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Deformation Prediction Using Linear Polynomial Functions

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Abstract

By Deformation, we mean change of shape of any structure from its original shape and by monitoring over time using Geodetic means, the change in shape, size and the overall structural dynamics behaviors of structure can be detected. Prediction is therefor based on the epochs measurement obtained during monitoring, the life, failure and when the structure is at unsafe condition may be forecast. The main purpose of structural deformation monitoring scheme and analysis is to detect any significant movements of the structure.The knowledge of behaviour of Tank Structure under uniaxial/biaxial tensile loads is necessary to predict the changes in the geometry of the structure.. The aim of this study is to predict the Deformation experience by the structure under continuous loading with data obtained in four epochs of measurement using linear polynomial technique. The predictions are compared with measured data reported in literature and the results are discussed. The computational aspects of implementation of the model are also discussed briefly.

Keywords: Linear Polynomial, Structural Deformation, Prediction.

1.0 Introduction

The security of civil engineering structures demands a periodical monitoring. In many civil structures like bridges, vertical oil storage tanks, tunnels and dams, the deformations are the most relevant parameters to be monitored. Monitoring the structural deformation and dynamic response to the large variety of external loadings has a great importance for maintaining structures safety and economical design of man-made structures [1].

Strain gauge, accelerometer, tilt meter, etc. are traditional tools used in measuring structure displacement, rotation and together with temperature, wind speed and direction allowing a comprehensive investigation of structure dynamics behaviors. These tools must be installed, maintained, and frequently recalibrated to produce reliable results. The collected data from these tools need to be interpreted to obtain direct geometric results which in many cases is very complicated and out of the control of the general structural engineers, therefore a flexible surveying technique is needed to overcome these obstacles, and make the process of measurements easier and more accurate [2].

This paper is based on crude oil deformation monitoring carried out in Forcados Terminal. Forcados is located in Burutu Local Government Area of Delta State of Nigeria. At the Terminal, there are eighteen (18) crude oil storage tanks of 76.2m nominal diameter and height at 22m. There are sixteen (16) monitoring points called (STUD) attached to the base of each Tank. In this paper, Tank number 6 is used as a case study. This measurement was carried out between year 2000 and 2008 for all the Tanks at the Terminal. It is important to note that no observation was carried out in year 2001, 2002, 2005, 2006 and 2007 because of the unrest in the Niger delta of Nigeria.

Circular liquid storage tanks are commonly used in industries for storing chemicals, petroleum products, etc. and for storing water in public water distribution systems. Such tanks require periodic surveys to monitor long-term movements and settlements or short-term deflections and deformations [3].

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2.0 Prediction of the deformation values of circular oil storage tanks

One of the main topics in Engineering Geodesy is monitoring structural deformation and the prediction of the deformation values. Time of observations for the purpose of structural deformation and the frequency of cycles can vary from a few hours, days to several months or even years. It is important that we not only determine the changes in the structure but also these changes have statistics on which to make predictions for the future, which will help to prevent disaster [4].
Deformation structures can be fully determined by the movement of points which are measured on the construction. Let the vector position of point P in three-dimensional coordinate system (X, Y, Z) before and after deformation be equal to r_p and $r^{/}_p$ respectively. Then $r^{/}_p$ may be expressed as:

$$r_P^{/} = f(x_p, y_p, z_p, t), \quad (1)$$

where t is the time variation between two cycles (epochs) of observations.

From equation (1) the displacement of the observed point depends on their initial position and time. The displacement vector dp at the point P is defined as:

$$d_P = r_P^{/} - r_P = f((x_p - x_0),(y_p - y_0),(z_p - z_0),(t - t_0)) \quad (2)$$

In this work we presented some functions to predict the deformation values of monitoring points on the outer surface of an oil storage tank.

3.0 Linear polynomial functions

Polynomial function can be used to determine the predicted values of deformation of monitoring points of oil tanks. The linear mathematical model for three-dimensional coordinate system is represented as follows [5]:

$$\begin{bmatrix} X_J^i \\ Y_J^i \\ Z_J^i \end{bmatrix} = \begin{bmatrix} X_J^0 \\ Y_J^0 \\ Z_J^0 \end{bmatrix} + a\ (t_i - t_0) + b, \quad (3)$$

Where $X^i_{J_J}, Y^i_{J_J}, Z^i_{J_J}$ – the coordinates of monitoring point J at time t_i; $X^0_{J_J}, Y^0_{J_J}, Z^0_{J_J}$ - the coordinates of the same point at initial time t_0, a, b – the coefficients; ***j= 1, …, n***, and n - the number of monitoring points; ***i = 1, …, m***, and m – the number of epochs of observations.

It is important to note that the prediction function by this model is applied to each monitoring point individual. Equation (3.0) can be written in the form [5]:

$$\begin{bmatrix} \Delta X_J^i \\ \Delta Y_J^i \\ \Delta Z_J^i \end{bmatrix} = a\ \Delta t_i + b, \quad (4.0)$$

where $\Delta X^i_{J_J}, \Delta Y^i_{J_J}, \Delta Z^i_{J_J}$ is the resulted displacement of point, Δt_i is the difference of time between the epochs of observations.

For determining the coefficients a, b for each monitoring point, least square theory must be used. The observation least square has the following matrix form:

[A] [X] + [L] = [V] (5)

where

Matrix [A] is of the form $A_{(3m,2)}$

Matrix [X] is of the form $X_{(2,1)}$

Matrix [L] is of the form $L_{(3m,1)}$
Matrix [V] is of the form $V_{(3m,1)}$
m is the number of epochs of observations.

In this case the solution can be found without assuming the approximated values of unknowns ***a*** and ***b***. Matrix **A** will be determined by differentiation the equation with respect to parameters a, b and has the form [6]:

$$A = \begin{bmatrix} \Delta t_1 & 1 \\ \Delta t_1 & 1 \\ \Delta t_1 & 1 \\ \Delta t_2 & 1 \\ \Delta t_2 & 1 \\ \Delta t_2 & 1 \\ \Delta t_3 & 1 \\ \Delta t_3 & 1 \\ \Delta t_3 & 1 \\ \dots & \\ \Delta t_m & 1 \end{bmatrix} \qquad L = \begin{bmatrix} \Delta X_1 \\ \Delta Y_1 \\ \Delta Z_1 \\ \Delta X_2 \\ \Delta Y_2 \\ \Delta Z_2 \\ \Delta X_3 \\ \Delta Y_3 \\ \Delta Z_3 \\ \dots\dots \\ \Delta Z_m \end{bmatrix} \tag{6}$$

Then the coefficients ***a*** and ***b*** can be directly calculated by using least square as follows:

$$\begin{bmatrix} a \\ b \end{bmatrix} = (A^T \ A)^{-1} \ (A^T \ L) \ . \tag{7}$$

The accuracy of coefficients can be determined form the variance-covariance matrix as follows [7]:

$$\begin{bmatrix} m_a^2 & m_{ab} \\ m_{ba} & m_b^2 \end{bmatrix} = (A^T \ A)^{-1}. \tag{8}$$

Note that this linear polynomial model can be used for one or two dimensional coordinates also. In the case of one - dimensional settlement of foundation resulting from leveling measurements, equation (4) becomes:

$$\Delta h_J^i = a \ \Delta t_J^i + b, \tag{9}$$

By method of using least square, the coefficients a and b can be determined. In this case matrix A will have dimension (m, 2).
On the other hand, in the case of two - dimensional coordinates (X, Y), equation (4) will have the form [7]:

$$\begin{bmatrix} \Delta X_J^i \\ \Delta Y_J^i \end{bmatrix} = a \ \Delta t_i + b, \tag{10}$$

In this case matrix A will have dimension (2 m, 2) in the form:

$$A=\begin{bmatrix} \Delta t_1 & 1 \\ \Delta t_1 & 1 \\ \\ \Delta t_2 & 1 \\ \Delta t_2 & 1 \\ \\ \Delta t_3 & 1 \\ \Delta t_3 & \\ \\ .... & \\ \Delta t_m & 1 \end{bmatrix} \qquad (11)$$

Equation (7) is 3D ca se. Using equation (7), the value of the unknown can be computed using Mathcad and the result is presented thus;

$$\mathbf{x1} = \begin{pmatrix} -1.53846\times10^{-4} & -6.15385\times10^{-5} & 2.15385\times10^{-4} \\ 1.11538\times10^{-3} & 6.46154\times10^{-4} & -7.61538\times10^{-4} \end{pmatrix}\mathbf{m}^3$$

Also from equation (8), the accuracy of the prediction is presented thus:

$$\mathbf{k} = \begin{pmatrix} 0.07385 & -0.37538 \\ -0.37538 & 2.24154 \end{pmatrix}$$

Table - 1 is the graph of prediction for tank 6 stud1 using linear polynomial function.

Table - 1: Velocity value of Deformation

Monitoring point	Velocity, mm/year Vertical values, mm/year		
	t= 3 years	t= 4.25 year	t= 8 years
	from 5/2000	from 5/2000	from 5/2000
	to	to 8/2004	to
	5/2003		May-08
STUD1	*3.84*	*3.68*	*2.87*
STUD9	*5.82*	*7.08*	*4.43*
STUD16	*4.67*	*4.75*	*3.69*
STUD8	*4.14*	*4.6*	*3.52*
STUD2	*3.69*	*3.99*	*3.18*
STUD10	*5.6*	*6.97*	*4.46*
STUD4	*0*	*0.64*	*1.24*
STUD12	*5.44*	*7.14*	*4.41*
STUD3	*0*	*0.76*	*1.32*
STUD11	*5.6*	*7.07*	*4.47*
STUD5	*1.33*	*2.35*	*2.07*
STUD13	*4.97*	*6.6*	*4.26*
STUD7	*1.3*	*2.35*	*2.2*
STUD15	*3.46*	*5.84*	*3.88*
STUD6	*1.07*	*2.19*	*2.04*
STUD14	*4.1*	*6.42*	*4.15*

Note: Studs are monitoring points around the Tank.

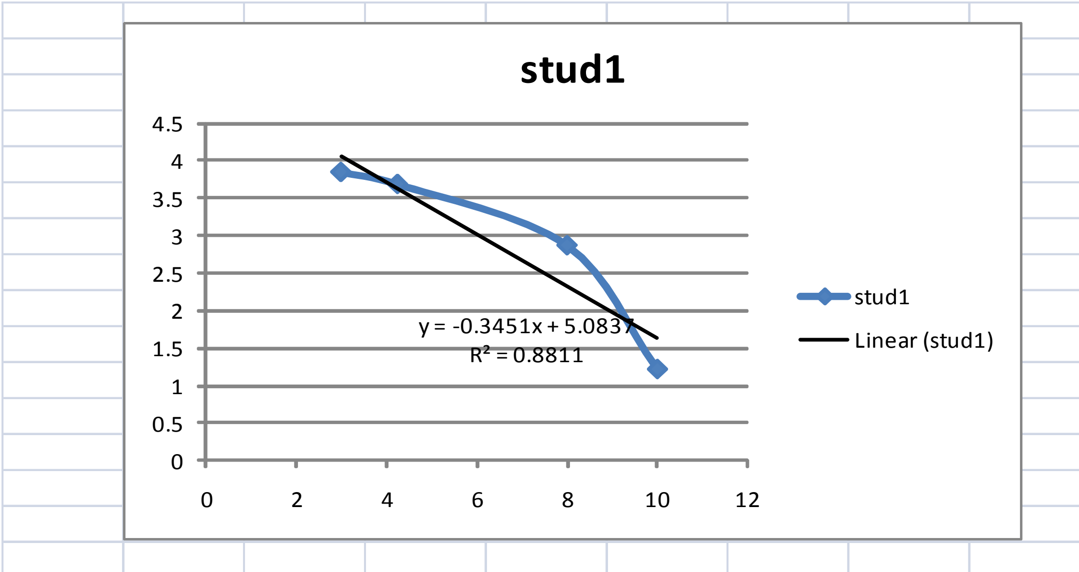

Figure 1.0 - shows plot of Time against velocity for Stud 1 tank 6 predictions

Note: Vertical axis is the Time, while horizontal axis is the Velocity

5.0 Analysis Of Results And Conclusion

Table 1 vertical deformation values while fig 1.0 is the plot of time against velocity for monitoring point stud 1 for tank № 6. From the above, the predicted deformation graph and the observed value intersected at two points with time 3.6 with a velocity of 4mm/yr and time 1.9 with velocity of 9mm/yr respectively.

At time equal to 3.6 with velocity of 2.5mm/yr and time equal to 2.9 with velocity of 8mm/yr, there was a uniform slope which gives an indication that the value of deformation is uniform. Also at time equal to 2.9 with velocity equal to 8mm/yr and time equal to 1.2 with velocity of 10mm/yr, there was sharp change in displacement value which we believe that the structure is at danger stage.

A further projection of the prediction graph and the observed value, there will be a point when both graphs will be further. This will be revealed with more observations and hence prediction.

The results obtained in this study may however be acceptable to the structural Engineer depending on the tank specifications and its properties at the design stage.

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