Journal of the Nigerian Association of Mathematical Physics
Volume **20 (March, 2012), pp 321 – 330**

Evaluating Thermal Properties of Rock

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Abstract

Application of thermography in material identification and characterization has been applied in this work. Nigeria geological set up comprises broadly sedimentary formation and crystalline basement complex, which occur more or less in equal proportion all over the country. The models generated in this work can be used to identify/characterise rock types. The coefficients of the generalized model give the thermal properties of each rock type. The chi-square test showed that there was no significant difference (p>0.05) between the expected and observed data for all the models. The model developed in this work enabled us to use simulation prediction as the basis for rock identification, which otherwise would be difficult or impossible to perform.

Keyword: Thermography, Quartz, Graphite, Limestone, Iron Ore.

1.0 Introduction

Nigeria lies very close to the equator (hot country) North eastern Africa between latitude 4^0 N and 14^0 N and longitude 5^0 E and 12^0 E. The country is located at the Northern end of Eastern branch of east Africa rift system [1]. Nigeria geological set up comprises broadly sedimentary formation and crystalline basement complex, which occur more or less in equal proportion all over the country. The sediment is mainly Upper Cretaceous to recent in age while the basement complex rocks are thought to be Precambrian.

2.0 Least Square Approximation

The basic idea of Least Square Approximation is to fit a polynomial function P(x) to a set of data (x,y) having a theoretical solution

$$y = f(x). \qquad (1)$$

Many problems arise in Engineering and Science where the dependent variable is a function of two or more independent variables, for example,

$$z = f(x,y) \qquad (2)$$

is a two-variable, or bivariate function. Least squares multivariate approximation is used to solve this type of problem.

Given N data points, $\{(x_i, y_i, z_i)\}$ $i = 1, 2, 3\ldots N$, the probability to fit the best linear bivariate polynomial through the set of data. Consider the linear polynomial:

$$z = a + bx + cy \qquad (3)$$

The sum of the squares of the deviations is given by

$$S(a, b, c) = \Sigma_i(e_i)^2 = \Sigma_i(Z_i - a - bx_i - cy_i)^2 \qquad (4)$$

The function S(a, b, c) is a minimum when

$$\frac{\delta S}{\delta a} = \Sigma_i 2(Z_i - a - bx_i - cy_i)(-1) = 0 \qquad (5a)$$

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$$\frac{\delta S}{\delta b} = \Sigma_i 2(Z_i - a - bx_i - cy_i)(-x_i) = 0 \tag{5b}$$

$$\frac{\delta S}{\delta c} = \Sigma_i 2(Z_i - a - bx_i - cy_i)(-y_i) = 0 \tag{5c}$$

Dividing equations (5) by 2 and rearranging yields the normal equations:

$$aN + b\sum_i x_i + c\sum_i y_i = \sum_i Z_i \tag{6a}$$

$$a\sum_i x_i + b\sum_i x_i^2 + c\sum_i x_i y_i = \sum_i x_i Z_i \tag{6b}$$

$$a\sum_i y_i + b\sum_i x_i y_i + c\sum_i y_i^2 = \sum_i y_i Z_i \tag{6c}$$

Equations (6) can be solved for a, b, and c by Gauss elimination.

A linear fit to a set of bivariate data may be inadequate. Consider the quadratic bivariate polynomial:

$$z = a + bx + cy + dx^2 + ey^2 + fxy \tag{7}$$

The sum of the squares of the deviations is given by

$$S(a,b,c,d,e,f) = \sum_i (Z_i - a - bx_i - cy_i - dx_i^2 - ey_i^2 - fx_i y_i)^2 \tag{8}$$

The function S(a, b, …, f) is a minimum when

$$\frac{\delta S}{\delta a} = \sum_i 2(Z_i - a - bx_i - cy_i - dx_i^2 - ey_i^2 - fx_i y_i)(-1) = 0 \tag{9a}$$

$$\frac{\delta S}{\delta b} = \sum_i 2(Z_i - a - bx_i - cy_i - dx_i^2 - ey_i^2 - fx_i y_i)(-x_i) = 0 \tag{9b}$$

$$\frac{\delta S}{\delta c} = \sum_i 2(Z_i - a - bx_i - cy_i - dx_i^2 - ey_i^2 - fx_i y_i)(-y_i) = 0 \tag{9c}$$

$$\frac{\delta S}{\delta d} = \sum_i 2(Z_i - a - bx_i - cy_i - dx_i^2 - ey_i^2 - fx_i y_i)(-x_i^2) = 0 \tag{9d}$$

$$\frac{\delta S}{\delta e} = \sum_i 2(Z_i - a - bx_i - cy_i - dx_i^2 - ey_i^2 - fx_i y_i)(-y_i^2) = 0 \tag{9e}$$

$$\frac{\delta S}{\delta f} = \sum_i 2(Z_i - a - bx_i - cy_i - dx_i^2 - ey_i^2 - fx_i y_i)(-x_i y_i) = 0 \tag{9f}$$

Dividing equations (9) by 2 and rearranging yields the normal equations:

$$aN + b\sum_i x_i + c\sum_i y_i + d\sum_i x_i^2 + e\sum_i y_i^2 + f\sum_i x_i y_i = \sum_i Z_i \tag{10a}$$

$$a\sum_i x_i + b\sum_i x_i^2 + c\sum_i x_i y_i + d\sum_i x_i^3 + e\sum_i x_i y_i^2 + f\sum_i x_i^2 y_i = \sum_i x_i Z_i \tag{10b}$$

$$a\sum_i y_i + b\sum_i x_i y_i + c\sum_i y_i^2 + d\sum_i x_i^2 y_i + e\sum_i y_i^3 + f\sum_i x_i y_i^2 = \sum_i y_i Z_i \tag{10c}$$

$$a\sum_i x_i^2 + b\sum_i x_i^3 + c\sum_i x_i^2 y_i + d\sum_i x_i^4 + e\sum_i x_i^2 y_i^2 + f\sum_i x_i^3 y_i = \sum_i x_i^2 Z_i \tag{10d}$$

$$a\sum_i y_i^2 + b\sum_i x_i y_i^2 + c\sum_i y_i^2 + d\sum_i x_i^2 y_i^2 + e\sum_i y_i^4 + f\sum_i x_i y_i^3 = \sum_i y_i^2 Z_i \tag{10e}$$

$$a\sum_i x_i y_i + b\sum_i x_i^2 y_i + c\sum_i x_i y_i^2 + d\sum_i x_i^3 y_i + e\sum_i x_i y_i^3 + f\sum_i x_i^2 y_i^2 = \sum_i x_i y_i Z_i \tag{10f}$$

Equations (10) can be written as the matrix equation

$$Ac = b \tag{11}$$

Where A is the 6 x 6 matrix, c is the 6 x 1 column vector of polynomial coefficients (i.e., *a* to *f*), and b is the 6 x 1 column vector of nonhomogeneous terms. The solution to equation (11) is

$$c = A^{-1}b \tag{12}$$

Where A^{-1} is the inverse of A [2].

3.0 Effect of Time and Soil Surface Temperature on the Temperature Emitted by Buried Rocks during Dry Season

This work looks at the effect of time and soil surface temperature on the temperature emitted by four different rocks

(Graphite, Quartz, limestone and Iron Ore) of dimension 12 cm x 12 cm surface area and thickness of 7.5 cm buried at a depth of 2cm in Abeokuta, South – West, Nigeria. The experiment was carried out in January 2009 which represents the dry season in the area.

The data from the experiment is presented in Table 1. The data was obtained using a Hoboware data logger and five temperature sensors manufactured by Onset Corporation, USA. One temperature sensor was placed on the soil surface and the rest four sensors placed each on the buried objects. Measurements of soil surface temperature and temperature of the buried objects were taken at one hour interval. A 6 x 6 matrix was generated from the data using equations (10) and the matrix solved using Microsoft Student Encarta.

3.1 Graphite

Let the variables x, y, and z in equations (10) correspond to t (time of the day), T (surface temperature), and $T_{graphite}$ (temperature of buried graphite). The matrix is presented in equation (13).

$$\begin{pmatrix} 24 & 276 & 825 & 4324 & 30716 & 8786 \\ 276 & 4324 & 8786 & 76176 & 300014 & 127401 \\ 825 & 8786 & 30716 & 127401 & 1239743 & 300014 \\ 4324 & 76176 & 127401 & 1431244 & 3937864 & 2141534 \\ 30716 & 300014 & 1239743 & 3937863 & 53887381 & 11141543 \\ 8786 & 127401 & 300014 & 2141534 & 11141543 & 3937863 \end{pmatrix}^{-1} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = \begin{pmatrix} 809 \\ 9020 \\ 29004 \\ 135612 \\ 1126811 \\ 297559 \end{pmatrix} \quad (13)$$

Solving the matrix equation (13) gives the values of *a* to *f* as:

$$a = 8.6343689816659, b = -0.3990120478046, c = 0.940812817151,$$

$$d = -0.0128839415059, e = -0.0082665127132, f = 0.0279258209712$$

Substituting the values of *a* to *f* above into equation (7) yields

$$T_{graphite}(T,t) = 8.6343689816659 - 0.3990120478046t + 0.940812817151T -$$

$$0.0128839415059t^2 - 0.0082665127132T^2 + 0.0279258209712tT \quad (14)$$

Where $T_{graphite}$ is the Temperature of Graphite (in degree centigrade), t is time of the day (in hour) and T is the temperature of the soil at the surface (in degree centigrade).

Evaluating equation (14) using the experimental data gives Table 2. Table 2 was obtained by substituting the values of time and soil surface temperature in Table 1 into equation (14).

3.2 Quartz

Let the variables x, y, and z in equations (10) correspond to t (time of the day), T (surface temperature), and T_{quartz} (temperature of buried quartz). The matrix is presented in equation (15).

$$\begin{pmatrix} 24 & 276 & 825 & 4324 & 30716 & 8786 \\ 276 & 4324 & 8786 & 76176 & 300014 & 127401 \\ 825 & 8786 & 30716 & 127401 & 1239743 & 300014 \\ 4324 & 76176 & 127401 & 1431244 & 3937864 & 2141534 \\ 30716 & 300014 & 1239743 & 3937863 & 53887381 & 11141543 \\ 8786 & 127401 & 300014 & 2141534 & 11141543 & 3937863 \end{pmatrix}^{-1} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = \begin{pmatrix} 810 \\ 9069 \\ 28921 \\ 136834 \\ 1119856 \\ 298305 \end{pmatrix} \quad (15)$$

Solving the matrix equation (15) gives the values of *a* to *f* as:

$$a = 17.9106808704127, b = -0.596752557929, c = 0.5076056616929,$$

$$d = -0.0156906283411, e = -0.0043301289226, f = 0.0372095619322$$

Substituting the values of *a* to *f* above into equation (7) yields

$$T_{quartz}(T,t) = 17.9106808704127 - 0.596752557929t + 0.5076056616929T -$$

$$0.0156906283411t^2 - 0.0043301289226T^2 + 0.0372095619322tT \quad (16)$$

Where T_{quartz} is the Temperature of Quartz (in degree centigrade), t is time of the day (in hour) and T is the temperature of the soil at the surface (in degree centigrade).
Evaluating equation (16) using the experimental data gives Table 3. Table 3 was obtained by substituting the values of time and soil surface temperature in Table 1 into equation (16).

3.3 Limestone

Let the variables x, y, and z in equations (10) correspond to t (time of the day), T (surface temperature), and $T_{\lim e\ stone}$ (temperature of buried limestone) respectively. The matrix is presented in equation (17).

$$\begin{pmatrix} 24 & 276 & 825 & 4324 & 30716 & 8786 \\ 276 & 4324 & 8786 & 76176 & 300014 & 127401 \\ 825 & 8786 & 30716 & 127401 & 1239743 & 300014 \\ 4324 & 76176 & 127401 & 1431244 & 3937864 & 2141534 \\ 30716 & 300014 & 1239743 & 3937863 & 53887381 & 11141543 \\ 8786 & 127401 & 300014 & 2141534 & 11141543 & 3937863 \end{pmatrix}^{-1} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = \begin{pmatrix} 814 \\ 9123 \\ 29061 \\ 137690 \\ 1125110 \\ 300054 \end{pmatrix} \quad (17)$$

Solving the matrix equation (17) gives the values of *a* to *f* as:

$a = 17.6777324940274, b = -0.756809838616, c = 0.5403517195049,$

$d = -0.0141690293546, e = -0.0051161047088, f = 0.0422532349391$

Substituting the values of *a* to *f* above into equation (7) yields

$$T_{\lim e\, stome}(T,t) = 17.6777324940274 - 0.756809838616t + 0.5403517195049T - 0.0141690293546t^2 - 0.0051161047088T^2 + 0.0422532349391tT \quad (18)$$

Where $T_{\lim e\ stone}$ is the Temperature of Lime Stone (in degree centigrade), t is time of the day (in hour) and T is the temperature of the soil at the surface (in degree centigrade).
Evaluating equation (18) using the experimental data gives Table 4. Table 4 was obtained by substituting the values of time and soil surface temperature in Table 1 into equation (18).

3.4 Iron Ore

Let the variables x, y, and z in equations (10) correspond to t (time of the day), T (surface temperature), and $T_{iron\ ore}$ (temperature of buried iron ore) respectively. The matrix is presented in equation (19).

$$\begin{pmatrix} 24 & 276 & 825 & 4324 & 30716 & 8786 \\ 276 & 4324 & 8786 & 76176 & 300014 & 127401 \\ 825 & 8786 & 30716 & 127401 & 1239743 & 300014 \\ 4324 & 76176 & 127401 & 1431244 & 3937864 & 2141534 \\ 30716 & 300014 & 1239743 & 3937863 & 53887381 & 11141543 \\ 8786 & 127401 & 300014 & 2141534 & 11141543 & 3937863 \end{pmatrix}^{-1} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = \begin{pmatrix} 814 \\ 9081 \\ 29215 \\ 136305 \\ 1136040 \\ 299909 \end{pmatrix} \quad (19)$$

Solving the matrix equation (19) gives the values of *a* to *f* as:

$a = 7.8094366307\ \ 579, b = -0.6348007003\ \ 331, c = 1.0034118712\ \ 083,$

$d = -0.0138968587\ \ 252, e = -0.0097622509\ \ 2, f = 0.0380061974\ \ 196$

Substituting the values of *a* to *f* above into equation (7) yields

$$T_{iron\,ore}(T,t) = 7.8094366307579 - 0.6348007003331t + 1.0034118712083T - 0.0138968587252t^2 - 0.0097626225092T^2 + 0.0380061974196tT \quad (20)$$

Where $T_{iron\ ore}$ is the Temperature of Iron Ore (in degree centigrade), t is time of the day (in hour) and T is the temperature of the soil at the surface (in degree centigrade).
Evaluating equation (20) using the experimental data gives Table 5. Table 5 was obtained by substituting the values of time and soil surface temperature in Table 1 into equation (20).
Equations (14), (16), (18) and (20) can be generalized into one equation given as equation (21)

$$T_{Rock\,type}(T,t) = 7.809436607579a - 0.3990120478046bt + 0.5076056616929cT - 0.0128839415059dt^2 - 0.0043301289226eT^2 + 0.0279258209712\,ftT \quad (21)$$

Where for graphite, $a = 1.105633, b = 1, c = 1.853432, d = 1, e = 1.909068, f = 1$

for quartz, $a = 2.293466, b = 1.495575, c = 1, d = 1.217844, e = 1, f = 1.332443$

for limestone, $a = 2.263637, b = 1.896709, c = 1.0644511, d = 1.099743, e = 1.181513, f = 1.513053$

for iron ore, $a = 1, b = 1.590931, c = 1.976755, d = 1.078619, e = 2.25458, f = 1.36097$

4.0 Chi – Square Test

Chi – Square distribution is one of the most widely used theoretical probability distributions in inferential statistics, e.g., in statistical significant tests. The best – known situations in which the chi – square distribution is used are the common chi – square tests for goodness of fit of an observed distribution to a theoretical one, and of the independence of two criteria of classification of qualitative data [3].

According to [4], chi – square test is used to test if a sample of data came from a population with a specific distribution. Chi – Square is a family of distributions commonly used for significance testing. Pearson's chi – square is by far the most common type of chi – square significance test [5].

In this work, the Observed Values and Expected Values were compared and subjected to statistical analysis using Chi Square to test if there were significant difference between the observed data and the data from the theoretical models.

The chi – square was computed for the models in equations (14), (16), (18) and (20) using equation (22)

$$\chi^2 = \sum \frac{(Observed\ data - Expected\ data)^2}{Expected\ data} \sim \chi^2_{0.05,n-1} \quad (22)$$

Where n = 24 (number of data points), and $\chi^2_{0.05,n-1}$ is the χ^2 tabulated which gives 35.507.

The chi – square calculated for the models was computed using equation (22) which gives 0.019064 for graphite, 0.100154 for quartz, 0.018756 for lime stone and 0.018943 for iron ore.

Comparing the χ^2 **Calculated** and the χ^2 **tabulated,** there was no significant difference between the expected and observed values for all the rock types examined in this work.

5.0 Conclusion

Nigeria is blessed with a lot of solid minerals deposit spread across the Country. Prospecting for these solid minerals has been a top priority of the Government of Nigeria in recent years in an attempt to diversify the Nation's economy which has been solely depended on oil. Application of thermography in material identification and characterization has been applied in this work. The models generated in this work can be used to identify/characterise rock types. The coefficients *a* to *f* in equation (21) give the thermal properties of each rock type.

The chi-square test showed that there was no significant difference (p>0.05) between the expected and observed data for all the models. The model developed in this work enabled us to use simulation prediction as the basis for rock identification, which otherwise would be difficult or impossible to perform.

Table 1: Experimental Data

Time of the Day (h)	Soil Surface Temperature (^{0}C)	Temperature of Buried Graphite (^{0}C)	Temperature of Buried Quartz (^{0}C)	Temperature of Buried Limestone (^{0}C)	Temperature of Buried Iron Ore (^{0}C)
0	25.404	27.801	28.196	28.345	27.727
1	26.109	28.048	28.295	28.468	27.850
2	29.540	29.515	29.464	29.565	29.240
3	33.131	31.484	31.306	31.230	31.331
4	38.365	34.308	33.678	33.652	34.124
5	48.504	39.601	38.393	38.254	39.431
6	49.309	40.804	40.142	40.057	41.385
7	53.553	43.013	42.208	42.386	43.465
8	56.898	44.165	43.556	44.073	44.472
9	50.059	41.825	41.648	41.854	42.654
10	41.123	39.375	39.234	39.545	40.286
11	37.178	37.618	37.563	37.838	38.337
12	33.287	35.743	35.904	36.146	36.444

13	30.722	34.045	34.360	34.598	34.598
14	29.414	32.846	33.183	33.443	33.261
15	28.891	32.073	32.407	32.665	32.355
16	28.122	31.306	31.714	31.919	31.561
17	27.210	30.545	30.976	31.179	30.722
18	26.573	29.916	30.343	30.571	30.041
19	26.275	29.565	29.941	30.142	29.590
20	26.402	29.240	29.590	29.790	29.215
21	26.598	29.090	29.490	29.640	29.065
22	26.134	28.742	29.115	29.265	28.667
23	26.475	28.667	29.015	29.165	28.568

Table 2: Table of expected values for buried graphite as a function of time and soil surface temperature during dry season

$T_{graphite}(t,T)$ (°C)	Expected Values (°C)	Actual Value (°C)
$T_{graphite}(0,25.404)$	27.200	27.801
$T_{graphite}(1,26.109)$	27.880	28.048
$T_{graphite}(2,29.540)$	30.013	29.515
$T_{graphite}(3,33.131)$	32.193	31.484
$T_{graphite}(4,38.365)$	35.045	34.308
$T_{graphite}(5,48.5.4)$	39.275	39.601
$T_{graphite}(6,49.309)$	40.330	40.804
$T_{graphite}(7,53.553)$	42.354	43.013
$T_{graphite}(8,56.898)$	44.098	44.165
$T_{graphite}(9,50.059)$	42.962	41.825
$T_{graphite}(10,41.123)$	39.549	39.375
$T_{graphite}(11,37.178)$	37.658	37.618
$T_{graphite}(12,33.287)$	35.303	35.743
$T_{graphite}(13,30.722)$	33.524	34.045
$T_{graphite}(14,29.414)$	32.544	32.846
$T_{graphite}(15,28.891)$	32.133	32.073
$T_{graphite}(16,28.122)$	31.437	31.306
$T_{graphite}(17,27.210)$	30.525	30.545
$T_{graphite}(18,26.573)$	29.798	29.916
$T_{graphite}(19,26.275)$	29.563	29.565
$T_{graphite}(20,26.402)$	29.324	29.240
$T_{graphite}(21,26.598)$	29.347	29.090
$T_{graphite}(22,26.134)$	28.618	28.742
$T_{graphite}(23,26.475)$	28.760	28.667

Table 3: Table of expected values for buried quartz as a function of time and soil surface temperature during dry season

$T_{quartz}(t,T)$ (°C)	Expected Values (°C)	Actual Value (°C)
$T_{quartz}(0,25.404)$	28.011	28.196
$T_{qurtz}(1,26.109)$	28.571	28.295
$T_{quartz}(2,29.540)$	30.069	29.464
$T_{quartz}(3,33.131)$	31.742	31.306
$T_{quartz}(4,38.365)$	34.084	33.678
$T_{quartz}(5,48.5.4)$	37.992	38.393
$T_{quartz}(6,49.309)$	39.275	40.142
$T_{quartz}(7,53.553)$	41.679	42.208
$T_{quartz}(8,56.898)$	43.933	43.556
$T_{quartz}(9,50.059)$	42.592	41.648
$T_{quartz}(10,41.123)$	39.227	39.234
$T_{quartz}(11,37.178)$	37.552	37.563
$T_{quartz}(12,33.287)$	35.452	35.904
$T_{quartz}(13,30.722)$	33.870	34.360
$T_{quartz}(14,29.414)$	32.988	33.183
$T_{quartz}(15,28.891)$	32.605	32.407
$T_{quartz}(16,28.122)$	31.939	31.714
$T_{quartz}(17,27.210)$	31.049	30.976
$T_{quartz}(18,26.573)$	30.314	30.343
$T_{quartz}(19,26.275)$	30.029	29.941
$T_{quartz}(20,26.402)$	29.731	29.590
$T_{quartz}(21,26.598)$	29.681	29.490
$T_{quartz}(22,26.134)$	28.890	29.115
$T_{quartz}(23,26.475)$	28.947	29.015

Table 4: Table of expected values for buried limestone as a function of time and soil surface temperature during dry season

$T_{\lim e\, stone}(t,T)$ (oC)	Expected Values (°C)	Actual Value (°C)
$T_{\lim e\, stone}(0,25.404)$	28.103	28.345
$T_{\lim e\, stone}(1,26.109)$	28.630	28.468
$T_{\lim e\, stone}(2,29.540)$	30.101	29.565
$T_{\lim e\, stone}(3,33.131)$	31.766	31.230
$T_{\lim e\, stone}(4,38.365)$	34.108	33.652
$T_{\lim e\, stone}(5,48.5.4)$	37.960	38.254
$T_{\lim e\, stone}(6,49.309)$	39.333	40.057
$T_{\lim e\, stone}(7,53.553)$	41.790	42.386
$T_{\lim e\, stone}(8,56.898)$	44.132	44.073
$T_{\lim e\, stone}(9,50.059)$	42.984	41.854
$T_{\lim e\, stone}(10,41.123)$	39.638	39.545
$T_{\lim e\, stone}(11,37.178)$	37.936	37.838
$T_{\lim e\, stone}(12,33.287)$	35.751	36.146
$T_{\lim e\, stone}(13,30.722)$	34.092	34.598
$T_{\lim e\, stone}(14,29.414)$	33.173	33.443
$T_{\lim e\, stone}(15,28.891)$	32.790	32.665
$T_{\lim e\, stone}(16,28.122)$	32.103	31.919
$T_{\lim e\, stone}(17,27.210)$	31.177	31.179
$T_{\lim e\, stone}(18,26.573)$	30.421	30.571
$T_{\lim e\, stone}(19,26.275)$	30.158	30.142
$T_{\lim e\, stone}(20,26.402)$	29.885	29.790
$T_{\lim e\, stone}(21,26.598)$	29.890	29.640
$T_{\lim e\, stone}(22,26.134)$	29.091	29.265
$T_{\lim e\, stone}(23,26.475)$	29.225	29.165

Table 5: Table of expected values for buried iron ore as a function of time and soil surface temperature during dry season

$T_{iron\ ore}(t,T)$ (°C)	Expected Values (°C)	Actual Value (°C)
$T_{iron\ ore}(0,25.404)$	26.700	27.727
$T_{iron\ ore}(1,26.109)$	27.696	27.850
$T_{iron\ ore}(2,29.540)$	29.851	29.240
$T_{iron\ ore}(3,33.131)$	32.086	31.331
$T_{iron\ ore}(4,38.365)$	35.007	34.124
$T_{iron\ ore}(5,48.5.4)$	39.207	39.431
$T_{iron\ ore}(6,49.309)$	40.485	41.385
$T_{iron\ ore}(7,53.553)$	42.670	43.465
$T_{iron\ ore}(8,56.898)$	44.628	44.472
$T_{iron\ ore}(9,50.059)$	43.859	42.654
$T_{iron\ ore}(10,41.123)$	40.455	40.286
$T_{iron\ ore}(11,37.178)$	38.449	38.337
$T_{iron\ ore}(12,33.287)$	35.995	36.444
$T_{iron\ ore}(13,30.722)$	34.000	34.598
$T_{iron\ ore}(14,29.414)$	32.917	33.261
$T_{iron\ ore}(15,28.891)$	32.472	32.355
$T_{iron\ ore}(16,28.122)$	31.693	31.561
$T_{iron\ ore}(17,27.210)$	30.657	30.722
$T_{iron\ ore}(18,26.573)$	29.829	30.041
$T_{iron\ ore}(19,26.275)$	29.572	29.590
$T_{iron\ ore}(20,26.402)$	29.310	29.215
$T_{iron\ ore}(21,26.598)$	29.361	29.065
$T_{iron\ ore}(22,26.134)$	28.525	28.667
$T_{iron\ ore}(23,26.475)$	28.723	28.568

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