e=\sum_{j=i}^{n}P_j^e\psi_j^e(x) \tag{7}$$

Where, ψ_j^e = Lagrange quadratic interpolation function at the jth node and p_j^e = pressure at the jth node of the element. Since we are applying Bubnov-Galerkin weighted residual finite element method in this paper, we assume that the weight function is equal to the interpolation function.

$$V=\psi_j^e(x) \tag{8}$$

Substituting Equation (7) and (8) into (6), we get

$$\int_o^L\left(\psi_j^e\frac{d}{dx}\sum_{j=1}^{n}P_j^e\psi_j^e-\frac{4\mu}{D}\psi_j^e\sum_{j=i}^{n}P_j^e\psi_j^e\right)dx=0 \tag{9}$$

$$\sum_{j=i}^{n}\left\{\int_o^L\left(\psi_j^e d\psi_j^e-4\mu\psi_j^e\psi_j^e\right)dx\right\}\left\{P_j^e\right\}=0 \tag{10}$$

It is imperative to mention that we recast equation (10) to equation (11) as the weighted residual finite element model as considered by Akpobi and Edobor [1]

$$\sum_{j=1}^{n}\left\{k_{ij}\right\}\left\{p_j^e\right\}=0 \tag{11}$$

where,

$$K_{ij}^e=\int_o^L\left(\psi_j^e\frac{d}{dx}\psi_j^e-\frac{4\mu}{D}\psi_j^e\psi_j^e\right)dx \tag{12}$$

Equation (11) is the weighted-residual finite element model of equation (4). Using the 1 – D Lagrange quadratic interpolation functions,

$$\psi_1^e=\left(1-\frac{X}{L}\right)\left(1-\frac{2X}{L}\right)$$

$$\psi_2^e=\frac{4x}{L}\left(1-\frac{x}{L}\right)$$

$$\psi_3^e=-\frac{x}{L}\left(1-\frac{2x}{L}\right)$$

We generate the matrix

$$K_{11} = \int_o^L \left(\psi_1^e \frac{d\psi_1^e}{dx} - \frac{4\mu}{D}\psi_1^e \psi_j^e \right) dx$$

$$= \frac{-15D - 16\mu L}{60D}$$

$$K_{12} = \int_o^L \left(\psi_1^e \frac{d\psi_2^e}{dx} - \frac{4\mu}{D}\psi_1^e \psi_2^e \right) dx$$

$$= \frac{-9D - 8\mu L}{60D}$$

$$K_{13} = \int_o^L \left(\psi_1^e \frac{d\psi_3^e}{dx} - \frac{4\mu}{D}\psi_1^e \psi_3^e \right) dx$$

$$= \frac{-5D - 64\mu L}{60D}$$

$$K_{21} = \int_o^L \left(\psi_2^e \frac{d\psi_1^e}{dx} - \frac{4\mu}{D}\psi_2^e \psi_1^e \right) dx$$

$$= \frac{-5D - \mu L}{60D}$$

$$K_{22} = \int_o^L \left(\psi_2^e \frac{d\psi_2^e}{dx} - \frac{4\mu}{D}\psi_2^e \psi_2^e \right) dx$$

$$= \frac{-128\mu L}{60D}$$

$$K_{23} = \int_o^L \left(\psi_2^e \frac{d\psi_3^e}{dx} - \frac{4\mu}{D}\psi_2^e \psi_3^e \right) dx$$

$$= \frac{5D - 2\mu L}{60D}$$

Due to symmetry

$$K_{23} = -K_{31} = \frac{5D - 64\mu L}{60D}$$

$$K_{12} = -K_{32} = \frac{9D - 8\mu L}{60D}$$

$$K_{11} = -K_{33} = \frac{15d + 16\mu L}{60D}$$

Hence, for one Lagrange quadratic element

$$K_{ij}^{e}=\frac{1}{60D}\begin{bmatrix}-15D-16\mu L & -9D-8\mu L & -5D+64\mu L\\ -5D-\mu L & -128\mu L & 5D-2\mu L\\ 5D-64\mu L & 9D+8\mu L & 15D+16\mu L\end{bmatrix} \tag{13}$$

In order to ensure high accuracy, we used a mesh of four quadratic elements (9 nodes) to get

$$K_{ij}^{e}=\begin{bmatrix}K_{11} & K_{12} & K_{13}\\ K_{21} & K_{22} & K_{23}\\ K_{31} & K_{32} & K_{33}\end{bmatrix}\begin{bmatrix}p_1\\ p_2\\ p_3\end{bmatrix}=\begin{bmatrix}Q_1\\ 0\\ Q_3\end{bmatrix} \tag{14}$$

For a mesh of four 1 – D quadratic elements, the assembled equations are:

$$K_{ij}^{e}=\frac{1}{60D}\begin{bmatrix}K_{11}^{1} & K_{12}^{1} & K_{13}^{1} & 0 & 0 & 0 & 0 & 0 & 0\\ K_{21}^{1} & K_{22}^{1} & K_{23}^{1} & 0 & 0 & 0 & 0 & 0 & 0\\ K_{31}^{1} & K_{32}^{1} & K_{33}^{1}+K_{11}^{2} & K_{12}^{2} & K_{13}^{2} & 0 & 0 & 0 & 0\\ 0 & 0 & K_{21}^{2} & K_{22}^{2} & K_{23}^{2} & 0 & 0 & 0 & 0\\ 0 & 0 & K_{31}^{2} & K_{32}^{2} & K_{33}^{2}+K_{11}^{3} & K_{12}^{3} & K_{13}^{3} & 0 & 0\\ 0 & 0 & 0 & 0 & K_{21}^{3} & K_{22}^{3} & K_{23}^{3} & 0 & 0\\ 0 & 0 & 0 & 0 & K_{31}^{3} & K_{32}^{3} & K_{33}^{3}+K_{11}^{4} & K_{12}^{4} & K_{13}^{4}\\ 0 & 0 & 0 & 0 & 0 & 0 & K_{21}^{4} & K_{22}^{4} & K_{23}^{4}\\ 0 & 0 & 0 & 0 & 0 & 0 & K_{431} & K_{32}^{4} & K_{33}^{4}\end{bmatrix}\begin{bmatrix}P_1\\ P_2\\ P_3\\ P_4\\ P_5\\ P_6\\ P_7\\ P_8\\ P_9\end{bmatrix}=\begin{bmatrix}Q_1\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ Q_9\end{bmatrix}.. \tag{15}$$

Substituting into Equation (15), it becomes,

$$K_{ij}^{e}=\frac{1}{60D}\begin{bmatrix}-5D-16\mu L & -9D-8\mu L & -5D+64\mu L & 0 & 0 & 0 & 0 & & 0\\ -5D-\mu L & -128\mu L & 5D-2\mu L & 0 & 0 & 0 & 0 & & 0\\ 5D-64\mu L & 9D+8\mu L & 32\mu L & -9D+8\mu L & -5D+64\mu L & 0 & 0 & & 0\\ 0 & 0 & -5D+\mu L & -128\mu L & 5D+2\mu L & 0 & 0 & & 0\\ 0 & 0 & 5D-64\mu L & 9D+8\mu L & 32\mu L & -9D+8\mu L & -5D+64\mu L & & 0\\ 0 & 0 & 0 & 0 & -5D+\mu L & -128\mu L & 5D-2\mu L & & 0\\ 0 & 0 & 0 & 5D-64\mu L & 9D+8\mu L & 32\mu L & -9D-8\mu L & & -5D+64\mu L\\ 0 & 0 & 0 & 0 & 0 & -5D-\mu L & -128\mu L & 15D+64 & 5D-2\mu L\\ 0 & 0 & 0 & 0 & 0 & 5D-64\mu L & 9S+8\mu L & & 15D+64\mu L\end{bmatrix}\begin{bmatrix}P_1\\ P_2\\ P_3\\ P_4\\ P_5\\ P_6\\ P_7\\ P_8\\ P_9\end{bmatrix} \tag{16}$$

The boundary condition is:
At x = L, $\sigma_x = 0$
From Tresca's yield criterion,
$\sigma_x + P = \sigma_o = 2k$
Therefore, x = L, $P_9 = \sigma_o = 2k$
Since there are now eight unknowns, P_1, P_2, P_3, P_4, P_5, P_6, P_7 and P_8,
Equation (16) becomes;

$$K_{ij}^{e}=\frac{1}{60D}\begin{bmatrix} -5D-16\mu L & -9D-8\mu L & -5D+64\mu L & 0 & 0 & 0 & 0 \\ -5D-\mu L & -128\mu L & 5D-2\mu L & 0 & 0 & 0 & 0 \\ 5D-64\mu L & 9D+8\mu L & 32\mu L & -9D+8\mu L & -5D+64\mu L & 0 & 0 \\ 0 & 0 & -5D+\mu L & -128\mu L & 5D+2\mu L & 0 & 0 \\ 0 & 0 & 5D-64\mu L & 9D+8\mu L & 32\mu L & -9D+8\mu L & -5D+64\mu L \\ 0 & 0 & 0 & 0 & -5D+\mu L & -128\mu L & 5D-2\mu L \\ 0 & 0 & 0 & 5D-64\mu L & 9D+8\mu L & 32\mu L & -9D-8\mu L \\ 0 & 0 & 0 & 0 & 0 & -5D-\mu L & -128\mu L \end{bmatrix}\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix}=\frac{\sigma_o}{60D}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -5D+64\mu L \\ 5D-2\mu L \end{bmatrix} \quad (17)$$

The stresses are obtained by substituting the values of the pressures into the equation:

$$\sigma_x - \sigma_o - P \qquad \ldots \qquad (18)$$

The pressure distribution over each element was obtained thus:

$$P(x) = P_1\psi_1^1 + P_2\psi_2^1 + P_3\psi_3^1 \quad For \;\; 0 \le x \le \frac{1}{4}$$

$$P(x) = P_3\psi_3^2 + P_4\psi_4^2 + P_5\psi_5^2 \quad For \;\; \frac{L}{4} \le x \le \frac{L}{2}$$

$$P(x) = P_5\psi_1^3 + P_6\psi_2^3 + P_7\psi_3^3 \quad For \;\; \frac{L}{2} \le x \le \frac{3L}{4}$$

$$P(x) = P_5\psi_1^3 + P_6\psi_2^3 + P_7\psi_3^3 \quad For \;\; \frac{L}{2} \le x \le \frac{3L}{4}$$

$$P(x) = P_7\psi_1^4 + P_8\psi_2^4 + P_9\psi_3^4 \quad For \;\; \frac{3L}{4} \le x \le L$$

Exact Solution

Recall the equation (4)

$$\frac{P}{P_o} = \ell\frac{(4\mu L)}{D} \qquad (19)$$

$$P = \sigma_o \ell^{\left[\frac{4\mu L}{D}\right]} \qquad (20)$$

Numerical example;

Consider - direct – extrusion operation in which the billet material 150mm long is extruded in 100mm diameter container, $\mu = 0.25$.

Solution:

Using the mathCAD software for the model developed in Equation (17), the pressures at the nodes as solved by weighted residual finite element method and exact solution method are as shown in Fig 2.

The pressure distribution under direct extrusion, initially rose rapidly from zero as the billet was been expanded to fill the container completely and the pressure then decreased as the ram moved along the container and total frictional force was reduced Fig 2. The solution obtained in the case considered can also be applied to indirect extrusion problem. The advantage was as a result of the fact that numerical values of pressure and stresses of such problem can be determined by simply substituting the appropriate value of the coefficient of friction μ, diameter of extruding billet and half length of the extruding billet into the difference between the pressure variations described by the finite element and exact solution was infinitesimal and negligible Fig 2.

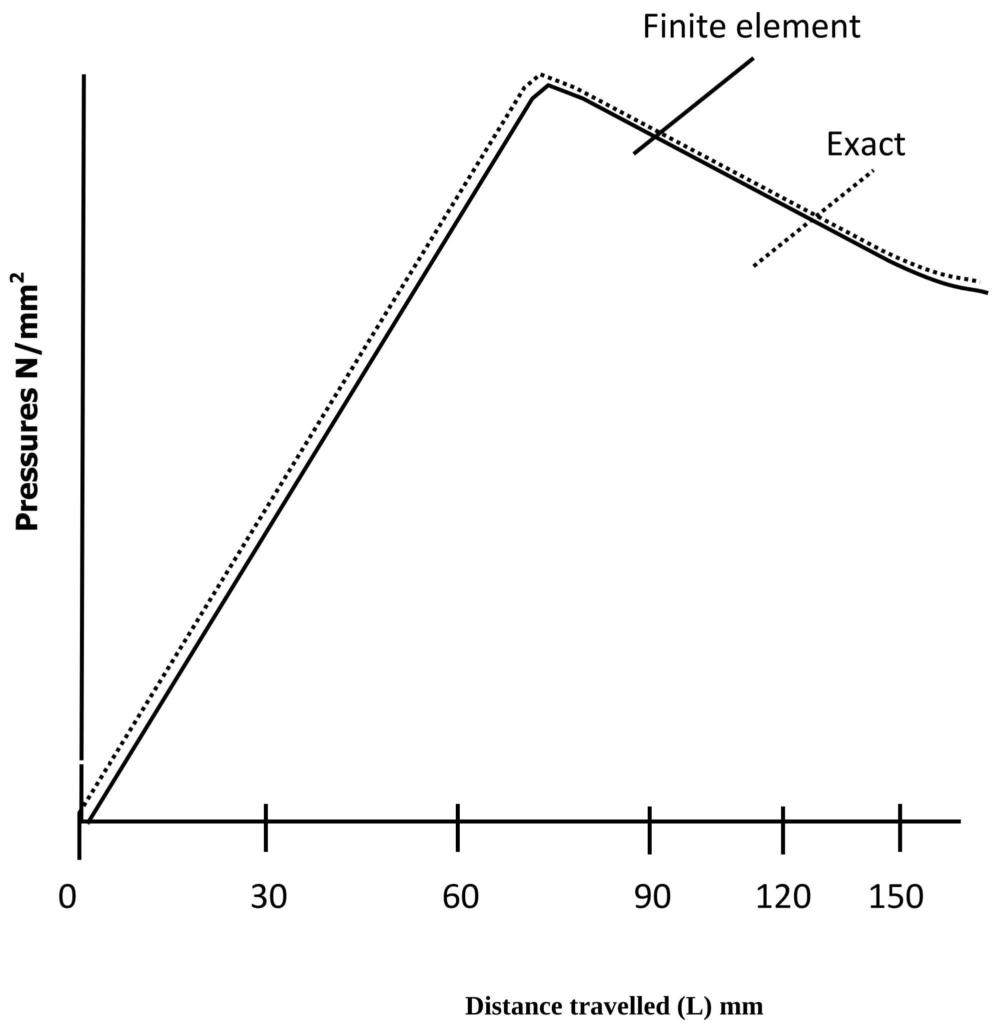

Fig 2: Graphical comparison of the exact solution and finite element solution

Extrusion pressure variation against distance traveled (L)

Conclusion

The weighted residual finite element method was capable of adequately and accurately analyzing the stress fields set up in a direct extrusion process.

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