

Scattering Phase Functions of Constituents of Mineral Dust Aerosols

*Sa'id R. S. and Kashimbila M. M.

**Department of Physics,
 Bayero University, PMB 3011, Kano, Nigeria**

Abstract

In this paper, a Mie-based numerical computation and visualization program has been used to compute the phase functions of typical constituents of mineral dust aerosols of Saharan origin in the visible range of the electromagnetic spectrum of wavelength 550nm. Computations were done for scattering angles ranging from 0 to 180 degrees and three radial sizes of 0.25, 0.5 and 1.0 microns. The results show that there is increasing asymmetry and complexity of the phase functions with increasing radial sizes for each of the selected constituents: Illite, Kaolinite, Montmorillonite, Hematite, Calcite and Quartz. The behaviour of these constituents as observed by their phase functions provide information on the optical properties and radiative effects of the mineral dust types and is therefore useful on regional and global scales in assessing radiative impacts of dust outbreak events.

1.0 Introduction

Generally, when light strikes a particle with an index of refraction different from the medium in which it is embedded, the light is refracted. The angle at which the light is bent is a function of the size and shape of the particle as well as the wavelength of the incident light and the incident angle. This results in each particle having a different scattering profile. The scattering profile is called the phase function [1]. The phase function is only a description of the distribution of the scattering by the particles.

2.0 The Scattering Phase Functions

The phase function is only a description of the distribution of the scattering by the particles [2]. The phase function satisfies the normalization condition

$$\frac{1}{4} \int_{4\pi} p(\hat{\Omega}', \hat{\Omega}) d\Omega' = 1 \tag{1}$$

A physical meaning to the scattering phase function is to regard $\frac{1}{4} p(\hat{\Omega}', \hat{\Omega})$ as a probability density. If we assume a photon arrives from a direction $\hat{\Omega}'$ and is scattered, then $(\hat{\Omega}', \hat{\Omega})$, the probability of its new direction is guaranteed to fall within an available 4π steradians of solid angle [3]. The scattering phase function then depends only on the angle θ between $\hat{\Omega}$ and $\hat{\Omega}'$, where

$$\cos \theta \equiv \hat{\Omega}' \cdot \hat{\Omega} \tag{2}$$

One can therefore replace $p(\hat{\Omega}', \hat{\Omega})$ with

$$p(\hat{\Omega}' \cdot \hat{\Omega}) \equiv p(\cos \theta) \tag{3}$$

The normalization condition then becomes:

*Corresponding author: Sa'id R. S., E-mail: : saidur2776@buk.edu.ng; rssaid2001@yahoo.co.uk, Tel. +2348033976393

$$\frac{1}{4} \int_0^{2\pi} \int_0^\pi p(\cos \theta) \sin \theta d\theta d\phi = 1 \tag{4}$$

The simplified notation of (4) commonly used in literature [4-8] is

$$\frac{1}{2} \int_{-1}^1 p(\cos \theta) d \cos \theta = 1 \tag{5}$$

The simplest scattering phase function is for isotropic scattering [9]

$$p(\cos \theta) = 1 \tag{6}$$

All directions in this case are equally likely for a scattered photon. The new direction of the photon cannot be predicted from the direction it was originally traveling before it was scattered. Van de Hulst [10] gives the scattering phase function as

$$p = \frac{1}{C_{sca}} \frac{dC_{sca}}{d\Omega} \tag{7}$$

where C_{sca} is the scattering cross-section. The phase function is an element of a matrix called the phase scattering matrix, which is a set of 4 x 4 matrix

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix} \tag{8}$$

The P_{11} component of the Phase Scattering Matrix is the Scattering Phase Function [11]. In terms of the Scattering

Functions S_1 and S_2 the phase function is written as:

$$P_{11}(\theta) = \frac{2\pi}{k^2 \sigma_{sca}} \left[|S_1(\theta)|^2 + |S_2(\theta)|^2 \right] \tag{9}$$

The scattering functions in equation (9), also known as the scattering amplitudes are given by

$$S_1(\cos \theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (a_n \pi_n + b_n \tau_n) \tag{10}$$

$$S_2(\cos \theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (a_n \tau_n + b_n \pi_n) \tag{11}$$

where π_n and τ_n are the angle dependent functions and the expansion of the incident plane wave in Legendre Polynomials yields the angle dependent functions. They describe the angular scattering patterns of the spherical harmonics used to describe scattering amplitudes. The angle dependent functions are given by

$$\pi_n(\cos \theta) = \frac{1}{\sin \theta} P_n^l(\cos \theta) \tag{12}$$

$$\tau_n(\cos \theta) = \frac{d}{d\theta} P_n^l(\cos \theta) \tag{13}$$

where the P_n^l are the associated Legendre polynomials of the first kind. The angle dependent functions are usually computed by upward recurrence [12] from the recurrence relations

$$\pi_n = \frac{2n-1}{n-1} \cos \theta \pi_{n-1} - \frac{n}{n-1} \pi_{n-2} \tag{14}$$

$$\tau_n = n \cos \theta \pi_n - (n+1) \pi_{n-1} \tag{15}$$

The angle dependent functions are alternatively even and odd functions of $\cos \theta$:

$$\begin{aligned} \pi_n(-\cos \theta) &= (-1)^{n-1} \pi_n \cos \theta \\ \tau_n(-\cos \theta) &= (-1)^n \tau_n \cos \theta \end{aligned} \tag{16}$$

Following the formulation in Deirmendjian [13],

$$\pi_0 = 0, \pi_1 = 1, \pi_2 = 3 \cos \theta \tag{17}$$

$$\tau_0 = 0, \tau_1 = \cos \theta, \tau_2 = 3 \cos(2\theta) \tag{18}$$

the scattering phase functions are expanded in terms of the Legendre polynomials in the form

$$P_{11}(\cos \theta) = \sum_{l=0}^N \beta_l P_l(\cos \theta) \tag{19}$$

where θ is the scattering angle and β_l is determined from the orthogonal properties of Legendre polynomials in the form

$$\beta_l = \frac{2l+1}{2} \int_{-1}^{+1} P_{11}(\cos \theta) P_l(\cos \theta) d \cos \theta \tag{20}$$

With the normalization condition of (1) and if N in (19) is made large enough to ensure accuracy and convergence, then

$$\begin{aligned} \beta_0 &= 1 \text{ and } \beta_1 = 3g \\ g &= \frac{\beta_1}{3} \end{aligned} \tag{21}$$

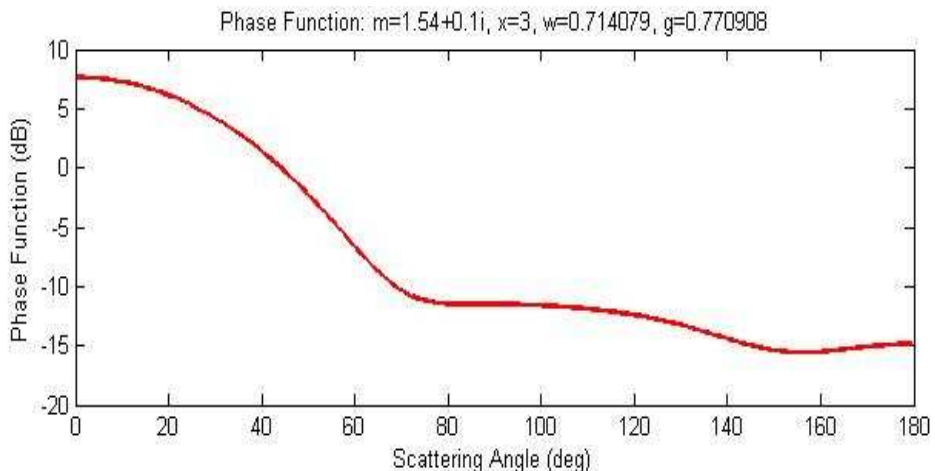
Thus, the asymmetry factor represented by g in (21) can be deduced from the phase function.

3.0 Results

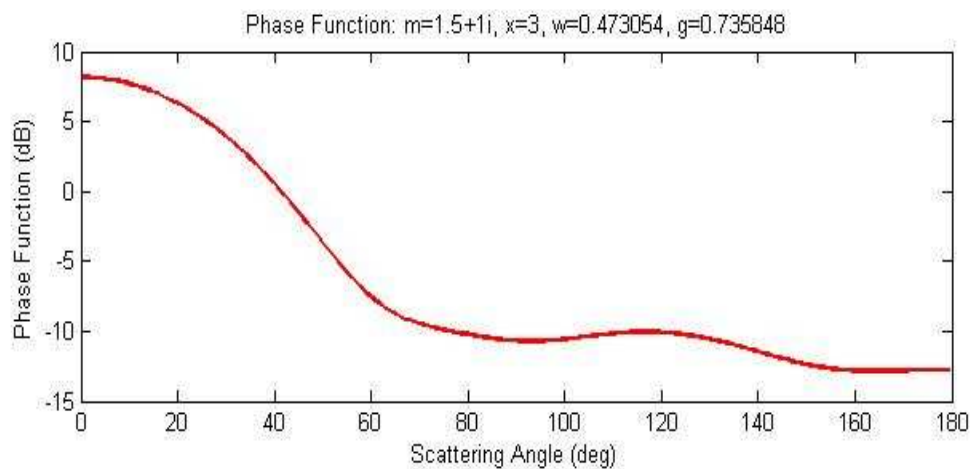
3.1 The phase function plots

The phase functions of the constituents of the dust aerosols are shown in Figures 1.5-1.7. The plots have specific complex refractive indices for the specific constituents of Calcite, Hematite, Illite, Kaolinite, Montmorillonite and Quartz.

CALCITE



HEMATITE



ILLITE

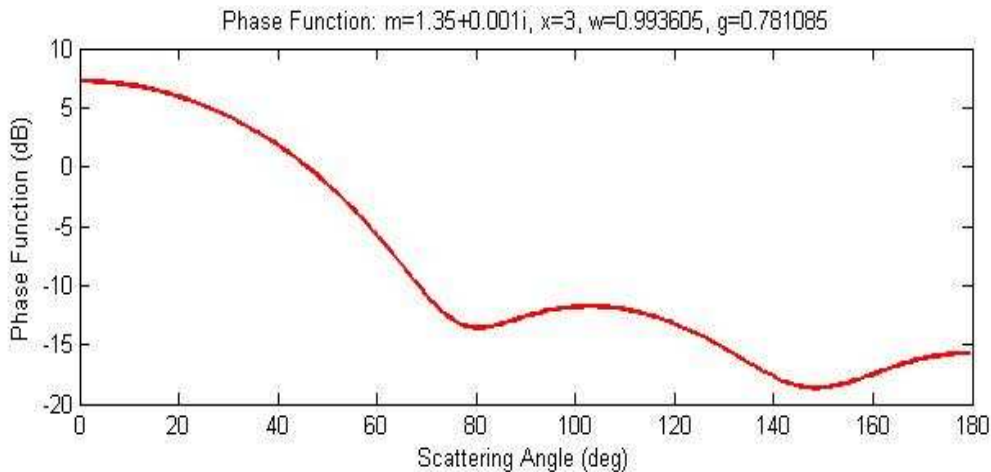


Figure 1.5a: Phase Function plots of Calcite, Hematite and Illite; radial sizes $0.25 \mu m$
Journal of the Nigerian Association of Mathematical Physics Volume 20 (March, 2012), 331 – 342

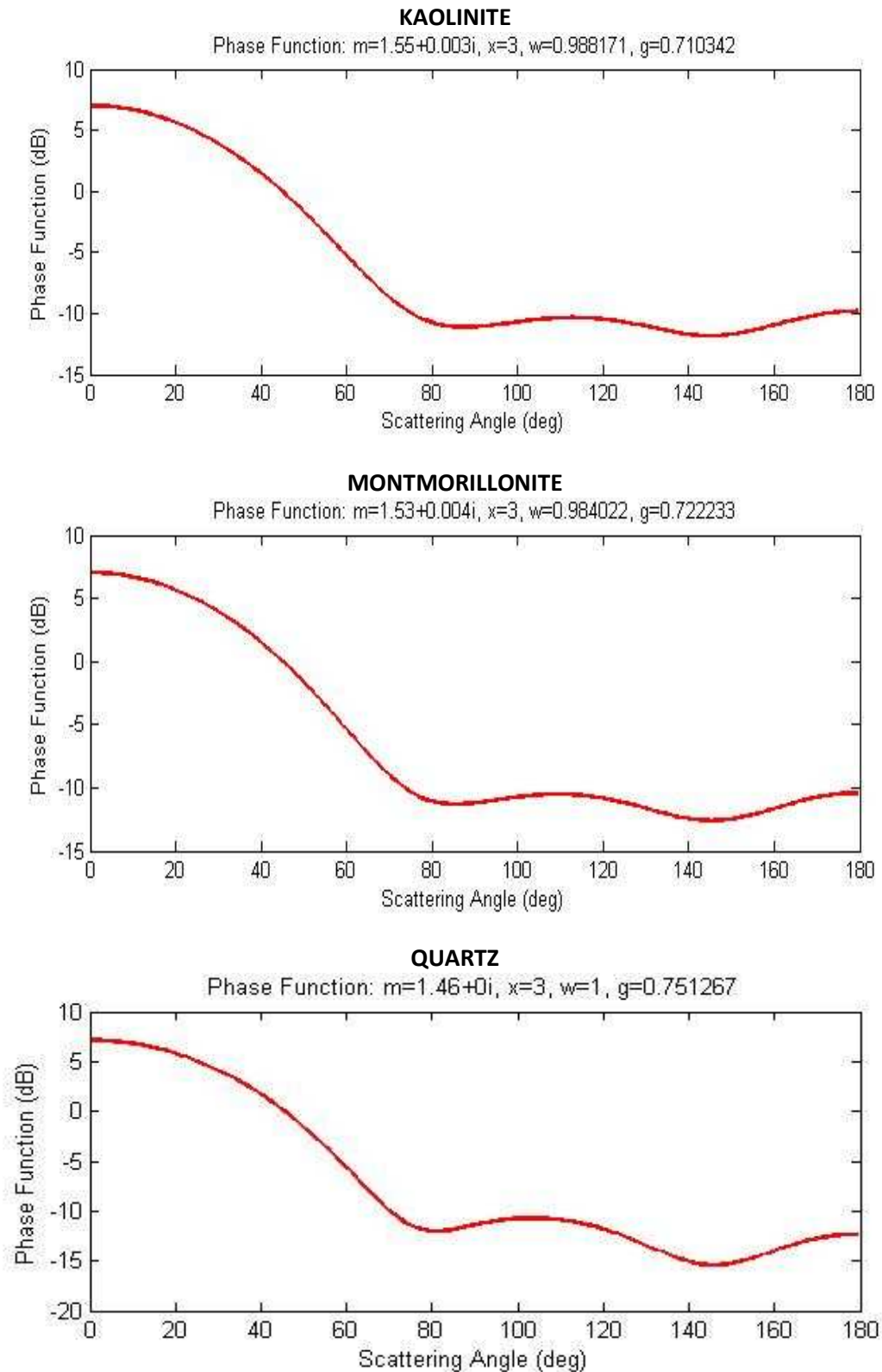
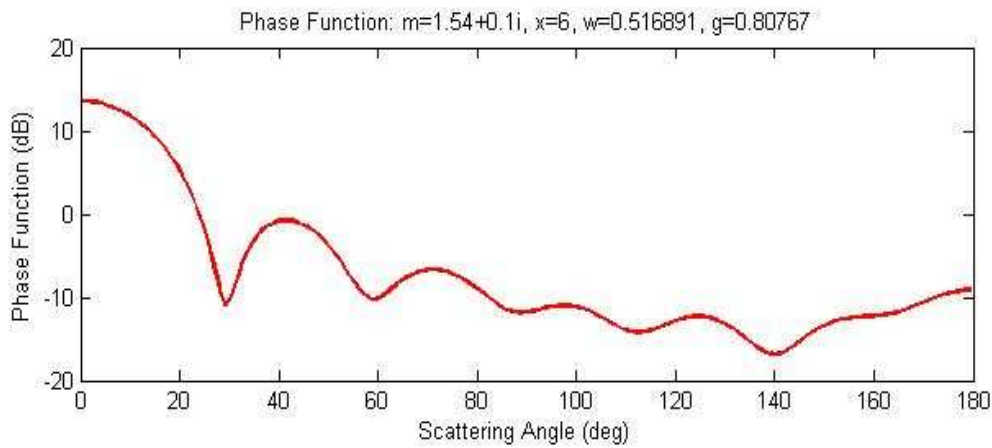
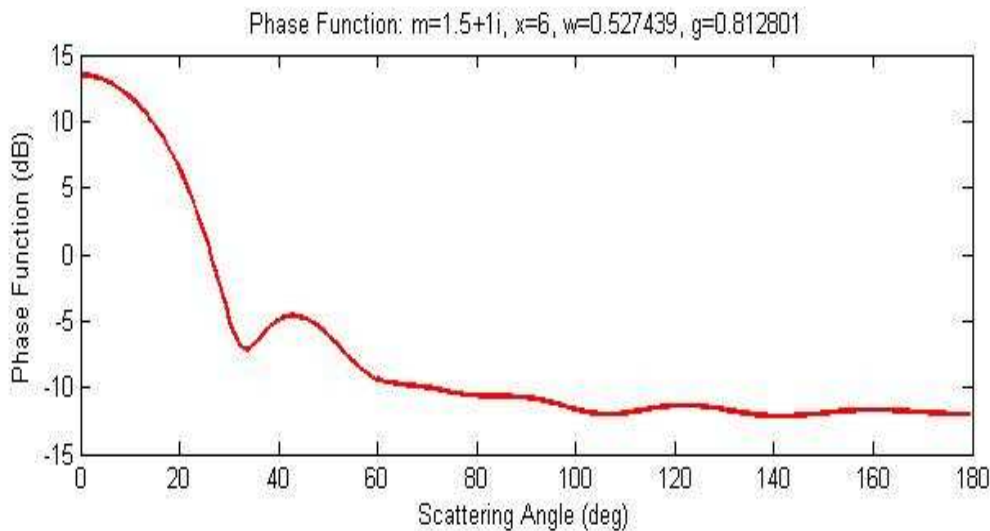


Figure 1.5b: Phase Function plots of Calcite, Hematite and Illite; radial sizes $0.25 \mu m$

CALCITE



HEMATITE



ILLITE

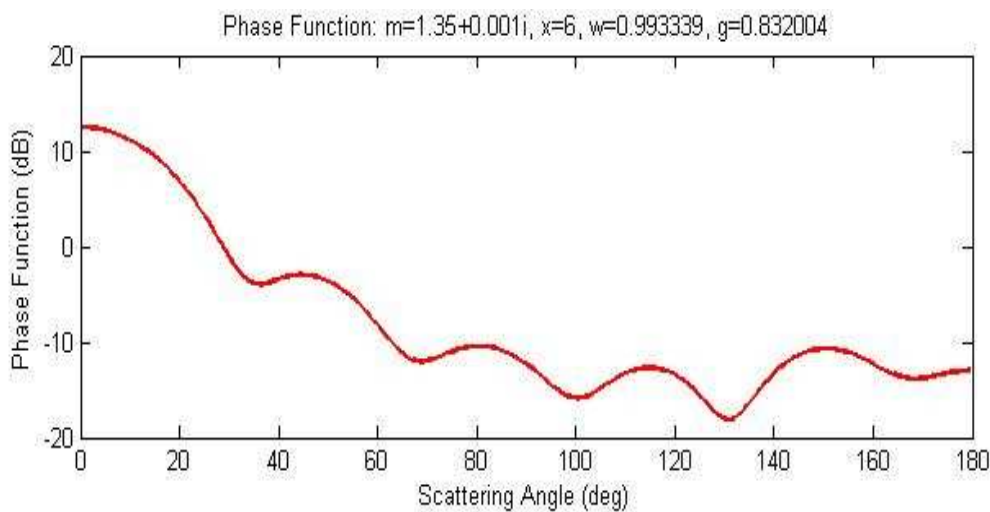


Figure 1.6a: Phase Function plots of Calcite, Hematite and Illite for radial sizes $0.5 \mu m$

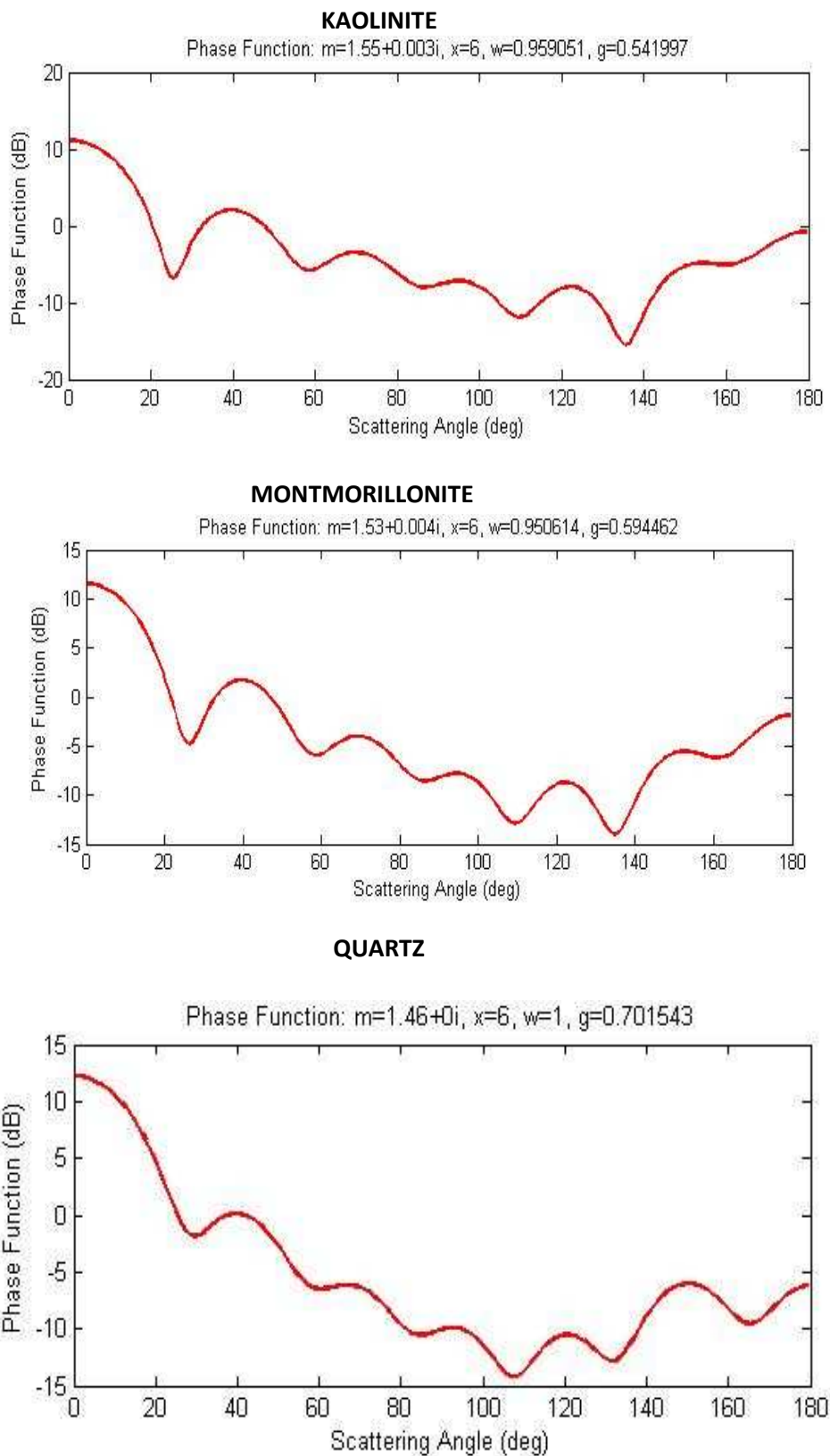


Figure 1.6b: Phase Function plots of Kaolinite, Montmorillonite and Quartz for radial sizes $0.5 \mu m$

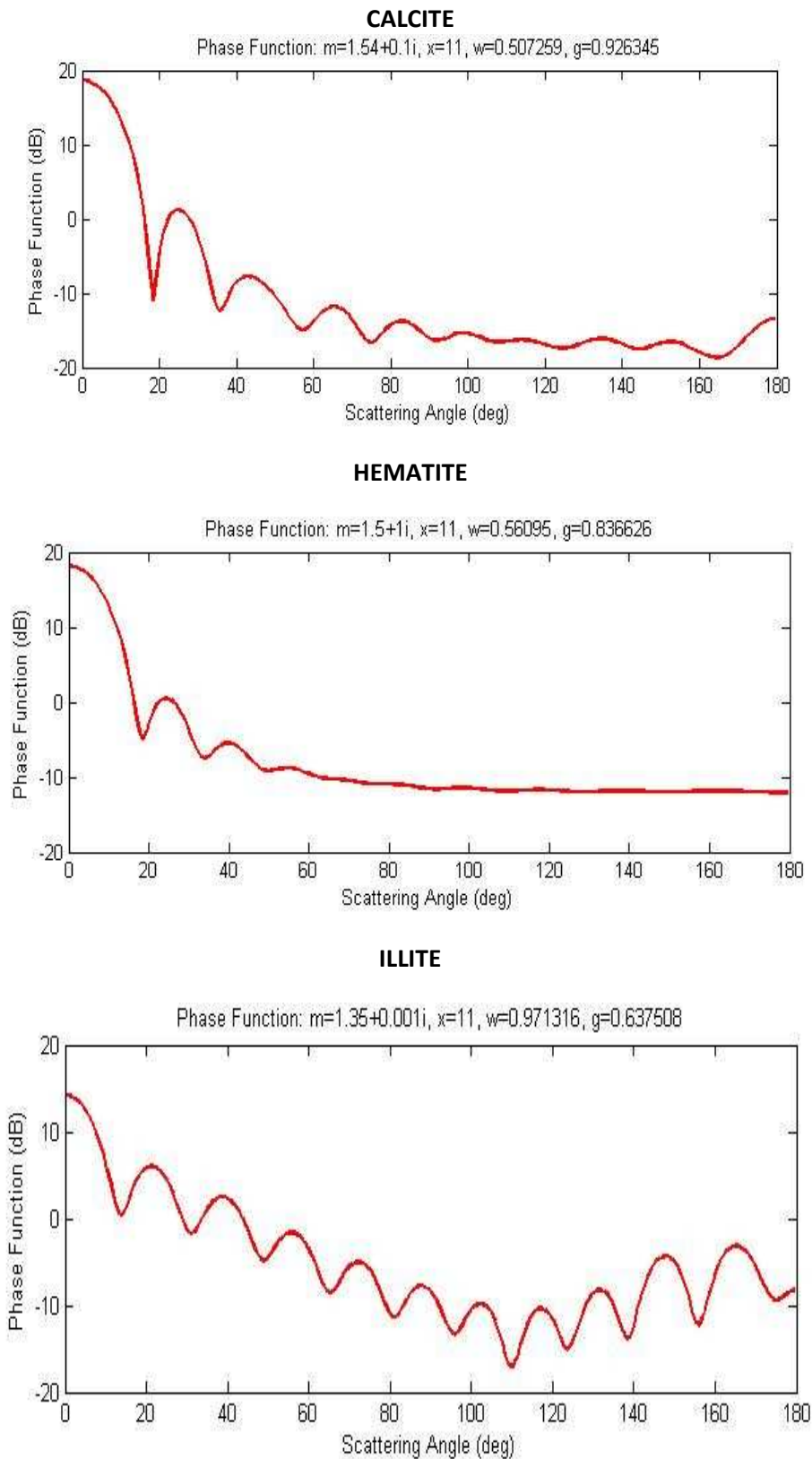
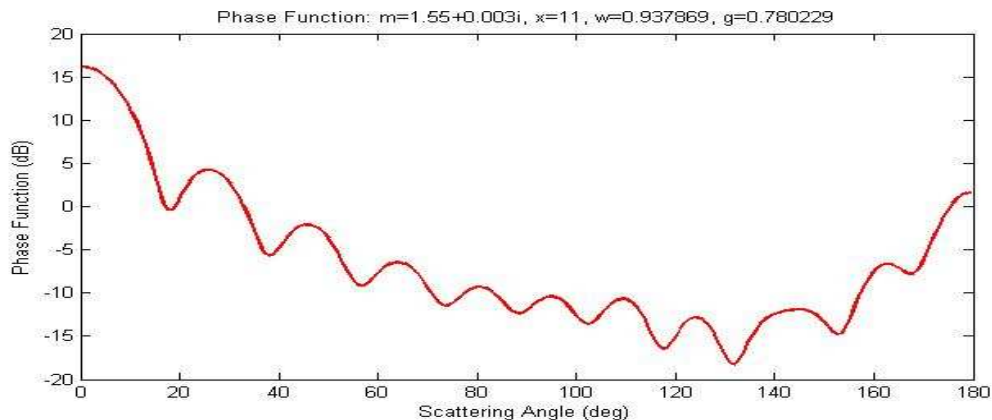
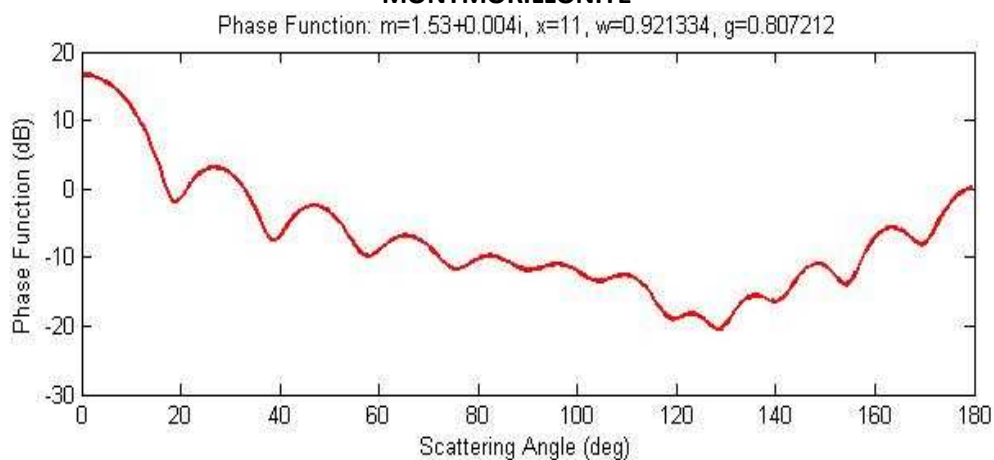


Figure 1.7a: Phase Function plots of Calcite, Hematite and Illite for; radial sizes $1.0 \mu m$

KAOLINITE



MONTMORILLONITE



QUARTZ

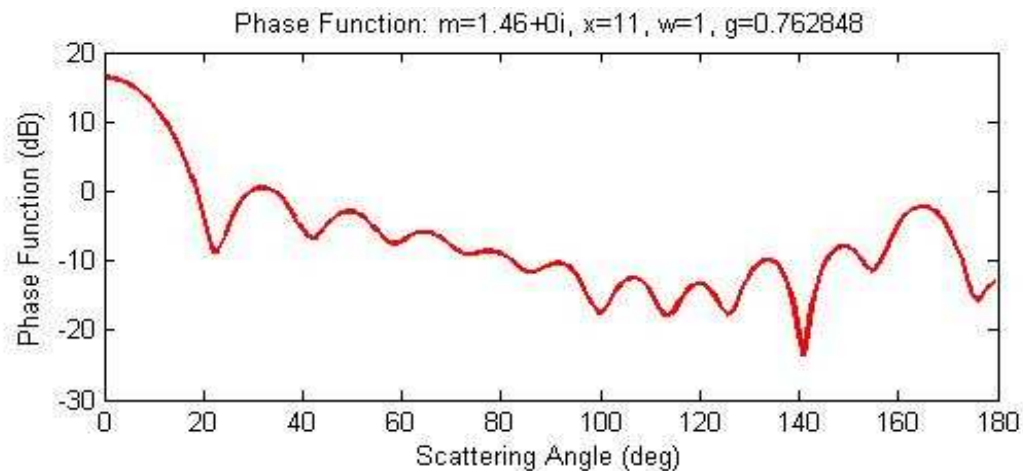


Figure 1.7b: Phase Function plots of Kaolinite, Montmorillonite and Quartz for; radial sizes $1.0 \mu m$

3.2 Discussion Of Results

The plots of the phase functions are in agreement with Mie-derived phase functions for various values of size parameters as recorded by Petty [14]. There is also a close agreement among the phase functions for the different size parameter. Gerber and Hindman [15] report that this suggests that the particle radius is the major contribution to the shape of the phase function. As the radial size increases, the phase functions in the forward direction (angles less than 90 degrees) have large amplitudes than in the backward directions (angles greater than 90 degrees). We notice for the radial size 0.25 micrometer (Figure 1.7), that there is a broad lobe of enhanced scattering between scattering angles (0-40 degrees) and a large variation of amplitude for scattering angles (120-180 degrees).

The phase functions plots show an increasing asymmetry and complexity of the phase functions with increasing radial size. There is more symmetry in the forward than backward directions for small size parameters, indicating that that small sized aerosols scatter radiation in the forward direction more efficiently. The phase function plots also yield values of single scattering albedo and asymmetry factor as shown at the top of each plot. Typical average value of Single Scattering Albedo for constituents of mineral dust aerosols with radial sizes of 0.25 micrometers is 0.85 in the visible wavelength of 550nm and the asymmetry parameter is 0.74. The single scattering albedo for radial size 0.5 micrometers and 1.0 micrometers is 0.81, while the average asymmetry factor is 0.71 and 0.79 respectively. Single scattering albedo for hematite increases steadily with increasing radial size, starting with a value of 0.47 for radial size 0.25 micrometers, 0.52 for 0.5 micrometers and 0.56 for 1.0 micrometers.

4.0 Conclusion

The three radial sizes computed: 0.25, 0.5 and 1.0 microns, show that at angles less than 90 degrees, scattering is more enhanced in the direction of propagation of the incident radiation. Averaged single scattering albedo for the three radial sizes as indicated at the top of each plot are 0.71, 0.74 and 0.85 for the constituents of the dust aerosol except hematite (0.47, 0.52 and 0.56). Hematite shows a flat out pattern after scattering angle of about 50 degrees. Since hematite is an absorbing constituent of the dust aerosol, this pattern shows that the absorption is more at scattering angles of more than 50 degrees at visible wavelengths.

In conclusion, one can infer that dust aerosols except for one with very high hematite content, will scatter more than absorb incident radiation. The inference of this for regional and climate impact is that with more radiation scattered by dust aerosols, dust aerosols are likely to result in negative radiative forcing (cooling) of the atmosphere. However, dust aerosols high in hematite, will exert positive radiative forcing (heating) of the atmosphere.

References

- [1] Prah, S. A. (1988): Light Transport in Tissue, PhD Thesis, University of Texas, Austin, USA.
- [2] Lenoble, J (ed.) (1985) Radiative Transfer in Scattering and Absorbing Atmospheres: Standard Computational Procedures. A. Deepak Publishing. Hampton, Virginia, USA.
- [3] Bohren, G.F. and Huffman D. R. (2004): Absorption and Scattering by Small Particles, John Wiley and Sons. (Inc., Wiley-VCH Verlag GmbH & co. Germany. ISBN13: 978-0-471-29340-8).
- [4] Kong, J.A. (1990): Electromagnetic Wave Theory, New York 1st Ed. (1986). BEWI: TEA 150, 2nd Ed. ETH Library.
- [5] McConnell, C. L., Highwood, E.J. Coe, H. Formenti, P. Anderson, B. Osborne, S. Nava, S. Desboefs, K. Chen, G. and Harrison, M.A.J. (2008), Seasonal Variations of the Physical and Optical Characteristics of Saharan Dust: Results from the Dust Outflow and Deposition to the Ocean (DODO) Experiment, *J. Geophys. Res.*, *113*, D14S05, doi:10.1029/2007JD009606.
- [6] Sokolik, I. N. Andronova, A. and Johnson, T. C. (1993): Complex Refractive Index of Atmospheric Dust Aerosols. *Atmos. Environ.*, **27A**, 2495 - 2502.
- [7] Tegen, I., Lacis, A.A and Fung, I. (1996): The influence on Climate Forcing of Mineral Aerosols from Disturbed Soils. *Nature*, **380**, 419–421.
- [8] Bayvel, L.P. and Jones, A. R. (1981): Electromagnetic Scattering and its Applications, published by: Applied Sciences Publishers Ltd. USA & UK. ISBN: 0-85334-955-X
- [9] Chandrashekar, S. (1960). Radiative Transfer. Dover Press. New York. Pp. 27-55.
- [10] Van de Hulst, H.C. (1957). Light Scattering by Small Particles, Dover Publications, New York
- [11] McLinden, C. A. (1998). Observations of Atmospheric Composition from NASA ER-2 Spectro-Radiometer Measurements: Ph.D. Thesis. New York University Toronto, Canada.
- [12] Abramowitz, M. and Stegun, I.A. (eds.), (1965): Handbook of Mathematical Functions. Dover Publication, New York, NY.

- [13] Diermendjian, D. (1969) Electromagnetic Scattering on Spherical Polydispersions. American Elsevier Publishing Company Inc. New York.
- [14] Petty, G.W. (2006). A First Course in Atmospheric Radiation. 2nd Edition. Sundog. Pp.459.
- [15] Gerber, H.E. and Hindman, E.E (eds.) (1982). Light Absorption by Aerosol Particles. Spectrum Press. Science and technology Corporation. Hampton, Virginia. Pp. 1 – 408.