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Multivariate Marshall and Olkin Exponential Minification Process

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Abstract

A stationary bivariate minification process with bivariate Marshall-Olkin exponential distribution that was earlier studied by Miroslav et al [15]is in this paper extended to multivariate minification process with multivariate Marshall and Olkin exponential distribution as its stationary marginal distribution. The innovation and the joint distributions of random vectors $\left(X_n^{(1)}, X_n^{(2)}, ..., X_n^{(k)}\right)$ ***and*** $\left(X_{n-j}^{(1)}, X_{n-j}^{(2)}, ..., X_{n-j}^{(k)}\right), \quad j > 0,$ ***are presented. The autocovariance and the autocorrelation matrices are developed. Lastly, the unknown parameters are estimated and their asymptotic properties are also investigated in this research work.***

Keywords: Ergodic; Estimation; Minification process; Multivariate Marshall and Olkin Exponential Distribution; uniformly mixing.

1.0 Introduction

Minification process can be of several orders. A minification process of the first order is given by $X_n = R \min(X_{n-1}, \varepsilon_n), \quad n \geq 1,$ where $R > 1$ and $\{\varepsilon_n, n \geq 1\}$, is an innovation process of independent and identically distributed random variables.

According to [1], a first order autoregressive minification process can be defined as a sequence having the general structure $X_n = \begin{cases} kX_{n-1} & with\ probability\ P \\ k\min(X_{n-1}, \in_n) & with\ probability\ 1-P \end{cases}$

Where $\{\in_n\}$ is an innovation process of independent and identically distributed random variables chosen to ensure that $\{X_n\}$ is a stationary Markov process with a specified marginal distribution function $F_X(X)$.

Another form of minification process is the one with structure

$$X_n = \begin{cases} k \in_{n-1} & with\ probability\ P \\ k\min(X_{n-1}, \in_n) & with\ probability\ 1-P. \end{cases}$$

Similarly, [2] defined first order autoregressive minification process model of random vectors $\{(X_n, Y_n)\}$ with Marshall and Olkin bivariate semi-Pareto distribution as

$$X_n = \begin{cases} U_n & with\ probability\ \ p \\ \min(X_{n-1}, U_n) & with\ probability\ \ 1-p \end{cases}$$

$$Y_n = \begin{cases} V_n & with\ probability\ \ p \\ \min(Y_{n-1}, V_n) & with\ probability\ \ 1-p \end{cases}$$

where $\{(U_n, V_n)\}$ are innovations, which are independent of $\{(X_{n-k}, Y_{n-k})\}$ for k=1,2,…,n.

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Several authors have introduced minification processes with given marginal distribution. Minification process with exponential marginal distribution was earlier presented by [3]. Minification process with Weibull marginal distribution was introduced by [4]. Pareto minification process was studied by [5]. Also, [6] presented a logistic minification process. Semi-Pareto minification processes were introduced and studied [7] and [8]. Recently, [9] considered some properties of the semi-Pareto minification process of [7] and estimated the unknown parameters of the model. Also, [1] introduced the minification process with marginal distribution function $F_X(x)$. Some bivariate and multivariate minification processes were studied by [10], [11], [12], [2] and [13]. Bivariate exponential distribution was presented by [14] with survival function

$$P(X > x, Y > y) = e^{-\lambda_1 x - \lambda_2 y - \lambda_{12}\max(x,y)}, \quad x, y > 0, \quad with\ \lambda_1 > 0, \lambda_2 > 0\ \ and\ \ \lambda_{12} > 0.$$

The random variables are constructed such that X *and* Y are dependent exponentially distributed random variables with parameters $\lambda_1 + \lambda_{12}$ and $\lambda_2 + \lambda_{12}$, respectively.

Recently, [15] presented a stationary bivariate minification process with Marshall and Olkin exponential distribution. The innovation and joint distributions of random vectors (X_n, Y_n) were also presented in that paper. Autocovariance and autocorrelation matrices were also developed. Unknown parameters in the model were estimated and asymptotic properties of the estimated parameters were also investigated.

In this paper, we consider a stationary multivariate minification process with Marshall and Olkin multivariate exponential distribution as an extension of bivariate minification process with Marshall and Olkin exponential distribution earlier presented by [15]. Marshall and Olkin multivariate exponential distribution, denoted by $MVE(\lambda_i, \lambda_{ij}),\ i = 1, 2, ..., n\ \ and\ \ j = 1, 2, ..., k$, has the survival function in the form

$$P\left(X_n^{(1)} > x_1, X_n^{(2)} > x_2, ..., X_n^{(k)} > x_k\right) = e^{-\sum_i \lambda_i x_i - \sum_i \sum_j \lambda_{ij} \max(x_1, x_2, ..., x_n)}, \ i \neq j, x_i > 0 \ \ \forall i \quad \text{with} \quad \lambda_i\ and\ \lambda_{ij}$$

positive.

The random vectors are designed such that $\left\{X_i^{(1)}, X_i^{(2)}, X_i^{(3)}, \ldots, X_i^{(k)}, i \geq 1\right\}$ are dependent exponentially distributed random vectors with parameters $\lambda_i + \lambda_{ij},\ \ i = 1, 2, \ldots, n,\ \ j = 1, 2, \ldots, k\ \ and\ \ i \neq j.$

This paper is organized as follows.

In section 2, the process and its properties are considered. Autocovariance and autocorrelation matrices are introduced and studied in section 3. In section 4, unknown parameters are estimated and its asymptotic properties are presented and discussed. Section 5 gives the conclusion of the entire paper.

2.0 Process and its Properties

In this section, we consider a stationary multivariate minification process with multivariate Marshall and Olkin exponential distribution, $MVE(\lambda_i, \lambda_{ij}),\ i = 1, 2, ..., n\ \ and\ \ j = 1, 2, ..., k.$

This process is given by

$$\begin{aligned} X_n^{(1)} &= R\min\left(X_{n-1}^{(1)}, X_{n-1}^{(2)}, ..., X_{n-1}^{(k)}, \varepsilon_{n1}\right) \\ X_n^{(2)} &= R\min\left(X_{n-1}^{(1)}, X_{n-1}^{(2)}, ..., X_{n-1}^{(k)}, \varepsilon_{n2}\right) \qquad (2.1) \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ X_n^{(k)} &= R\min\left(X_{n-1}^{(1)}, X_{n-1}^{(2)}, ..., X_{n-1}^{(k)}, \varepsilon_{nk}\right) \end{aligned}$$

where $\lambda_i > 0, \ \forall i$ and $\lambda_{ij} > 0, \ \forall i \ \ and \ \ j,$ $\quad R = \frac{\sum_{i=1}^{n}\left(\lambda_i + \lambda_{ij}\right)}{\sum_{i=1}^{n}\lambda_{ij}}, \quad j = 1, 2, ..., k.$ The sequence $\left\{\left(\varepsilon_{n1}, \varepsilon_{n2}, ..., \varepsilon_{nk}\right), n \geq 1\right\}$ is independently and identically distributed random vectors. Also, the random vectors $\left(X_m^{(1)}, X_m^{(2)}, ..., X_m^{(k)}\right)$ and $\left(\varepsilon_{n1}, \varepsilon_{n2}, ..., \varepsilon_{nk}\right)$ are independent for $m < n.$

The innovation distribution of random vector $\left(\varepsilon_{n1}, \varepsilon_{n2}\right)$ was developed by [13] and represented by [15].

The random vector $\left(\varepsilon_{n1}, \varepsilon_{n2}\right)$ has the bivariate Marshall and Olkin exponential distribution $BVE\left(\lambda_1 R, \lambda_2 R, \lambda_{12} R - \lambda\right)$. The marginal distributions of random variables $\varepsilon_{n1} \ and \ \varepsilon_{n2}$ are $f\left(c_1\right) \ and \ f\left(c_2\right)$, respectively, where $c_1 = \left(\lambda_1 + \lambda_{12}\right)R - \lambda \ \ and \ \ c_2 = \left(\lambda_2 + \lambda_{12}\right)R - \lambda.$ Following this, the innovation distribution of random vectors $\left(\varepsilon_{n1}, \varepsilon_{n2}, ..., \varepsilon_{nk}\right)$ is the multivariate Marshall and Olkin exponential distribution $MVE\left(\lambda_1 R, \lambda_2 R, ..., \lambda_n R, \lambda_{1n} R - \lambda\right).$ The marginal distribution of random variables $\varepsilon_{n1}, \varepsilon_{n2}, ..., \varepsilon_{nk}$ are $f\left(c_i\right), \ i = 1, 2, 3, ..., n,$ respectively where $c_i = \left(\lambda_i + \lambda_{ij}\right)R - \lambda, \ i = 1, 2, ...n \quad j = 1, 2, ..., k, \ i \neq j.$

From the above information, we obtain the joint survival function of random vectors $\left(X_n^{(1)}, X_n^{(2)}, ..., X_n^{(k)}\right)$ and $\left(X_{n-j}^{(1)}, X_{n-j}^{(2)}, ..., X_{n-j}^{(k)}\right), \ \ j > 0.$ We denote joint survival function of $\left(X_n^{(1)}, X_n^{(2)}, ..., X_n^{(k)}\right)$ and $\left(X_{n-j}^{(1)}, X_{n-j}^{(2)}, ..., X_{n-j}^{(k)}\right)$ by

$$T_j\left(x_1, x_2, ..., x_n, x_1^*, x_2^*, ..., x_n^*, R\right) = P\left(X_n^{(1)} > x_1, ..., X_n^{(k)} > x_n, X_{n-j}^{(1)} > x_1^*, ..., X_{n-j}^{(k)} > x_n^*\right). \quad ...(2.2)$$

The joint survival function $T_j\left(x_1, x_2, ..., x_n, x_1^*, x_2^*, ..., x_n^*, R\right), \ \ j \geq 1,$ can be obtained recursively as

$$T_j\left(x_1, x_2, ..., x_n, x_1^*, x_2^*, ..., x_n^*, R\right) = \frac{\overline{F}\left(\max\left(\frac{x_1}{R_j}, \frac{x_2}{R_j}, ..., \frac{x_n}{R_j}, x_1^*\right), ..., \max\left(\frac{x_1}{R_j}, \frac{x_2}{R_j}, ..., \frac{x_n}{R_j}, x_n^*\right)\right) \bullet \overline{F}\left(x_1, x_2, ..., x_n\right)}{\overline{F}\left(\max\left(\frac{x_1}{R_j}, \frac{x_2}{R_j}, ..., \frac{x_n}{R_j}\right), \max\left(\frac{x_1}{R_j}, \frac{x_2}{R_j}, ..., \frac{x_n}{R_j}\right)\right)}$$

$$= T_1\left(x_1, x_2, ..., x_n, x_1^*, x_2^*, ..., x_n^*; R_j\right).$$

It is clear that the joint distribution and properties of the random vector $\left(X_n^{(1)}, X_n^{(2)}, ..., X_n^{(k)}, X_{n-j}^{(1)}, X_{n-j}^{(2)}, ..., X_{n-j}^{(k)}\right)$ can be obtained from the joint distribution and properties of the random vector $\left(X_n^{(1)}, X_n^{(2)}, ..., X_n^{(k)}, X_{n-1}^{(1)}, X_{n-1}^{(2)}, ..., X_{n-1}^{(k)}\right)$ by replacing $R \ \ with \ \ R_j.$

3.0 Autocovariance and Autocorrelation

The autocovariance structure of the multivariate Marshall and Olkin exponential minification process $\left\{\left(X_n^{(1)}, X_n^{(2)}, ..., X_n^{(k)}, \ n \geq 0\right)\right\}$ is given by

$$\Gamma(j)=\begin{bmatrix} \operatorname{cov}\left(X_n^{(1)},X_{n-j}^{(1)}\right) & \operatorname{cov}\left(X_n^{(1)},X_{n-j}^{(2)}\right) & \dots & \operatorname{cov}\left(X_n^{(1)},X_{n-j}^{(k)}\right) \\ \operatorname{cov}\left(X_n^{(2)},X_{n-j}^{(1)}\right) & \operatorname{cov}\left(X_n^{(2)},X_{n-j}^{(2)}\right) & \dots & \operatorname{cov}\left(X_n^{(2)},X_{n-j}^{(k)}\right) \\ \dots & \dots & \dots & \dots \\ \operatorname{cov}\left(X_n^{(k)},X_{n-j}^{(1)}\right) & \operatorname{cov}\left(X_n^{(k)},X_{n-j}^{(2)}\right) & \dots & \operatorname{cov}\left(X_n^{(k)},X_{n-j}^{(k)}\right) \end{bmatrix}$$

To derive the autocovariance matrix $\Gamma(j)$, it is enough to obtain the autocovariance matrix $\Gamma(1)$.

To compute the moment $E\left[X_n^{(1)}X_{n-1}^{(1)}\right]$, we consider the conditional expectation

$E\left[X_n^{(1)} \big/ X_{n-1}^{(1)}=x_1, X_{n-1}^{(2)}=x_2,\dots,X_{n-1}^{(k)}=x_k\right]$. From the definition of the process

$\left\{\left(X_n^{(1)},X_n^{(2)},\dots,X_n^{(k)},\ n\ge 0\right)\right\}$, we have the conditional distribution for $X_n^{(1)}$ given

$X_{n-1}^{(1)}=x_1, X_{n-1}^{(2)}=x_2,\dots,X_{n-1}^{(k)}=x_k$ as

$$P\left\{X_n \le z \big/ X_{n-1}^{(1)}=x_1, X_{n-1}^{(2)}=x_2,\dots,X_{n-1}^{(k)}=x_k\right\}=\begin{cases} 1-\dfrac{e^{-c_1 z}}{R}, & z<R\min\left(x_1,x_2,\dots x_k\right) \\ 1, & z\ge R\min\left(x_1,x_2,\dots x_k\right) \end{cases}.$$

Note that this is not an absolute continuous distribution, since the probability

$$P\left\{X_n^{(1)}=R\min\left(X_{n-1}^{(1)},X_{n-1}^{(2)},\dots,X_{n-1}^{(k)}\right) \big/ X_{n-1}^{(1)}=x_1, X_{n-1}^{(2)}=x_2,\dots,X_{n-1}^{(k)}=x_k\right\}$$

$$=P\left\{\varepsilon_{n1}>\min\left(x_1,x_2,\dots,x_k\right)\right\}=e^{-c_1\min\left(x_1,x_2,\dots,x_k\right)}$$

is non negative. The conditional expectation is given by $E\left[X_n^{(1)} \big/ X_{n-1}^{(1)}=x_1, X_{n-1}^{(2)}=x_2,\dots,X_{n-1}^{(k)}=x_k\right]$

$$=\frac{c_1}{R}\int_0^{R\min\left(x_1,x_2,\dots,x_k\right)} ze^{-\frac{c_1 z}{R}}\,dz+R\min\left(x_1,x_2,\dots,x_k\right)e^{-c_1\min\left(x_1,x_2,\dots,x_k\right)}$$

$$=\frac{R}{c_1}\left(1-e^{-c_1\min\left(x_1,x_2,\dots,x_k\right)}\right).$$

From the above, we have

$$E\left[X_n^{(1)}X_{n-1}^{(1)}\right]=\frac{R}{c_1}E\left[X_{n-1}^{(1)}\left(1-e^{-c_1\min\left(X_{n-1}^{(1)},X_{n-1}^{(2)},\dots,X_{n-1}^{(k)}\right)}\right)\right] \qquad (3.1)$$

The following lemma leads to $E\left[X_n^{(1)}X_{n-1}^{(1)}\right]$.

Lemma 3.1: Let $\left(X^{(1)}, X^{(2)}, \ldots, X^{(k)}\right)$ be a random vector with multivariate Marshall and Olkin exponential distribution. Let $U_i = X^{(i)},\ i = 1,2,\ldots,k \ \ and \ \ V = \min\left(X^{(i)}\right),\ i = 1,2,\ldots,k.$ Then the random vector $\left(U_i, V\right)$ has the survival function

$$P\left(U_1 > x_1, U_2 > x_2, \ldots, U_k > x_k, V > \min\left(x_1, x_2, \ldots, x_k\right)\right)$$
$$= e^{-\left(\lambda_i + \lambda_{ij}\right)\max\left(x_1, x_2, \ldots, x_k\right) - \lambda_{i+1} x_k}$$

and $P\left(U_i = V\right) = \dfrac{\lambda_i + \lambda_{i,i+j}}{\lambda}, \quad \forall i \ \ and \ \ j.$

Proof: From the definition of the random variables $U_i \ and \ V$, we have

$$P\left(U_1 > x_1, U_2 > x_2, \ldots, U_k > x_k, V > \min\left(x_1, x_2, \ldots, x_k\right)\right) = P\left(X^{(k-1)} > \max\left(x_1, x_2, \ldots, x_k\right), X^{(k)} > x_k\right)$$
$$= e^{-\lambda_1 \max\left(x_1, x_2, \ldots, x_k\right) - \lambda_2 x_k - \lambda_{12}\max\left(\max\left(x_1, x_2, \ldots, x_k\right), x_k\right)} = e^{-\left(\lambda_1 + \lambda_{12}\right)\max\left(x_1, x_2, \ldots, x_k\right) - \lambda_2 x_k}$$

and $P\left(U_1 = V\right) = P\left(X^{(1)} \le X^{(k)}\right) = \dfrac{\lambda_1 + \lambda_{12}}{\lambda}.$

Letting $U_1 = X_{n-1}^{(1)} \ \ and \ \ V = \min\left(X_{n-1}^{(1)}, X_{n-1}^{(2)}, \ldots, X_{n-1}^{(k)}\right)$ in (3.1) and using this lemma, we have

$$E\left[X_n^{(1)} X_{n-1}^{(1)}\right] = \frac{R}{c_1} E\left[U_1\left(1 - e^{-c_1 V}\right)\right]$$
$$= \frac{R}{c_1}\lambda_2\left(\lambda_1 + \lambda_{12}\right)\int_0^\infty \int_0^{u_1} u_1\left(1 - e^{-c_1 v}\right) e^{-\left(\lambda_1 + \lambda_{12}\right)u_1 - \lambda_2 v} dv du_1 + \frac{R}{c_1}\left(\lambda_1 + \lambda_{12}\right)\int_0^\infty U_1\left(1 - e^{-c_1 u_1}\right) e^{-\lambda u_1} du_1$$
$$= \frac{R+1}{R\left(\lambda_1 + \lambda_{12}\right)^2}.$$

From this result, $\operatorname{cov}\left(X_n^{(1)}, X_{n-1}^{(1)}\right) = \dfrac{1}{R\left(\lambda_1 + \lambda_{12}\right)^2}$, similarly, $\operatorname{cov}\left(X_n^{(1)}, X_{n-1}^{(2)}\right) = \dfrac{1}{R\left(\lambda_1 + \lambda_{12}\right)^2}$. Using the same argument, $\operatorname{cov}\left(X_n^{(2)}, X_{n-1}^{(1)}\right) = \dfrac{1}{R\left(\lambda_2 + \lambda_{12}\right)^2}$ and so on. Thus, the autocovariance matrix $\Gamma(1)$ is given as:

$$\Gamma(1) = \frac{1}{R}\begin{bmatrix} \frac{1}{\left(\lambda_1 + \lambda_{12}\right)^2} & \frac{1}{\left(\lambda_1 + \lambda_{12}\right)^2} & \cdots & \frac{1}{\left(\lambda_1 + \lambda_{12}\right)^2} \\ \frac{1}{\left(\lambda_2 + \lambda_{12}\right)^2} & \frac{1}{\left(\lambda_2 + \lambda_{12}\right)^2} & \cdots & \frac{1}{\left(\lambda_2 + \lambda_{12}\right)^2} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{1}{\left(\lambda_k + \lambda_{12}\right)^2} & \frac{1}{\left(\lambda_k + \lambda_{12}\right)^2} & \cdots & \frac{1}{\left(\lambda_k + \lambda_{12}\right)^2} \end{bmatrix}.$$

If we replace R by R_j in $\Gamma(1)$, we have antocovariance matrix $\Gamma(j)$ as

$$\Gamma(j)=\frac{1}{R_j}\begin{bmatrix} \frac{1}{(\lambda_1+\lambda_{12})^2} & \frac{1}{(\lambda_1+\lambda_{12})^2} & \cdots & \frac{1}{(\lambda_1+\lambda_{12})^2} \\ \frac{1}{(\lambda_2+\lambda_{12})^2} & \frac{1}{(\lambda_2+\lambda_{12})^2} & \cdots & \frac{1}{(\lambda_2+\lambda_{12})^2} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{1}{(\lambda_k+\lambda_{12})^2} & \frac{1}{(\lambda_k+\lambda_{12})^2} & \cdots & \frac{1}{(\lambda_k+\lambda_{12})^2} \end{bmatrix}.$$

Now, let us discuss the autocorrelation structure of multivariance minification process with multivariate Marshall and Olkin exponential distribution. We define the autocorrelation matrix by

$$H(j)=\begin{bmatrix} \operatorname{corr}\left(X_n^{(1)},X_{n-j}^{(1)}\right) & \operatorname{corr}\left(X_n^{(1)},X_{n-j}^{(2)}\right) & \cdots & \operatorname{corr}\left(X_n^{(1)},X_{n-j}^{(k)}\right) \\ \operatorname{corr}\left(X_n^{(2)},X_{n-j}^{(1)}\right) & \operatorname{corr}\left(X_n^{(2)},X_{n-j}^{(2)}\right) & \cdots & \operatorname{corr}\left(X_n^{(2)},X_{n-j}^{(k)}\right) \\ \cdots & \cdots & \cdots & \cdots \\ \operatorname{corr}\left(X_n^{(k)},X_{n-j}^{(1)}\right) & \operatorname{corr}\left(X_n^{(k)},X_{n-j}^{(2)}\right) & \cdots & \operatorname{corr}\left(X_n^{(k)},X_{n-j}^{(k)}\right) \end{bmatrix}$$

After simplification, we have

$$H(j)=\frac{1}{R_j}\begin{bmatrix} 1 & \frac{\lambda_2+\lambda_{12}}{\lambda_1+\lambda_{12}} & \cdots & \frac{\lambda_k+\lambda_{12}}{\lambda_1+\lambda_{12}} \\ \frac{\lambda_1+\lambda_{12}}{\lambda_2+\lambda_{12}} & 1 & \cdots & \frac{\lambda_k+\lambda_{12}}{\lambda_2+\lambda_{12}} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\lambda_1+\lambda_{12}}{\lambda_k+\lambda_{12}} & \frac{\lambda_2+\lambda_{12}}{\lambda_k+\lambda_{12}} & \cdots & 1 \end{bmatrix}$$

The autocorrelation matrix H(1), can be obtained by replacing R(j) with R(1). Hence,

$$H(1)=\frac{1}{R}\begin{bmatrix} 1 & \frac{\lambda_2+\lambda_{12}}{\lambda_1+\lambda_{12}} & \cdots & \frac{\lambda_k+\lambda_{12}}{\lambda_1+\lambda_{12}} \\ \frac{\lambda_1+\lambda_{12}}{\lambda_2+\lambda_{12}} & 1 & \cdots & \frac{\lambda_k+\lambda_{12}}{\lambda_2+\lambda_{12}} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\lambda_1+\lambda_{12}}{\lambda_k+\lambda_{12}} & \frac{\lambda_2+\lambda_{12}}{\lambda_k+\lambda_{12}} & \cdots & 1 \end{bmatrix}$$

$$Note\ \ that:\ 0\le corr\left(X_n^{(i)},X_{n-1}^{i}\right)\le 1,\quad i=1,2,3,\ldots,k.$$

4.0 Estimation of the Parameters

In this section, we will estimate the unknown parameters $R,\lambda_1,\lambda_2,\ldots,\lambda_k\ \ and\ \ \lambda_{12}$.

From [11], [13] and [15], we can see that multivariate minification process is ergodic and uniformly mixing. Now, let $\left\{\left(X_0^{(1)},X_0^{(2)},\ldots,X_0^{(k)}\right),\left(X_1^{(1)},X_1^{(2)},\ldots,X_1^{(k)}\right),\ldots,\left(X_{N-1}^{(1)},X_{N-1}^{(2)},\ldots,X_{N-1}^{(k)}\right)\right\}$ be a sample of size N. First, we estimate the parameter R. The estimate of R is given by

$\hat{R}_N = \max_{1 \le n \le N-1} \left\{ \frac{X_n^{(1)}}{\min\left(X_{n-1}^{(1)}, X_{n-1}^{(2)}, \dots, X_{n-1}^{(k)}\right)} \right\}$. This estimate is strongly consistent estimate and is not asymptotically normal. Similarly, we can use the estimate

$\hat{R}_N = \max_{1 \le n \le N-1} \left\{ \frac{X_n^{(2)}}{\min\left(X_{n-1}^{(1)}, X_{n-1}^{(2)}, \dots, X_{n-1}^{(k)}\right)} \right\}$ as another strongly consistent estimator of R. Following the same argument, we can have k- different estimators of R.

All the estimators can be used in practical situation, since the true values of the parameters can be obtained for small N.

For the estimation of the remaining parameters, we use the estimates

$$\overline{X}_N^{(1)} = \tfrac{1}{N}\sum_{i=0}^{N-1} X_i^{(1)}, \overline{X}_N^{(2)} = \tfrac{1}{N}\sum_{i=0}^{N-1} X_i^{(2)}, \dots, \overline{X}_N^{(k)} = \tfrac{1}{N}\sum_{i=0}^{N-1} X_i^{(k)}.$$

Since multivariate minification process with multivariate Marshall and Olkin exponential distribution is ergodic, it follows that the estimates $\overline{X}_N^{(1)}, \overline{X}_N^{(2)}, \dots, \overline{X}_N^{(k)}$ are strongly consistent estimates of the parameters $\frac{1}{\lambda_m + \lambda_{12}}, m = 1, 2, \dots k$ respectively.

Now, we consider the asymptotic properties of the estimated parameters. As the multivariate minification process is stationary and uniformly mixing and $\sum_{h=1}^{\infty} \psi^{1/n}(h) < \infty,$ it follows from [16] that $\sqrt{N}\begin{bmatrix} \overline{X}_N^{(1)} - \frac{1}{\lambda_1+\lambda_{12}} \\ \overline{X}_N^{(2)} - \frac{1}{\lambda_2+\lambda_{12}} \\ \dots\dots\dots\dots \\ \overline{X}_N^{(k)} - \frac{1}{\lambda_k+\lambda_{12}} \end{bmatrix}$ has asymptotically multivariate normal distribution as $N \to \infty.$

So, we can take the estimates of the parameters $\lambda_1, \lambda_2, \dots, \lambda_k$ *and* $\lambda_{12},$ as the solutions of the system of equations

$$\overline{X}_N^{(1)} = \tfrac{1}{\lambda_1+\lambda_{12}},$$

$$\overline{X}_N^{(2)} = \tfrac{1}{\lambda_2+\lambda_{12}},$$

$$\dots\dots\dots\dots$$

$$\overline{X}_N^{(k)} = \tfrac{1}{\lambda_k+\lambda_{12}}.$$

5.0 Conclusion

From the foregoing, we can conclude that the bivariate minification process with bivariate Marshall and Olkin exponential distribution as its stationary marginal distribution can be extended to multivariate case. Multivariate autocovariate and autocorrelation matrices can also be derived as an extension of bivariate ones earlier presented by [15]. Parameters that are unknown in this research work can be estimated. Its asymptotic properties can also be investigated.

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References

[1] Lewis, P.A.W and Mckenzie, E (1991): Minification Process and their transformations. J App Prob 28:45-57.
[2] Thomas, A and Jose, K.K (2004): Bivariate semi-Pareto minification processes, Metrica 59: 305-313.
[3] Tavares, L.V (1980): An exponential Markovian stationary process. J. Appl. Prob. 17: 1117-1120.
[4] Sim, C.H (1986): Simulation of Weibull and Gamma autoregressive stationaryprocess. Commun. Stat. Simul. Computat. 15: 1141-1146.
[5] Yeh, H.C, Arnold, B.C and Robertson, C.A (1988): Pareto process. J. Appl. Prob. 25: 291-301.
[6] Arnold B.C and Robertson, C.A (1989): Autoregressive logistic Processes. J. App Prob 26:524-531.
[7] Pillai, R.N (1991): Semi-Pareto processes. J. Appl. Prob. 28: 461-465.
[8] Pillai, R.N, Jose, K.K and Jayakumar, K (1995): Autoregressive minification processes and the class of distributions of universal geometric minima. Journal Ind- Stat- Assoc 33:53-61.
[9] Balakrishna, N (1998): Estimation for the semi-Pareto processes. Statist. Theory Meth. 27: 2307-2323.
[10] Balakrishna, N and Jayakumar, K (1997): Bivariate semi-Pareto distributions and processes, statistics paper 38: 149-165.
[11] Balakrishna, N and Jacob, T.M (2003): Parameter estimation in minification processes. Commun. Statist. Theory Meth. 32: 2139-2152.
[12] Thomas, A and Jose, K.K (2002): Multivariate minification processes, STARS internet. J 3: 1-9.
[13] Ristic, M.M (2006): Stationary bivariate minification processes, Statist. Probab. Lett.76:439-445.
[14] Marshall, A.W and Olkin, I (1967): A multivariate exponential distribution. J. Amer.Stat. Assoc. 62: 30-44.
[15] Miroslav, M.R, Biljana, C.P, Aleksandar, N and Miodrag, D (2008): A bivariate Marshall and Olkin exponential minification Process, Filomat 22:69-77.
[16] Billingsley, P (1968): Convergence of Probability Measures, Wiley, Now York.