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Kronecker Product Analytical Approach to ANOVA of Surface Roughness Optimization

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Abstract

The Fishers-Yates algorithm has remained the most widely used statistical approach that involves the use of sum of squares of treatments or blocks in the determination of mean square errors (MSEs) needful for the computation of F-statistic prior to the decision making based on the acceptance or rejection of the null hypothesis. A review of literature on design of experiments shows a trend away from uncritical acceptance of the approach, thus confirming that sustained effort is being made to develop a new method. As part of this effort, this paper attempts to develop a novel approach for determining the MSEs in designed experiment. Using the new method, the combination of controllable variables that optimized most the surface finish of machined workpiece materials was determined with Kronecker product analysis which was enhanced by the use of MATLAB software package. The response value for the surface roughness, $\widehat{X}_{ijklmn}$***, obtained from the model developed was 1.5368µm. Residual analysis carried out indicates that the model output was adequate. The analytical method explored can be used to develop a statistical software package that will be helpful in the computation of sums of squares of observation as well as make decision on the null hypothesis without recourse to Fisher's table.***

Keywords: Kronecker product, Sum of Squares, Mean sum of squares, Optimization.

1.0 Introduction

The surface textures of locally machined products have to be of good quality in order to compete favourably with imported ones. Achievement of this quality level has remained a challenge to local machinist in developing countries particularly Nigeria. The most cost-beneficial of existing statistical methods used in industry for quality and productivity improvement is statistical design of experiments [1]. The machining operation is a production process which generates output subject to certain controllable variables. These variables are pre-selected by the machinist based on experience and operating standard of the machining company. The appropriate selection of these controllable variables is a major factor that determines the degree of surface finish of the machined workpiece. The traditional approach involving the use of Yate's algorithm in conjunction with Fisher's ratio in making decision in analysis of variance are in some cases, very computationally demanding and complicated. The traditional approach works very well when the problem being solved is well behaved. However, under certain situations such as nested design involving several treatments and blocks, the traditional approach fails to be very effective, thereby calling for a better approach.

The Kronecker product proposed in the current study has a more intuitive appeal in the sense that it makes use of matrix algebra to achieve linear transformation of experimental observations thereby lending computational expediency, possibly through the use of software. In this regard, minimization of errors in the measurement of response variables is more readily achieved. These attributes combine to make this novel approach very attractive and user-friendly.

A selective review of relevant work on design of experiment (DoE) is contained in [2]. The review made tangential reference to the seminal work done by Fisher which was improved upon in[3] and later [4]. Many studies had focused on the optimization of design to achieve the desired response. For example, [5] examined some statistical properties of designed experiment. The author explained that design robustness is pursued when the errors in certain factor levels cannot be measured. Further, [6] applied design of experiment to optimize models that estimate reservoir of hydrocarbon

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volume. Also [7] proposed the use of design keys in dealing with the following:

(i) identifying treatment effect with particular plot;
(ii) contrasting factorial experiment;
(iii) constructing design.

They also proposed development of blocked structure for symmetric and asymmetric designs.

Moreover, [8] researched the use of simulation techniques in estimating computational efforts required to obtain desired statistical decision for contemplated statistical estimators that can deal with asymptotic variance and asymptotic bias.

It is instructive to note that a new approach for dealing with designed experiment has been studied by several authors. For instance [9] employed tensor product space ANOVA models to deal with the course dimensionality in high-dimensional nonparametric problems that are able to capture interaction of order. They also examined many properties of the tensor product space of Sobolev-Hilbert spaces. Also, [10] had proposed the use of Kronecker product in ANOVA. The paper noted that [11] was the first paper to look into the use of Kronecker product in analysis of variance, and that thereafter[12], and [13] followed up the issue. Moreover, [14] studied orthogonality in factorial designs. They explained that orthogonality means that all the level combination of any two factors occur equally often. They leaned on the standard criterion for optimal factorial design that can engender minimum aberration as proposed in[15]. Other works that deal with Kronecker product algorithm include[16], and[17]. Finally, [18] carried out a comprehensive study of the frictional chatter occurring during metal cutting processes. They found out that some of the bifurcation diagrams cannot be classified into standard route to chaos, noting that crisis type transition to chaos is dominating.

Although the work on experimental design is plentiful in DoE literature, little had been devoted to the use of Kronecker product especially in minimizing surface roughness of machined workpiece materials. This work therefore seeks to employ Kronecker product in determining the sum of squares of sources of variation. The work also demonstrates the use of Kronecker product in establishing hypothesis. Our results indicate that the method advocated can do away with the use of Yates algorithm that involves great computational efforts in some cases.

2.0 Materials and Methods

Four types of workpiece materials namely, aluminium, copper, mild steel and stainless steel, were machined using a centre lathe machine with standardized process parameters as shown in **Table1.**

Table 1: Process Parameters

Process Parameters	Low	High
Rake Angle	10^0	60^0
Speed	140 rev/min	210 rev/min
Feed Rate	0.04 mm/sec	0.12 mm/sec
Depth of Cut (DoC)	0.05 mm	0.25 mm

Source: [19]

The experimental design data were obtained by using TR100 Surface Roughness Tester to measure the surface roughness of the workpiece materials after the machining operation. The full factorial design matrix was developed based on 2^n array where n is the number of controllable factors. The total runs carried out for each workpiece material are sixteen (16) indicating 2^4. The four controllable factors were taken at two different levels. The design matrix for HSS cutting tool is shown in Figure 1 and similar layouts were used for the other three cutting tools namely, ceramic, carbide and cobalt.

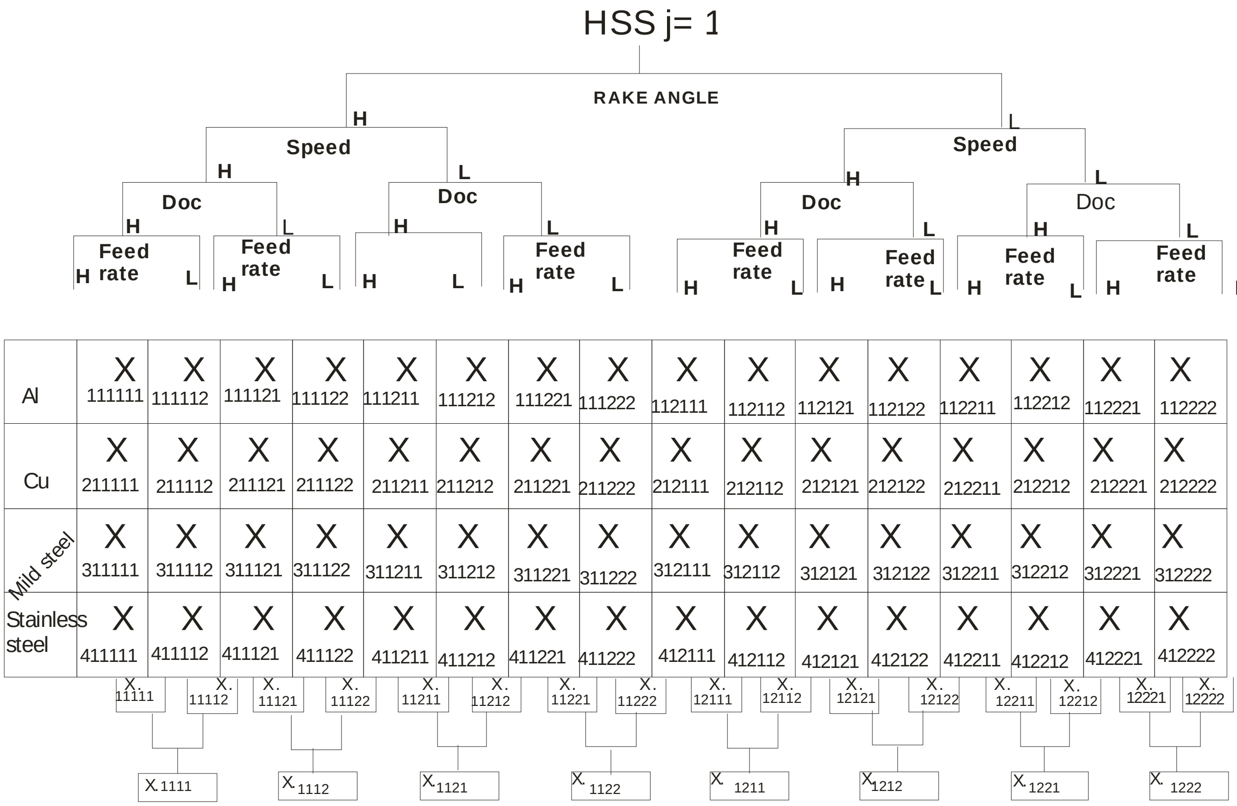

Fig.1: Design Matrix for HSS Cutting Tool

The cutting tool type was taken as the treatment while the workpiece material was taken as the block. The matrix in Fig 1 was reduced to the form shown in Table 2.

Table 2: Design matrix for Blocked Design

Tool Type Workpiece	HSS j=1	Ceramic j=2	Carbide j=3	Cobalt j=4
Aluminium, i=1	$\bar{X}_{11....}$	$\bar{X}_{12....}$	$\bar{X}_{13....}$	$\bar{X}_{14....}$
Copper, i=2	$\bar{X}_{21....}$	$\bar{X}_{22....}$	$\bar{X}_{23....}$	$\bar{X}_{24....}$
Mild Steel, i =3	$\bar{X}_{31....}$	$\bar{X}_{32....}$	$\bar{X}_{33....}$	$\bar{X}_{34....}$
Stainless Steel, i=4	$\bar{X}_{41....}$	$\bar{X}_{42....}$	$\bar{X}_{43....}$	$\bar{X}_{44....}$

3.0 Analytical Computations

Computation of Basic Sum of Squares

The data matrix of the design matrix for the blocked design is depicted in Table 3.

Table 3: Blocked Design Data Matrix

Tool Type Workpiece	HSS j=1	Ceramic j=2	Carbide j=3	Cobalt j=4
Aluminium, i=1	*0.89*	*1.06*	*0.97*	*1.00*
Copper, i=2	*1.03*	*1.25*	*0.99*	*1.22*
Mild Steel, i =3	*0.96*	*1.24*	*0.96*	*0.93*
Stainless Steel, i=4	*0.97*	*1.25*	*1.15*	*0.93*

The computation of the elements of the first basic sum of squares using

$\sum_{i=1}^{r}\sum_{j=1}^{c} X_{ij}^2 = Y^T[I_r \otimes I_c]Y$, is shown in equations (1), (2), (3), (4).and (5).

$$Y=\begin{bmatrix} 0.89 \\ 1.06 \\ 0.97 \\ 1.00 \\ 1.03 \\ 1.25 \\ 0.99 \\ 1.22 \\ 0.96 \\ 1.24 \\ 0.96 \\ 0.93 \\ 0.97 \\ 1.25 \\ 1.15 \\ 0.93 \end{bmatrix}; \qquad (1)$$

$$Y^T = [0.89 \quad 1.06 \quad 0.97 \quad 1.00 \quad 1.03 \quad 1.25 \quad 0.99 \quad 1.22 \quad 0.96 \quad 1.24 \quad 0.96 \quad 0.93 \quad 0.97 \quad 1.25 \quad 1.15 \quad 0.93] \qquad (2)$$

$$I_r = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

$$I_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (4)$$

$$I_r \otimes I_c = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \qquad (5)$$

Equation (6) shows the computed value of the first basic sum of squares. The observations in Table 3 were stacked row-wise to form a column matrix, Y, shown in equation (1) and the transpose of this matrix is shown equation (2).

$$Y^T[I_r \otimes I_c]Y = 17.8850 \qquad (6)$$

The computation of the elements of the second basic sum of squares, which is the sum of squares for constant using $\frac{\sum_{i=1}^{r}\sum_{j=1}^{c} X_{..}^2}{rc} = (rc)^{-1}\left[\sum_{i=1}^{r}\sum_{j=1}^{c} X_{ij}\right]^2 = Y^T[(rc)^{-1} J_r \otimes J_c]Y$, is shown in equations (1), (2), (7), (8).and (9).

$$\mathrm{J_r} = \begin{bmatrix} 1&1&1&1 \\ 1&1&1&1 \\ 1&1&1&1 \\ 1&1&1&1 \end{bmatrix} \qquad (7)$$

$$\mathrm{J_c} = \begin{bmatrix} 1&1&1&1 \\ 1&1&1&1 \\ 1&1&1&1 \\ 1&1&1&1 \end{bmatrix} \qquad (8)$$

$$J_r \otimes J_c = \begin{bmatrix} 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \end{bmatrix} \quad (9)$$

Equation (10) shows the computed value of second basic sum of squares.

$$Y^T[(rc)^{-1} J_r \otimes J_c]Y = (4\text{x}4)^{-1}(282.24) \quad = 17.625 \quad (10)$$

The computation of the elements of the third basic sum of squares using

$$\frac{\sum_{i=1}^{r}\sum_{j=1}^{c} X_{i.}^2}{c} = c^{-1}\sum_{i=1}^{r}\left[\sum_{j=1}^{c} X_{ij}\right]^2 = Y^T[(c^{-1})I_r \otimes J_c]Y$$ is shown in equations (1), (2), (3), (8).and (11).

$$I_r \otimes J_c = \begin{bmatrix} 1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1 \end{bmatrix} \quad (11)$$

Equation (12) shows the computed value of the second basic sum of squares.

$$Y^T[(c^{-1})I_r \otimes J_c]Y = (4)^{-1(}70.7446) = 17.686 \quad (12)$$

4.0 Computation of Final Basic Sum of Squares

The computation of the elements of the final basic sum of squares using

$\frac{\sum_{i=1}^{r}\sum_{j=1}^{c} X_{.j}^2}{r} = r^{-1}\sum_{j=1}^{c}\left[\sum_{i=1}^{r} X_{ij}\right]^2 = Y^T[(r^{-1})J_r \otimes I_c]Y$ is shown in equations (1), (2), (3), (8).and (13).

$$J_r \otimes I_c = \begin{bmatrix} 1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0 \\ 0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0 \\ 0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0 \\ 0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1 \\ 1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0 \\ 0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0 \\ 0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0 \\ 0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1 \\ 1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0 \\ 0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0 \\ 0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0 \\ 0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1 \\ 1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0 \\ 0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0 \\ 0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0 \\ 0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1 \end{bmatrix} \quad (13)$$

Equation (14) shows the computed value of the final basic sum of squares.

$Y^T[(r^{-1})J_r \otimes I_c]Y = (4)^{-1}(71.0738) = 17.7645$ (14)

The computational result using traditional split plot sum of squares approach gave a value of 27.602 as against 17.7645 obtained with the proposed Kronecker product.

5.0 Data Analysis for the One-Way ANOVA

The data matrix in Table 3 was analysed without considering any interaction effect between cutting tool types and workpiece materials. The cutting tool type was taken as the treatment and the workpiece materials as replications under various cutting tool types. This analysis was meant to determine the existence of variation in the cutting tool type at various levels. The Sum of Square of Treatment for the observed data is obtained by the expression shown in equation (14), which is the difference between $Y^T[(c^{-1})I_r \otimes J_c]Y = (4)^{-1}(70.7446) = 17.686$, equations (12), and $Y^T[(rc)^{-1}J_r \otimes J_c]Y = (4\text{x}4)^{-1}(282.24) = 17.625$, equation (10), and is fully expressed in

$$c^{-1}\sum_{i=1}^{r}\left[\sum_{j=1}^{c} X_{ij}\right]^2 - (rc)^{-1}\left[\sum_{i=1}^{r}\sum_{j=1}^{c} X_{ij}\right]^2 = Y^T[c^{-1}(I_r \otimes J_c)]Y - Y^T[(rc)^{-1}(J_r \otimes J_c)]Y$$

Sum of Square of Treatment = $Y'[(c^{-1})I_r \otimes J_c]Y - Y'[(rc)^{-1}J_r \otimes J_c]Y$ (14)

$= 17.686 - 17.625 = 0.061$

The sum of squares of errors for the observed data is obtained with the expression

$\frac{\sum_{i=1}^{r}\sum_{j=1}^{c} X_{i.}^2}{c} = c^{-1}\sum_{i=1}^{r}\left[\sum_{j=1}^{c} X_{ij}\right]^2 = Y^T[(c^{-1})I_r \otimes J_c]Y$. The left hand side (LHS) of the equation is the first basic sum of squares as computed and shown in equation (6). The sum of squares for mean (SS_{mean}) is the second basic sum of squares as computed and shown in equation (10). Hence, the expression $\sum_{r=1}^{r}\sum_{j=1}^{c} X_{ij}^2$ = SS_{mean} + $SS_{treatment}$ + $SS_{residual}$ was used to compute the error sum of squares as shown in equation (25).

$$SS_{residual} = 17.885-17.625-0.06 = 0.2 \qquad (15)$$

The degree of freedom for the various sources of variation and the resultant mean squares are tabulated and presented in Table 4.

6.0 Data Analysis for Two-Way Blocked Design ANOVA

The data matrix in Table 3 was further analysed by considering the workpiece materials as a block and the cutting tool types as the treatment. The expression for the sum of square for block effect computation is

$$\sum_{i=1}^{r}\sum_{j=1}^{c} X_{ij}^2 - c^{-1}\sum_{i=1}^{r}\left[\sum_{j=1}^{c} X_{ij}\right]^2 - r^{-1}\sum_{j=1}^{c}\left[\sum_{i=1}^{r} X_{ij}\right]^2 + (rc)^{-1}\left[\sum_{i=1}^{r}\sum_{j=1}^{c} X_{ij}\right]^2 = Y^T[G_r \otimes H_c]Y \ , \qquad .$$

SS_{block} = $Y'[G_r \otimes H_c]Y$ but $G_r = r^{-1}J_r$ and $H_c = I_c - G_c$. From equations (4) and (7).

Therefore, $G_r = (4^{-1})\begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix} = \begin{bmatrix}\frac{1}{4}&\frac{1}{4}&\frac{1}{4}&\frac{1}{4}\\\frac{1}{4}&\frac{1}{4}&\frac{1}{4}&\frac{1}{4}\\\frac{1}{4}&\frac{1}{4}&\frac{1}{4}&\frac{1}{4}\\\frac{1}{4}&\frac{1}{4}&\frac{1}{4}&\frac{1}{4}\end{bmatrix}$.

Since r=c, then $H_c = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} - \begin{bmatrix}\frac{1}{4}&\frac{1}{4}&\frac{1}{4}&\frac{1}{4}\\\frac{1}{4}&\frac{1}{4}&\frac{1}{4}&\frac{1}{4}\\\frac{1}{4}&\frac{1}{4}&\frac{1}{4}&\frac{1}{4}\\\frac{1}{4}&\frac{1}{4}&\frac{1}{4}&\frac{1}{4}\end{bmatrix} = \begin{bmatrix}\frac{3}{4}&\frac{-1}{4}&\frac{-1}{4}&\frac{-1}{4}\\\frac{-1}{4}&\frac{3}{4}&\frac{-1}{4}&\frac{-1}{4}\\\frac{-1}{4}&\frac{-1}{4}&\frac{3}{4}&\frac{-1}{4}\\\frac{-1}{4}&\frac{-1}{4}&\frac{-1}{4}&\frac{3}{4}\end{bmatrix}$.

The sum of square for block effect, SS_{block} = $Y'[G_r \otimes H_c]Y$ was obtained by computing kronecker product $[G_r \otimes H_c]$.

$$[G_r \otimes H_c] = \begin{bmatrix}
\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}\\
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\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}\\
\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}\\
\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}\\
\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{-1}{16}&\frac{3}{16}
\end{bmatrix} \qquad (16)$$

The value for the sum of squares for block effect, shown in equation (17), was obtained using

$$\sum_{i=1}^{r}\sum_{j=1}^{c} X_{ij}^{2} - c^{-1}\sum_{i=1}^{r}\left[\sum_{j=1}^{c} X_{ij}\right]^{2} - r^{-1}\sum_{j=1}^{c}\left[\sum_{i=1}^{r} X_{ij}\right]^{2} + (rc)^{-1}\left[\sum_{i=1}^{r}\sum_{j=1}^{c} X_{ij}\right]^{2} = Y^{T}[G_r \otimes H_c]Y$$ and the detailed computation was carried out using MATLAB software..

$$SS_{block} = Y'[G_r \otimes H_c]Y = 0.1284 \qquad (17)$$

The sum squares of error for the observed data is obtained with the expression:
shown in $SS_{error} = Y^{T}[(I_r \otimes I_c) - (I_r \otimes G_c) - (G_r \otimes I_c) + (G_r \otimes G_c)]Y \; = Y^{T}[H_r \otimes H_c]Y$
Kronecker product element of the expression were computed as shown in equation (18). Accordingly, the value for the sum of squares of errors was obtained as shown in equation (19) and the detailed computation was carried out using MATLAB software.

$$H_r \otimes H_c = \begin{bmatrix}
\frac{9}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\
\frac{-3}{16} & \frac{9}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} \\
\frac{-3}{16} & \frac{-3}{16} & \frac{9}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} \\
\frac{-3}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{9}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} \\
\frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{9}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\
\frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{9}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} \\
\frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{9}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} \\
\frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{9}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} \\
\frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{9}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\
\frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{9}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} \\
\frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{9}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} \\
\frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{9}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} \\
\frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{9}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{-3}{16} \\
\frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{9}{16} & \frac{-3}{16} & \frac{-3}{16} \\
\frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{9}{16} & \frac{-3}{16} \\
\frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{-3}{16} & \frac{9}{16}
\end{bmatrix} \qquad (18)$$

$$SS_{error} = Y'[H_r \otimes H_c]Y = 0.2974 \qquad (19)$$

7.0 Kronecker Product Hypothesis Matrix Construction

The data matrix for the observed experimental data was represented as a 4 x 4 ANOVA by taking the row means of each cell under various cutting tool types. In this regard, the original data were analysed for various effects, namely row, column and interaction effects. The null hypothesis, $\begin{matrix}\mu_{1,1}+\mu_{1,2}+\mu_{1,3}+\mu_{1,4}-(\mu_{2,1}+\mu_{2,2}+\mu_{2,3}+\mu_{2,4}) \\ +\mu_{3,1}+\mu_{3,2}+\mu_{3,3}+\mu_{3,4}-(\mu_{4,1}+\mu_{4,2}+\mu_{4,3}+\mu_{4,4})=0\end{matrix}$, for row effect, for this data matrix, can now be written as shown in equation (20).

$$H_o: \ (\mathbf{O}\otimes\mathbf{1}^1)\boldsymbol{\mu} \quad = [1\ 1\ 1\ 1\ \text{-}1\ \text{-}1\ \text{-}1\ \text{-}1\ 1\ 1\ 1\ 1\ \text{-}1\ \text{-}1\ \text{-}1\ \text{-}1]\begin{bmatrix}0.89\\1.06\\0.97\\1.00\\1.03\\1.25\\0.99\\1.22\\0.96\\1.24\\0.96\\0.93\\0.97\\1.25\\1.15\\0.93\end{bmatrix}$$

$$= 0.89+1.06+0.97+1.00-(1.03+1.25+0.99+1.22)+0.96+1.24+0.96+0.93-(0.97+1.25+1.15+0.93) = 3.54 \quad (20)$$

The null hypothesis, $= \mu_{1,1}-\mu_{1,2}+\mu_{1,3}-\mu_{1,4}+\mu_{2,1}-\mu_{2,2}+\mu_{2,3}-\mu_{2,4} + \mu_{3,1}-\mu_{3,2}+\mu_{3,3}-\mu_{3,4}+\mu_{4,1}-\mu_{4,2}+\mu_{4,3}-\mu_{4,4} = 0$'

for column effect for this data matrix, can now be written as shown in equation (30).

$H_o:$

$$(\mathbf{1}\otimes\mathbf{O}^1)\boldsymbol{\mu} \quad = [1\ \text{-}1\ 1\ \text{-}1\ 1\ \text{-}1\ 1\ \text{-}1\ 1\ \text{-}1\ 1\ \text{-}1\ 1\ \text{-}1\ 1\ \text{-}1]\begin{bmatrix}0.89\\1.06\\0.97\\1.00\\1.03\\1.25\\0.99\\1.22\\0.96\\1.24\\0.96\\0.93\\0.97\\1.25\\1.15\\0.93\end{bmatrix}$$

$$= 0.89\text{-}1.06\text{+}0.97\text{-}1.00+1.03\text{-}1.25\text{+}0.99-1.22+0.96\text{-}1.24\text{+}0.96\text{-}0.93+0.97\text{-}1.25\text{+}1.15\text{-}0.93 = -0.96 \quad (21)$$

The null hypothesis for the interaction effect is that there are no differential effect and is analysed in what follows.

$$H_o: \mathbf{O}_A\otimes\mathbf{O}_B = \mathbf{0}$$

$\mathbf{O_A}$ = [$a_{1.}$ $-a_{2.}$ $a_{3.}$ $-a_{4.}$]
$\mathbf{O_B}$ = [$b_{.1}$ $-b_{.2}$ $b_{.3}$ $-b_{.4}$]
Where [$a_{1.}$ $a_{2.}$ $a_{3.}$ $a_{4.}$] and [$b_{.1}$ $-b_{.2}$ $b_{.3}$ $-b_{.4}$] are the respective row and column sums.

$\mathbf{O_A}\otimes\mathbf{O_B}$ = [(0.98)(0.97 -1.20 1.02 -1.02) (-1.12)(0.97 -1.20 1.02 -1.02) (2.1)(0.97 -1.20 1.02 -1.02) (-1.08)(0.97 -1.2 1.02 -1.02)]

= [0.9506-1.176+0.9996-0.9996-1.0864+1.344-1.1424+1.1424+2.037-2.52+2.142-2.142-1.0476+1.296-1.1016+1.1016]

= -0.4508 (22)

8.0 Results

The results of data computations carried out are presented hereunder.
The following hypotheses were employed.

(i) Nature of Workpiece Material

(a) $H_{workpiece}^{(0)}$: all α_i =0; the four types of workpiece specimens, (aluminum, copper, mild steel and stainless steel) employed showed no significant differential effect under the cutting conditions adopted.

(b) $H_{workpiece}^{(1)}$: all α_i =0; surface roughness observed on the workpiece varied according to the strength of material used.

(ii) Tool Type

(a) $H_{tooltype}^{(0)}$: all β_j =0; the four tool specimens (HSS, ceramic, carbide and cobalt) employed in the experiment impact similar surface texture under the same cutting conditions.

(b) $H_{tooltype}^{(1)}$: all β_j =0; the four tool specimens exhibit different surface roughness characteristics under the experimental conditions observed.

Results for One-Way ANOVA

The sums of squares, for the various sources of variation, are tabulated as in Table 4. The mean square and the degree of freedom and number of independent variables, were also computed. The decisions on the null hypothesis for the various sources of variations were also established.

Table 4: One-Way ANOVA Table for Surface Roughness of Machined Workpiece

Sources of Variation	Sums of Squares (SS)	Degrees of Freedom (DoF)	Mean of Squares $= \frac{SS}{DoF}$	$F_{cal} = \frac{MS_{variation}}{MS_{error}}$	F_{tab} (at $\alpha = 0.05$)	Decision	F_{tab} (at $\alpha = 0.01$)	Decision
Treatment (Cutting tool Types)	0.061	(J-1) = 4-1 = 3	0.02033	0.91494	3.86	$F_{cal} < F_{tab}$ Accept H_0	6.99	$F_{cal} < F_{tab}$ Accept H_0
Error	0.2	(I-1)(J-1)=(4-1)(4-1) = 9	0.02222					

Results for Two-Way Blocked Design ANOVA

The sums of squares, for the various sources of variation are tabulated as in Table 5. The mean sum of squares and the degree of freedom and number of independent variables, were also computed. Moreover, the null hypothesis for the various sources of variations was determined.

Table 5: Two-Way ANOVA Table for Surface Roughness of Machined Workpiece

Sources of Variation	Sums of Squares (SS)	Degrees of Freedom (DoF)	Mean of Squares $= \frac{SS}{DoF}$	$F_{cal} = \frac{MS_{variation}}{MS_{error}}$	F_{tab} (at $\alpha = 0.05$)	Decision	F_{tab} (at $\alpha = 0.01$)	Decision
Treatment (Cutting tool Types)	0.061	(J-1) = 4-1 = 3	0.02033	0.0684	3.86	$F_{cal} < F_{tab}$ Accept H_0	6.99	$F_{cal} < F_{tab}$ Accept H_0
Block (Workpiece Materials)	0.1284	(I-1) = 4-1 = 3	0.0428	0.1439	3.86	$F_{cal} < F_{tab}$ Accept H_0	6.99	$F_{cal} < F_{tab}$ Accept H_0
Error	0.2	(I-1)(J-1)=(4-1)(4-1) = 9	0.2974					

By this approach, both null hypotheses were accepted at 0.01 and 0.05 levels of significance respectively. The import is that workpiece material and tool type contribute less in the determination of the final texture of machined workpiece in relation to other factors.

However, using Kronecker Product Hypothesis Matrix, the same null hypotheses, including their interaction were rejected. The approach is superior to the Yates algorithm because of its use of cell means and omnibus matrices that is more robust. The method is therefore confirmatory and more reliable.

Our final conclusion therefore is that workpiece material as well as tool type play a significant role in the determination of the surface texture of machined workpiece. These results collaborate with literature results and

9.0 Final Model Developed

The numerical value for the model was obtained by substituting the values of overall mean (μ), sum of squares for cutting tool type, sum of squares for the workpiece cutting tool type interaction and the error sum of squares.

From equations (14), (17), (19) and the computed value for the overall mean, the numerical value for the model is expressed follows:

$$\hat{\mathbf{X}}_{\mathbf{ijklmn}} = 1.05 + 0.1284 + 0.061 + 0.2974 = 1.5368\mu m \quad (23)$$

10.0 Residual Analysis and Model Checking

The residual associated with the model developed is obtained using, Residual = $X_{ijklmn} - \hat{X}_{ijklmn}$, and the numerical values are shown in table 6.

Table 6: Residuals Associated with the Model

Workpiece Materials	Residuals			
Aluminium, i=1	*-0.64*	*-0.47*	*-0.57*	*-0.54*
Copper, i=2	*-0.50*	*-0.29*	*-0.55*	*-0.32*
Mild Steel, i =3	*-0.57*	*-0.30*	*-0.58*	*-0.61*
Stainless Steel, i=4	*-0.57*	*-0.29*	*-0.38*	*-17.00*

The normality plot of the residuals from the surface roughness experiment is shown in Figure 2. Figures 3 and 4 present the residual plotted against the factor level of workpiece material and fitted value of $\hat{X}_{ijklmn}$.

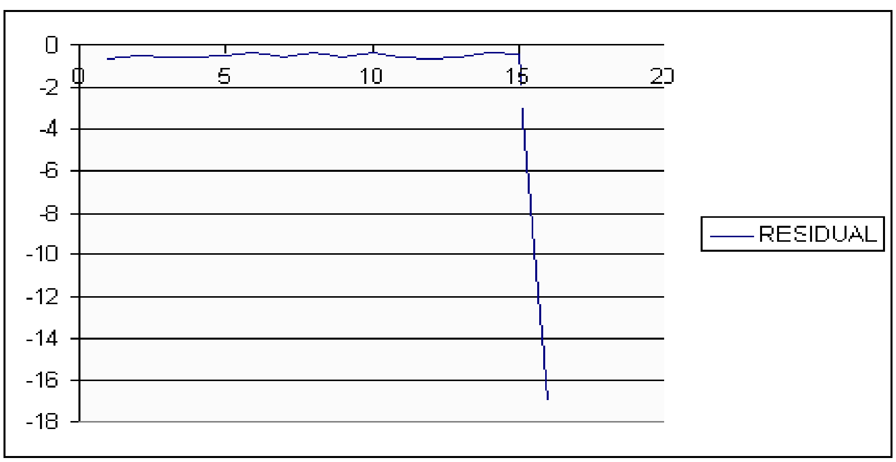

Fig. 2: Normality plot of the Residuals

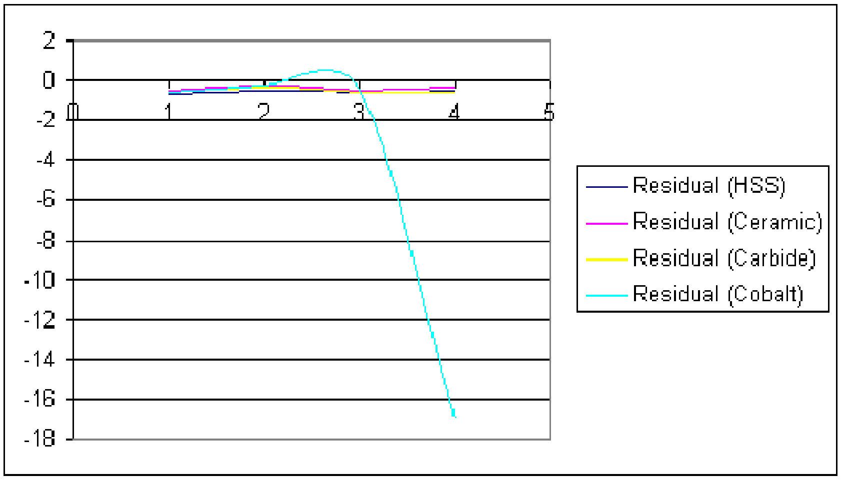

Fig. 3: Plot of Residual versus Workpiece Material Factor Level

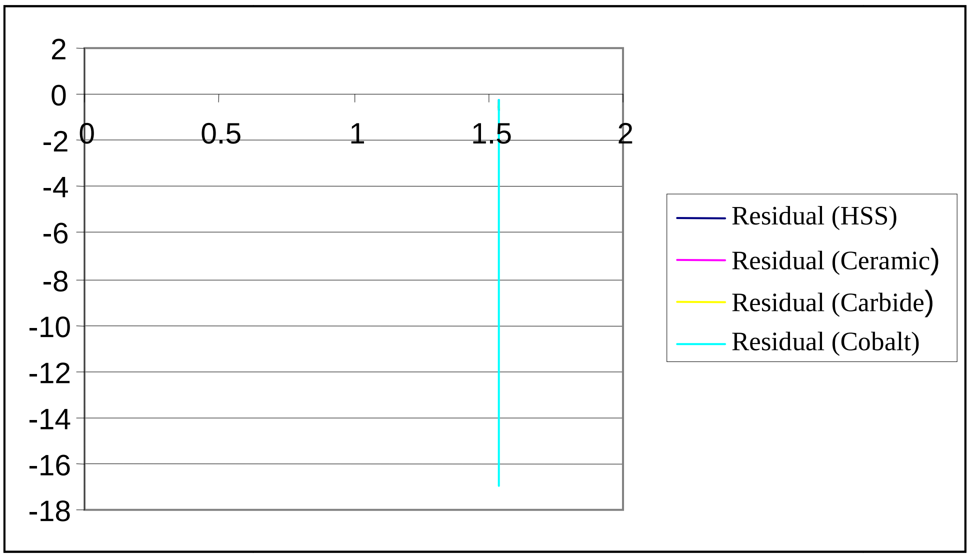

Fig. 4: Plot of Model against Residuals

These plots do not reveal any model inadequacy or unusual problem with the assumption.

11.0 Discussion

The kronecker product analytical tool employed in this research project has facilitated the decomposition of sources of variance into their contributing components without performing the rigorous Fishers-Yates algorithm. The algorithm involves computation of sum of squares of these various sources of variation. The cutting tool type was selected as one of the factors that influence the workpiece materials surface finish. The model was able to state the particular combination of these factors that gave rise to certain degree of surface roughness of 1.536µm. MATLAB software was used as a computational aid and this eliminated the rigorous steps involved in matrix algebra, which is one of the major mathematical theories applied to this research work. Furthermore, it was claimed at the beginning that Kronecker product analytical technique proposed is a new paradigm that obviates the use of complex computations associated with the Yates Algorithm. The research outcomes have justified this claim. Thus the overall aim of the research study has been fruitfully achieved.

12.0 Conclusion

The application of right Kronecker product to fractional experimental design of surface roughness data has been established in this study. This analytical method explored can be used to develop a statistical software package that will be helpful in the computation of sums of square of observations as well as take decision on the null hypothesis without comparing the values in Fisher's table with the computed values.

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