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Localization of Electrons in a One-Dimensional Disordered Crystal

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Abstract

Localization of electrons in a one-dimensional disorder crystal and suppression of conduction were investigated in the research work. These were achieved by modeling the disorders within the crystal lattice with random potential barriers. The transmission coefficients of the electron (non-interacting electron) in the random potential barriers were determined and analyzed. All the electronic eigenstates were found to be localized and decay asymptotically exponentially. Also, using Landeur formula [3], the conductance of the disordered crystal was found to tend to zero.

1.0 Introduction

Disorder (due to e.g impurities and defects) within a crystal lattice leads to localization of the electron eigenstates [2]. An electron is said to be localized, if the probability of finding it over the entire system is not the same. In other words, localization of the eigenstates implies that the probability density does not vanish only in a limited spatial region. All electronic states in a one-Dimensional disorder systems are exponentially localized [4].

We want to show in this work that all the eigenstates in a one-dimensional disordered crystal are localized no matter the amount of the disorder and that these states decay asymptotically exponentially. Also, to show that at low temperature, conduction is suppressed in the one-dimensional disordered crystal.

In this paper, disorder within a fixed lattice is modeled with random rectangular potential barriers and we assumed that the electrons are non-interacting at low temperature.

2.0 Theory

We considered 3-random potential barriers, 4-random potential barriers and 7-random potential barriers and determined the transmission coefficient for each of them

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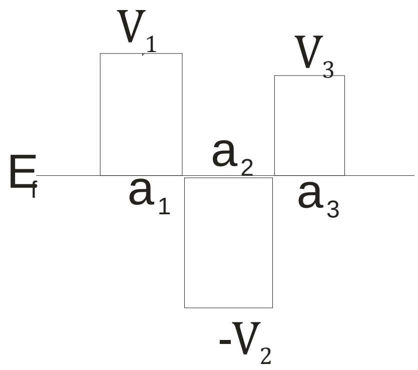
2.1 Transmission Coefficient for Three-Random Potential Barriers



$V(x) = 0 \quad x < 0,$ $\quad V(x) = V_1 \quad 0 < x < a_1,$

$V(x) = -V_2 \quad a_1 < x < a_{2,}$ $\quad V(x) = V_3 \quad a_2 < x < a_3$,

$V(x) = 0 \quad x > a_3.$

Fig 1 Three -Random Potential Barriers

Considering fig1, the potentials $V_1, V_2, and\ V_3$ with corresponding width a_1, a_2, and a_3 are random. The potential V_1, V_3 are positive while V_2 is negative.

The model is subject to the condition

$$\frac{a_1 v_1 + a_2 v_2 + a_3 v_3}{a_1 + a_2 + a_3} = 0 \qquad (2.1)$$

This is a condition for the probability of transmission through the barriers. The zero is the Fermi energy, E_f value [2]. Considering a non-interacting electron, the one-dimensional time independent Schrödinger equation can be solved for it in each of the region.

The one-dimensional time independent Schrödinger equation is as follows;

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + V(x)\Psi(x) = E\Psi(x) \qquad (2.2)$$

Assuming that the incident particle of energy E is coming from $x = -\infty$, the solutions to the equations for each region are as follows;

$_1(x) = Ae^{i(k_1 x)} + Be^{-i(k_1 x)}$ $\qquad x < 0,$

$_2(x) = Ce^{(k_2 x)} + De^{-(k_2 x)}$ (If and only if $0 < E < V_1$) $\qquad 0 < x < a_{1,}$

$_3(x) = Fe^{i(k_3 x)} + Ge^{-i(k_3 x)}$ $\qquad a_1 < x < \sum_{i=1}^{2} a_i$

$_4(x) = He^{(k_4 x)} + Ie^{-(k_4 x)}$ (If and only if $0 < E < V_3$) $\qquad \sum_{i=1}^{2} a_i < x < \sum_{i=1}^{3} a_i$

$_5(x) = Je^{i(k_1 x)}$ $\qquad \sum_{i=1}^{3} a_i < x < \infty$

Where $k_1 = \sqrt{\frac{2mE}{\hbar^2}}$ $\qquad V = 0$

$k_i = \sqrt{\frac{2m(-1)^j(E - V_j)}{\hbar^2}}$ $\qquad V_j > E \qquad i = 2,3,4, \quad j = i - 1$

$A, B, C, D, F, G, H, I, J$ are the amplitudes in the different regions. Applying the boundary conditions for Ψ and the $\frac{d\Psi(x)}{dx}$ at the boundaries $x = 0,\ x = a_1, x = P_2(\sum_{i=1}^{2} a_i), x = P_3(\sum_{i=1}^{3} a_i)$.

We have the following equations;

$$A + B = C + D$$

$$ik_1 A - ik_1 B = ik_2 C - ik_2 D$$

$$Ce^{k_2 a_1} + De^{-k_2 a_1} = Fe^{i(k_3 a_1)} + Ge^{-i(k_3 a_1)}$$

$$k_2 Ce^{k_2 a_1} - k_2 De^{-k_2 a_1} = k_3 Fe^{i(k_3 a_1)} - k_3 Ge^{-i(k_3 a_1)}$$

$$Fe^{i(k_3 P_2)} + Ge^{-i(k_3 P_2)} = He^{(k_4 P_2)} + Ie^{-(k_4 P_2)}$$

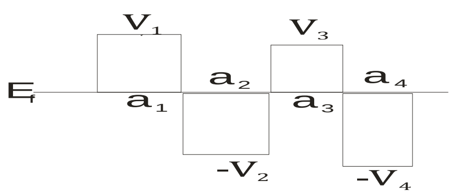
$$ik_3Fe^{i(k_3P_2)} - ik_3Ge^{-i(k_3P_2)} = k_4He^{(k_4P_2)} - k_4Ie^{-(k_4P_2)}$$
$$He^{(k_4P_3)} + Ie^{-(k_4P_3)} = Je^{i(k_1P_3)}$$
$$k_4He^{(k_4P_3)} - k_4Ie^{-k_4P_3} = ik_1Je^{i(k_1P_3)}$$

For each pair of equation, a $2x2$ matrix was set up. Then, solving using the transfer matrix method, the transmission coefficient was determined using;

$$T = \frac{YY^*}{AA^*} \qquad (2.3)$$

Where Y is the amplitude of the transmitted wave, Y* is the complex conjugate of Y, A is the amplitude of the reflected wave and A* is the complex conjugate of A.

2.2 Transmission Coefficient for 4-Random Potential Barriers.



$V(x) = 0$ $\quad x < 0,$ $\qquad V(x) = V_1$ $\quad 0 < x < a_1,$
$V(x) = -V_2$ $\quad a_1 < x < a_2,$ $\qquad V(x) = V_3$ $\quad a_2 < x < a_3$,
$V(x) = -V_4$ $\quad a_3 < x < a_4,$ $\qquad V(x) = 0$ $\quad x > a_4.$

Fig2 Four-random potential barriers

Considering fig2, the potentials V_1, V_2, V_3, and V_4 with corresponding width a_1, a_2, a_3, and a_4 are random. The potentials V_1 and V_3 are positive while V_2 and V_4 are negative.

The model is subject to the condition

$$\frac{a_1V_1 + a_2V_2 + a_3V_3 + a_4V_4}{a_1 + a_2 + a_3 + a_4} = 0 \qquad (2.4)$$

This is a condition for the probability of transmission through the barriers. The zero is the Fermi energy, E_f value [2].

The one-dimensional time independent Schrödinger equation was solved for each of the region. The one-dimensional time independent Schrödinger equation is as follows;

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + V(x)\Psi(x) = E\Psi(x) \qquad (2.5)$$

Assuming that the incident particle of energy E is coming from $x = -\infty$, the solutions to the equations for each region are as follows;

$\Psi_1(x) = Ae^{i(k_1x)} + Be^{-i(k_1x)}$ $\qquad x < 0,$
$\Psi_2(x) = Ce^{(k_2x)} + De^{-(k_2x)}$ (If and only if $0 < E < V_1$) $\qquad 0 < x < a_1,$
$\Psi_3(x) = Fe^{i(k_3x)} + Ge^{-i(k_3x)}$ $\qquad a_1 < x < \sum_{i=1}^{2} a_i$
$\Psi_4(x) = He^{(k_4x)} + Ie^{-(k_4x)}$ (If and only if $0 < E < V_3$) $\qquad \sum_{i=1}^{2} a_i < x < \sum_{i=1}^{3} a_i$
$\Psi_5(x) = Je^{i(k_5x)} + Ke^{-i(k_5x)}$ $\qquad \sum_{i=1}^{3} a_i < x < \sum_{i=1}^{4} a_i$
$\Psi_6(x) = Me^{(k_1x)}$ $\qquad \sum_{i=1}^{4} a_i < x < \infty$

Where $k_1 = \sqrt{\frac{2mE}{\hbar^2}}$ $\qquad V = 0$

$$k_i = \sqrt{\frac{2m(-1)^j(E-V_j)}{\hbar^2}} \qquad V_j>E \qquad i = 2,3,4,5, \quad j = I-1$$

$A, B, C, D, F, G, H, I, J, K, M$ are the amplitude in different region. Applying the boundary conditions for Ψ and the $\frac{d\psi(x)}{dx}$ at the boundaries $x = 0, x = a_1, x = P_2(\sum_{i=1}^{2} a_i), x = P_3(\sum_{i=1}^{3} a_i), x = P_4(\sum_{i=1}^{4} a_i)$.

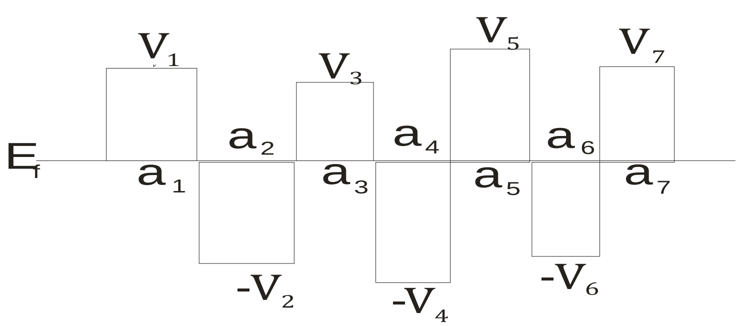
We have the following equations;

$$A + B = C + D$$
$$ik_1 A - ik_1 B = ik_2 C - ik_2 D$$
$$Ce^{k_2a_1} + De^{-k_2a_1} = Fe^{i(k_3a_1)} + Ge^{-i(k_3a_1)}$$
$$k_2Ce^{k_2a_1} - k_2De^{-k_2a_1} = k_3Fe^{i(k_3a_1)} - k_3Ge^{-i(k_3a_1)}$$
$$Fe^{i(k_3P_2)} + Ge^{-i(k_3P_2)} = He^{(k_4P_2)} + Ie^{-(k_4P_2)}$$
$$ik_3Fe^{i(k_3P_2)} - ik_3Ge^{-i(k_3P_2)} = k_4He^{(k_4P_2)} - k_4Ie^{-(k_4P_2)}$$
$$He^{(k_4P_3)} + Ie^{-(k_4P_3)} = Je^{i(k_5P_3)} + Ke^{-i(k_5P_3)}$$
$$k_4He^{(k_4P_3)} - k_4Ie^{-k_4P_3} = ik_5Je^{i(k_5P_3)} - ik_5Ke^{-i(k_5P_3)}$$
$$Je^{i(k_5P_4)} + Ke^{-i(k_5P_4)} = Me^{(k_1P_4)}$$
$$ik_5Je^{i(k_5P_4)} - ik_5Ke^{-i(k_5P_4)} = k_1Me^{(k_1P_4)}$$

For each pair of equation, a $2x2$ matrix was set up. Then, solving using the transfer matrix method, the transmission coefficient was determined using

$$T = \frac{YY^*}{AA^*} \tag{2.6}$$

2.3 Transmission Coefficient for 7-Random Potential Barriers.



$V(x) = 0 \quad x < 0,$ $\qquad V(x) = V_1 \quad 0 < x < a_1,$
$V(x) = -V_2 \quad a_1 < x < a_2,$ $\qquad V(x) = V_3 \quad a_2 < x < a_3,$
$V(x) = -V_4 \quad a_3 < x < a_4,$ $\qquad V(x) = V_5 \quad a_4 < x < a_5,$
$V(x) = -V_6 \quad a_5 < x < a_6,$ $\qquad V(x) = V_7 \quad a_6 < x < a_7,$
$V(x) = 0 \quad x > a_7.$

Fig 3 Seven-random potential barriers

Considering fig 3, the potentials $V_1, V_2, V_3, V_4, V_5, V_6, and\ V_7$ with corresponding width $a_1, a_2, a_3, a_4, a_5, a_6$ and a_7 are random. The potential V_1, V_3, V_5, and V_7 are positive while V_2, V_4, and V_6 are negative.
The model is subject to the condition;

$$\frac{a_1V_1 + a_2V_2 + a_3V_3 + a_4V_4 + a_5V_5 + a_6V_6 + a_7V_7}{a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7} = 0 \qquad (2.7)$$

This is a condition for the probability of transmission through the barriers. The zero is the Fermi energy, E_f value [2].

The one-dimensional time independent Schrödinger equation was solved for each of the region. The one-dimensional time independent Schrödinger equation is as follows;

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + V(x)\Psi(x) = E\Psi(x) \qquad (2.8)$$

Assuming that the incident particle of energy E is coming from $x = -\infty$, the solutions to the equations for each region are as follows;

$\Psi_1(x) = Ae^{i(k_1x)} + Be^{-i(k_1x)}$ $\quad x < 0,$

$\Psi_2(x) = Ce^{(k_2x)} + De^{-(k_2x)}$ (If and only if $0 < E < V_1$) $\quad 0 < x < a_1,$

$\Psi_3(x) = Fe^{i(k_3x)} + Ge^{-i(k_3x)}$ $\quad a_1 < x < \sum_{i=1}^{2} a_i$

$\Psi_4(x) = He^{(k_4x)} + Ie^{-(k_4x)}$ (If and only if $0 < E < V_3$) $\quad \sum_{i=1}^{2} a_i < x < \sum_{i=1}^{3} a_i$

$\Psi_5(x) = Je^{i(k_5x)} + Ke^{-i(k_5x)}$ $\quad \sum_{i=1}^{3} a_i < x < \sum_{i=1}^{4} a_i$

$\Psi_6(x) = Me^{(k_6x)} + Ne^{-(k_6x)}$ (If and only if $0 < E < V_5$) $\quad \sum_{i=1}^{4} a_i < x < \sum_{i=1}^{5} a_i$

$\Psi_7(x) = Pe^{i(k_7x)} + Qe^{-i(k_7x)}$ $\quad \sum_{i=1}^{5} a_i < x < \sum_{i=1}^{6} a_i$

$\Psi_8(x) = Re^{(k_8x)} + Se^{-(k_8x)}$ (If and only if $0 < E < V_7$) $\quad \sum_{i=1}^{6} a_i < x < \sum_{i=1}^{7} a_i$

$\Psi_9(x) = Te^{i(k_1x)}$ $\quad \sum_{i=1}^{7} a_i < x < \infty$

Where $k_1 = \sqrt{\frac{2mE}{\hbar^2}}$ $\qquad V=0$

$k_i = \sqrt{\frac{2m(-1)^j(E-V_j)}{\hbar^2}}$ $\qquad V_j > E \qquad i = 2,3,4\ldots 8$, j= I – 1

$A, B, C, D, F, G, H, I, J, K, M, N, P, Q, R, S, T$ are the amplitude in different region. Applying the boundary conditions for Ψ and the $\frac{d\Psi(x)}{dx}$ at the boundaries $x = 0, x = a_1, \ x = P_2(\sum_{i=1}^{2} a_i), x = P_3(\sum_{i=1}^{3} a_i)$ $x = P_4(\sum_{i=1}^{4} a_i), x = P_5(\sum_{i=1}^{5} a_i)$ $x = P_6(\sum_{i=1}^{6} a_i), x = P_7(\sum_{i=1}^{7} a_i)$.

We have the following equations;

$$A + B = C + D$$
$$ik_1A - ik_1B = ik_2C - ik_2D$$
$$Ce^{k_2a_1} + De^{-k_2a_1} = Fe^{i(k_3a_1)} + Ge^{-i(k_3a_1)}$$
$$k_2Ce^{k_2a_1} - k_2De^{-k_2a_1} = k_3Fe^{i(k_3a_1)} - k_3Ge^{-i(k_3a_1)}$$
$$Fe^{i(k_3P_2)} + Ge^{-i(k_3P_2)} = He^{(k_4P_2)} + Ie^{-(k_4P_2)}$$
$$ik_3Fe^{i(k_3P_2)} - ik_3Ge^{-i(k_3P_2)} = k_4He^{(k_4P_2)} - k_4Ie^{-(k_4P_2)}$$
$$He^{(k_4P_3)} + Ie^{-(k_4P_3)} = Je^{i(k_5P_3)} + Ke^{-i(k_5P_3)}$$
$$k_4He^{(k_4P_3)} - k_4Ie^{-k_4P_3} = ik_5Je^{i(k_5P_3)} - ik_5Ke^{-i(k_5P_3)}$$

$$Je^{i(k_5P_4)} + Ke^{-i(k_5P_4)} = Me^{(k_6P_4)} + Ne^{-(k_6P_4)}$$
$$ik_5Je^{i(k_5P_4)} - ik_5Ke^{-i(k_5P_4)} = k_6Me^{(k_6P_4)} - k_6Ne^{-(k_6P_4)}$$
$$Me^{(k_6P_5)} + Ne^{-(k_6P_5)} = Pe^{i(k_7P_5)} + Qe^{-i(k_7P_5)}$$
$$k_6Me^{(k_6P_5)} - k_6Ne^{-(k_6P_5)} = ik_7Pe^{i(k_7P_5)} - ik_7Qe^{-i(k_7P_5)}$$
$$Pe^{i(k_7P_6)} + Qe^{-i(k_7P_6)} = Re^{(k_8P_6)} + Se^{-(k_8P_6)}$$
$$ik_7Pe^{i(k_7P_6)} - ik_7Qe^{-i(k_7P_6)} = k_8Re^{(k_8P_6)} - k_8Se^{-(k_8P_4)}$$
$$Re^{(k_8P_7)} + Se^{-(k_8P_7)} = Te^{i(k_1P_7)}$$
$$k_8Re^{(k_8P_7)} - k_8Se^{-(k_8P_7)} = ik_1Te^{i(k_1P_7)}$$

For each pair of equation, a $2x2$ matrix was set up. Then, solving using the transfer matrix method, the transmission coefficient was determined using;

$$T = \frac{YY^*}{AA^*} \quad (2.9)$$

2.4 Conductance in a Disordered System

Landauer[3] concluded that the conductance of a one- dimensional guide in which the carriers are scattered by static obstacles is given by

$$G = \frac{e^2T}{2\pi\hbar R} \quad (3.0)$$

Where T and $R = 1 - T$ are the probabilities of the transmission and reflection respectively. The characteristic coefficient e^2/h is the quantum of conductance and equivalently its inverse is the quantum resistance h/e^2= 25812.8Ω[1]. For a disorder system, $G \to 0$ and for an ordered system, $G \to \infty$ [1].

3.0 METHODOLOGY

A C++ program was written to generate random potential (height) and width for the barriers and also was used to calculate the transmission coefficient for every random generation subject to the conditions given (equation 2.1, 2.4 and 2.7).

The energy of the electron was taken, $E = 100meV$, mass of electron, $m \approx 10^{-31}kg$, the electronic charge $1.6 * 10^{-19}c$ and planck constant, $h \approx 10^{-34}Js$.

The results were tabulated and graphs of transmission coefficient against area a_1V_1 of the barrier were plotted. Also, graphs transmission coefficient against reflection coefficient were plotted and their slope multiplied by a constant (quantum resistance value=25812.8Ω [1]) to yield conductance of the system.

4.0 DISCUSSION OF RESULTS

The fig4, fig 6 and fig 8 are the graphs of transmission coefficient against a_1V_1 for the 3, 7and 4 random potential barriers respectively. It can be seen from these graphs and the table of values that the transmission coefficients differ from point to point, that is, no two points on the graphs have the same value of transmission coefficient though there are points with very close values. Thus, all the eigenstates (eigenstate transmissions) are strongly localized. It can also be observed that the transmission coefficient decrease (decay) with the area of the barrier and the trend line of the decay is asymptotically exponential.

The fig5 is the graph of transmission coefficient against reflection coefficient. The slope of the graph gives -1.0000029. Using equation 3.0,

Conductance, G $= \frac{-1.0000029}{25812.8} = -3.87 * 10^{-5}\Omega^{-1}\mathrm{m}^{-1}$

The fig9 is the graph of transmission coefficient against reflection coefficient. The slope of the graph gives -1.0000000. Using equation 3.0, Conductance, G $= \frac{-1.0000000}{25812.8} = -3.87 * 10^{-5}\Omega^{-1}m^{-1}$

The fig7 is the graph of transmission coefficient against reflection coefficient. The slope of the graph gives -1.0000031.

Table 1. Values of the three random Potential barriers

$a_1$nm	$a_2$nm	$a_3$nm	V_1 nV	V_2nV	V_3 nV	a_1V_1 nmnV	T	$R = 1 - T$	lnT
4.10	5.80	0.61	5.40	-0.70	8.60	22.10	0.036388	0.963612	-3.31353
4.50	6.80	0.59	7.10	-1.30	9.30	32.00	0.027878	0.972122	-3.57993
4.80	7.70	0.58	8.70	-2.30	0.30	41.80	0.02292	0.97708	-3.77574
5.40	9.70	0.56	2.00	-5.20	3.20	10.80	0.125476	0.874524	-2.07564
5.80	0.70	0.64	3.60	-7.20	5.20	20.90	0.064187	0.935813	-2.74596
6.10	1.60	0.63	5.20	-9.50	7.40	31.70	0.042989	0.957011	-3.14681
6.40	2.60	0.62	6.90	-2.10	0.10	44.20	0.02909	0.97091	-3.53737
6.80	3.60	0.60	8.50	-5.00	3.00	57.80	0.024075	0.975925	-3.72659
7.10	4.60	0.59	0.10	-8.30	6.30	0.710	0.260815	0.739185	-1.34394
8.10	7.50	0.55	5.00	-0.10	8.00	40.50	0.038228	0.961772	-3.26419
8.40	8.50	0.54	6.70	-4.60	2.60	56.30	0.030931	0.969069	-3.47599
3.30	3.20	0.64	1.20	-2.40	0.40	3.960	0.20623	0.79377	-1.57876
4.30	6.10	0.60	6.10	-2.80	0.70	26.20	0.033449	0.966551	-3.39774
4.60	7.10	0.59	7.70	-3.50	1.50	35.40	0.026375	0.973625	-3.63535
4.90	8.10	0.58	9.30	-4.60	2.60	45.60	0.021823	0.978177	-3.8248
5.60	0.10	0.65	2.60	-7.80	5.80	14.60	0.096531	0.903469	-2.33789
5.90	1.00	0.64	4.20	-9.90	7.90	24.80	0.055429	0.944571	-2.89266
6.20	2.00	0.62	5.90	-2.30	0.30	36.60	0.034381	0.965619	-3.37024
6.50	3.00	0.61	7.50	-5.10	3.00	48.80	0.027536	0.972464	-3.59225
6.90	4.00	0.60	9.10	-8.10	6.10	62.80	0.022848	0.977152	-3.77891

Conductance, G= $\frac{-1.0000031}{25812.8} = -3.87 * 10^{-5}\Omega^{-1}m^{-1}$

The conductance, G in both cases is the same and modulus of which gave a small value (close to zero). This implies that conduction is suppressed in the disordered system (crystal).

5.0 Conclusion

From our results, we can conclude that all the eigenstates in a one-dimensional disordered crystal (random potential system) are completely localized with an asymptotic exponential decay trend no matter the amount of the disorder, that is, the probability of finding an electron over the entire system is not the same. This results to the suppression of conduction in the disordered crystals. The suppression was justified by determining the conductance in each of the system. Also, comparing the value of the conductance obtained in each case, they were found to be same and close zero. Thus, one-dimensional disordered crystals behave like insulators. These results are consistent with what Mott and Twose[4] and Thouless[5] got in their works. They showed that in a one-dimensional material with random distribution of potentials, all the electronic states which are solution of Schrödinger equation are localized.

The model (random potential barrier) used can to an extent be instrumental for analyzing and studying of other electronic properties of a disorder system (one, two or three dimension).

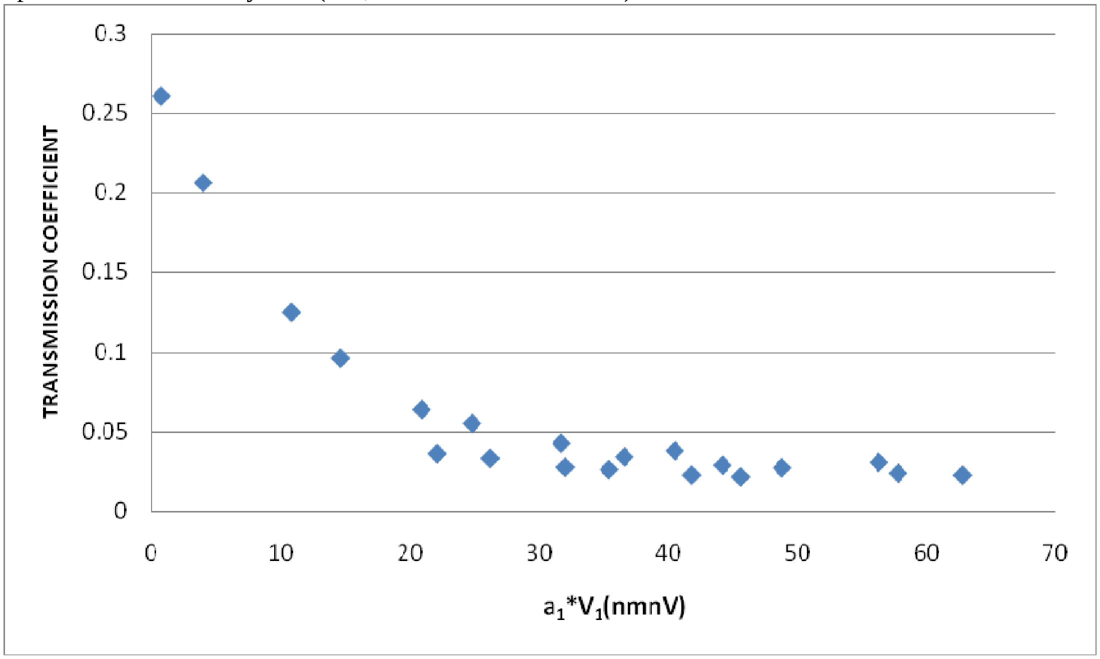


Fig 4: A graph of Transmission coefficient against area a_1V_1 for three random potential barriers

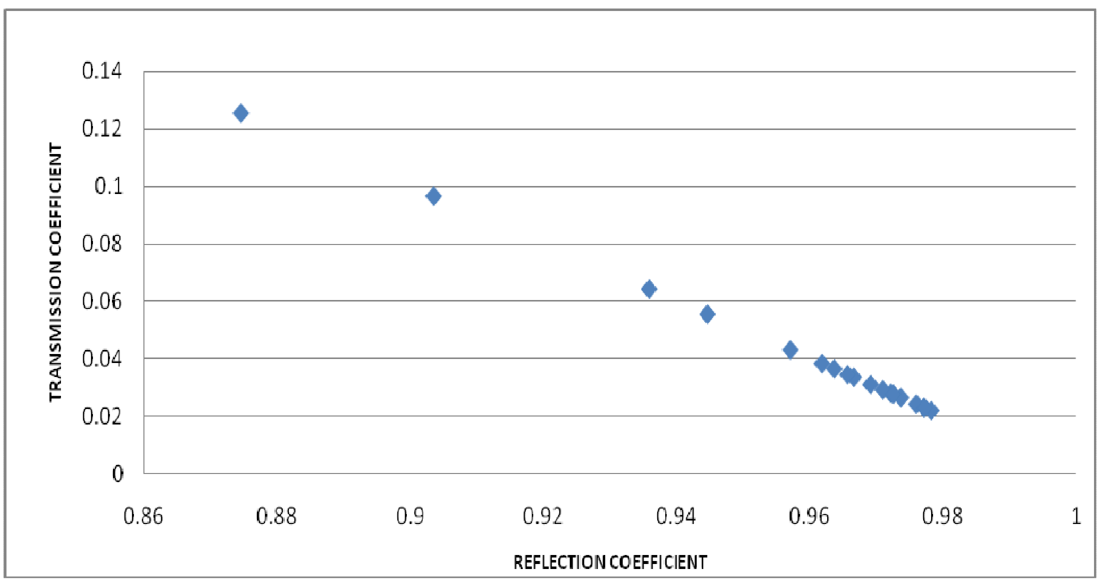


Fig 5 A graph of Transmission coefficient against Reflection coefficient for three random potential barrier

Table 2 Values for the seven random potential barriers

a_1 nm	a_2 nm	a_3 nm	a_4 nm	a_5 nm	a_6 nm	a_7 nm	V_1 nV	V_2 nV	V_3 nV	V_4 nV	V_5 nV	V_6 nV	V_7 nV	a_1V_1 nmnV	T	$R = 1 - T$	lnT
4.50	0.70	7.20	3.70	2.30	0.30	0.52	4.50	-3.90	0.50	-7.00	5.40	-8.80	7.10	23.0	0.04722	0.952781	-3.05295
5.10	1.30	7.90	4.40	3.00	0.90	0.49	5.10	-4.60	1.10	-7.70	7.10	-9.50	0.30	26.0	0.0416	0.958404	-3.17976
5.80	2.30	8.80	5.40	3.90	1.90	0.43	5.80	-5.60	2.10	-8.60	8.70	-0.50	3.60	33.6	0.03656	0.96344	-3.3088
6.40	3.60	0.10	6.70	5.20	3.20	0.46	6.40	-6.90	3.40	-9.90	0.30	-1.80	6.90	41.0	0.03323	0.966768	-3.40424
7.10	5.20	1.80	8.30	6.90	4.80	0.37	7.10	-8.50	5.00	-1.60	2.00	-3.40	0.10	50.4	0.03003	0.969974	-3.5057
7.70	7.20	3.70	0.30	8.80	6.80	0.37	7.70	-0.50	7.00	-3.50	3.60	-5.40	3.40	59.3	0.02523	0.974768	-3.67966
8.40	9.50	6.00	2.50	1.10	9.10	0.35	8.40	-2.80	9.30	-5.80	5.20	-7.70	6.70	70.6	0.0239	0.976101	-3.73393
1.60	5.80	2.30	8.90	7.40	5.40	0.39	1.60	-9.10	5.60	-2.10	3.40	-4.00	3.00	2.56	0.19228	0.807718	-1.64879
2.30	0.10	6.60	3.10	1.70	9.70	0.48	2.30	-3.30	9.90	-6.40	5.00	-8.20	6.30	5.29	0.09933	0.900666	-2.30927
3.00	4.60	1.20	7.70	6.30	4.20	0.44	3.00	-7.90	4.40	-1.00	6.70	-2.80	9.50	9.0	0.08092	0.919085	-2.51435
3.60	9.50	6.10	2.60	1.20	9.10	0.39	3.60	-2.80	9.30	-5.90	8.30	-7.70	2.80	13.0	0.05902	0.940984	-2.82994
4.30	4.80	1.30	7.80	6.40	4.30	0.42	4.30	-8.00	4.60	-1.10	9.90	-2.90	6.10	18.5	0.0527	0.947304	-2.94322
4.90	0.30	6.80	3.40	1.90	9.90	0.44	4.90	-3.60	0.10	-6.60	1.60	-8.50	9.30	24.0	0.04281	0.957185	-3.15088
5.60	6.20	2.70	9.20	7.80	5.80	0.34	5.60	-9.50	6.00	-2.50	3.20	-4.30	2.60	31.4	0.03949	0.960509	-3.23169
6.20	2.40	8.90	5.50	4.00	2.00	0.42	6.20	-5.70	2.20	-8.70	4.80	-0.60	5.90	38.4	0.03402	0.96598	-3.3808
6.90	8.90	5.50	2.00	0.60	8.50	0.39	6.90	-2.20	8.70	-5.20	6.50	-7.10	9.10	47.6	0.02913	0.970871	-3.53603
7.50	5.80	2.30	8.80	7.40	5.40	0.34	7.50	-9.00	5.60	-2.10	8.10	-3.90	2.40	56.3	0.02836	0.971637	-3.56267
1.40	3.80	0.30	6.80	5.40	3.40	0.5	1.40	-7.00	3.60	-0.10	7.90	-1.90	2.00	1.96	0.21572	0.784276	-1.53375
2.10	2.90	9.50	6.00	4.60	2.50	0.43	2.10	-6.20	2.70	-9.30	9.50	-1.10	5.20	4.41	0.12167	0.878332	-2.10646
2.80	2.40	8.90	5.50	4.00	2.00	0.46	2.80	-5.70	2.20	-8.70	1.20	-0.60	8.50	7.84	0.08417	0.915827	-2.47488
3.40	2.20	8.70	5.30	3.80	1.80	0.45	3.40	-5.50	2.00	-8.50	2.80	-0.40	1.80	11.60	0.06666	0.93334	-2.70815

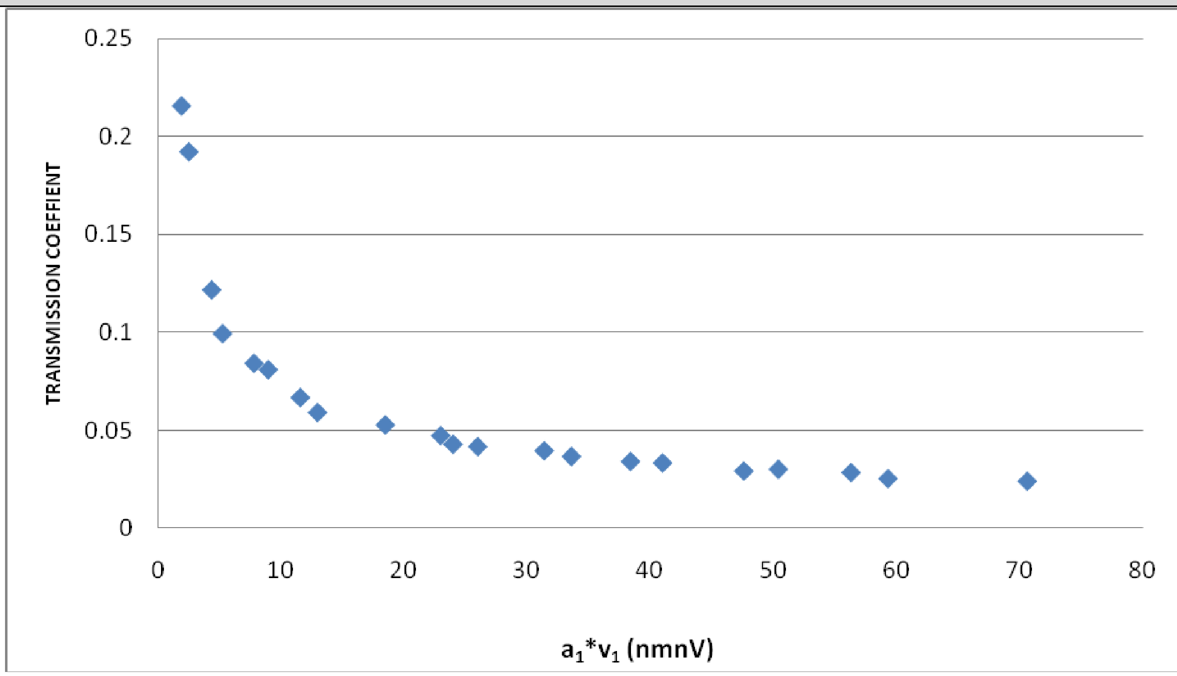


Fig 6 A graph of Transmission coefficient against area a_1V_1 for seven-random potential barriers.

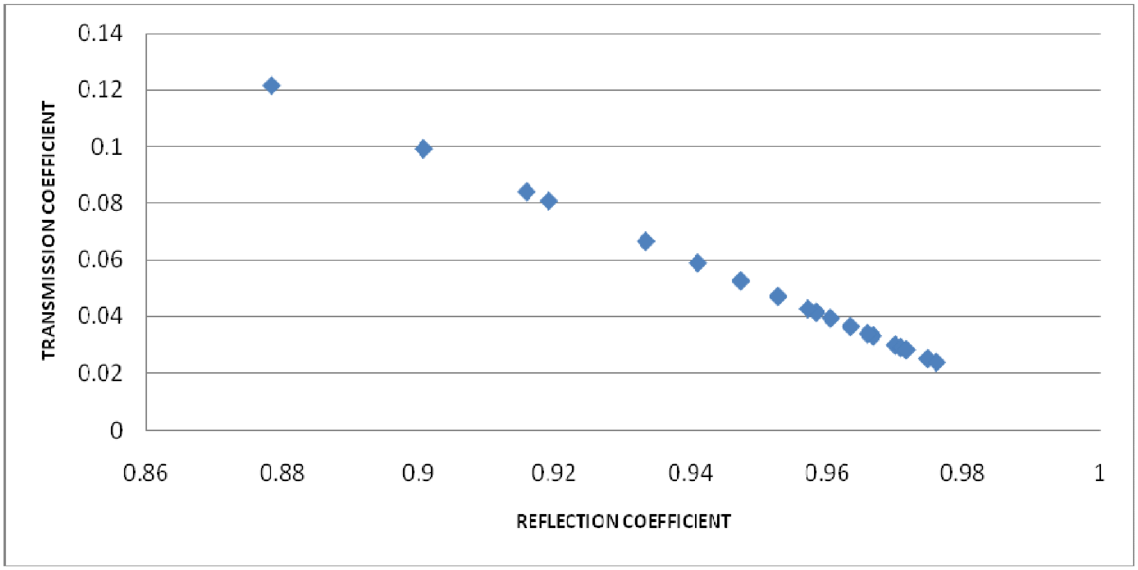


Fig 7 A graph of Transmission coefficient against Reflection coefficient for seven-random potential barriers.

Table 3 Values for the four random potential barriers

$a_1$nm	$a_2$nm	$a_3$nm	$a_4$nm	V_1nV	V_2nV	V_3nV	V_4nV	a_1*V_1 nmnV	T	R=1-T	ln(T)
4.50	4.80	4.10	11.8	4.80	-4.50	4.10	-5.10	21.6	0.237049	0.762951	-1.43949
5.10	5.40	4.50	10.2	5.80	-4.80	4.50	-6.40	29.6	0.193238	0.806762	-1.64383
5.80	6.10	4.80	8.50	6.80	-5.10	4.80	-7.70	39.4	0.163067	0.836933	-1.81359
6.40	6.80	5.10	6.90	7.70	-5.40	5.10	-9.00	49.3	0.142993	0.857007	-1.94496
7.10	7.40	5.40	5.30	8.70	-5.80	5.40	-0.30	61.8	0.125826	0.874174	-2.07286
7.70	8.10	5.80	3.60	9.70	-6.10	5.80	-1.60	74.7	0.11227	0.88773	-2.18685
3.60	3.90	8.70	9.00	8.50	-9.00	8.70	-3.40	30.6	0.131361	0.868639	-2.0298
4.30	4.60	9.00	7.30	9.50	-9.40	9.00	-4.70	40.9	0.116678	0.883322	-2.14833
7.50	7.90	7.00	9.10	4.40	-1.00	0.70	-1.20	33.0	0.24431	0.75569	-1.40932
8.20	8.50	1.00	7.50	5.40	-1.30	1.00	-2.50	44.3	0.198636	0.801364	-1.61628
8.80	9.20	1.30	5.90	6.30	-1.60	1.30	-3.90	55.4	0.170018	0.829982	-1.77185
9.50	9.80	1.60	4.30	7.30	-2.00	1.60	-5.20	69.4	0.146612	0.853388	-1.91996
2.80	3.10	3.30	16.00	2.20	-3.60	3.30	-1.70	6.16	0.571757	0.428243	-0.55904
3.40	3.70	3.60	14.50	3.20	-3.90	3.60	-3.00	10.9	0.370565	0.629435	-0.99273
4.10	4.40	3.90	12.80	4.20	-4.30	3.90	-4.30	17.2	0.274239	0.725761	-1.29376
4.70	5.00	4.30	11.2	5.20	-4.60	4.30	-5.60	24.4	0.217295	0.782705	-1.5265
5.40	5.70	4.60	9.50	6.10	-4.90	4.60	-6.90	32.9	0.183097	0.816903	-1.69774
6.00	6.30	4.90	8.00	7.10	-5.20	4.90	-8.20	42.6	0.155779	0.844221	-1.85931
6.70	7.00	5.20	6.30	8.10	-5.60	5.20	-9.50	54.3	0.135633	0.864367	-1.9978
7.30	7.70	5.60	4.60	9.10	-5.90	5.60	-0.80	66.4	0.120015	0.879985	-2.12014

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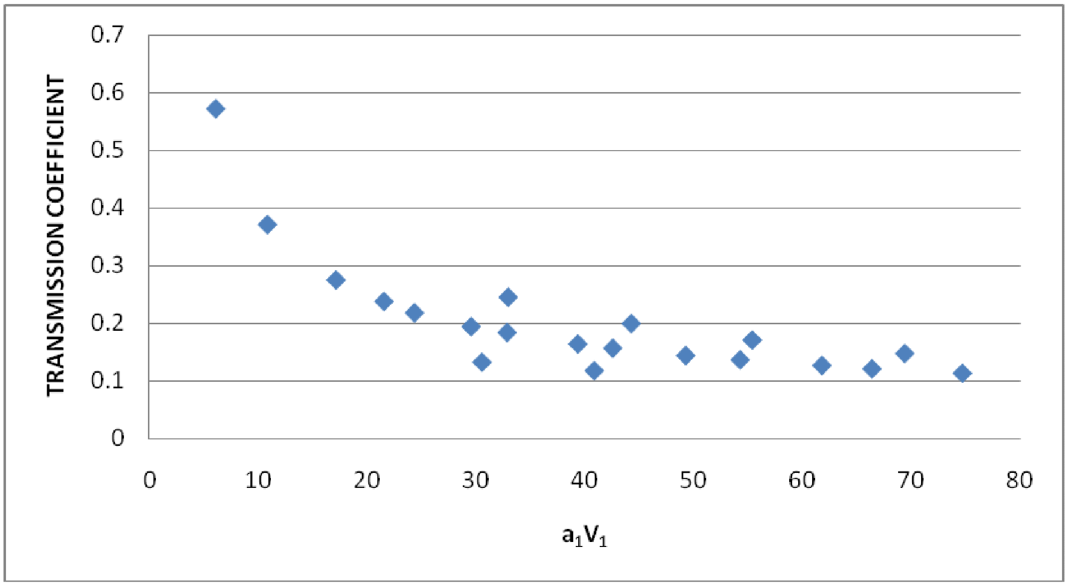

Fig 8 A graph of Transmission coefficient against area $a_1 V_1$ for four- random potential barriers

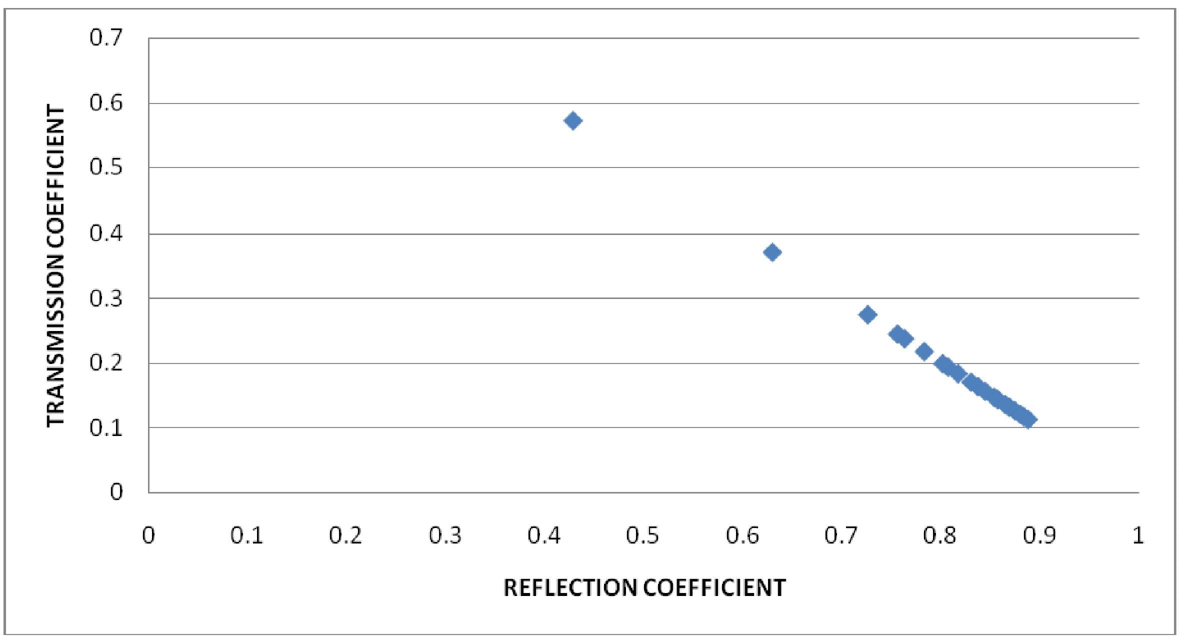

Fig 9 A graph of Transmission coefficient against Reflection coefficient for four-random potential barriers.

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