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Mathematical Modelling of a Two – Link Planar Manipulator Arm Using The Denavit – Hartenberg Matrices

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Abstract

The Remote Manipulator System (R.M.S) is a typical example of a robotic manipulator. It is a two – link planar system which is made up of the base link (link 0), the upper arm link (link 1) and the end – effector or forearm link (link 2). The two joints are revolute and are respectively located at the shoulder and the elbow. In this work, frames are assigned to all the links; the direct and inverse kinematics modelling of the manipulator arm is also being discussed. The complete homogeneous transformation matrices relating gripper's (generally known as the end – effector) frame with the base / reference frame have being derived using Denavit – Hartenberg matrix.

Keywords: Manipulator Arm, Frame, Link, Joint, End – Effector, Mapping, Kinematics.

1.0 Introduction

Kinematics is the science of motion which treats motion without regard to the forces which causes it. The study of manipulator kinematics refers to all the geometrical (the position and orientation) and time based property (velocity, Acceleration) of the motion. The manipulator architecture of a robot is composed of an arm mostly for movements of translation, a wrist for movement of orientation and an end – effector for interaction with the environment and / or external objects. Generally, the term manipulator refers to the mechanical structure a robot must have in order to move an object around in the working volume [1]. According to Marco [2], the term manipulator refers specifically to the arm design, but it can also include the wrist when attention is addressed to the overall manipulation characteristics of a robot. In this work, only the arm design is discussed.

The mobility of a manipulator is due to the degrees of freedom (D.O.F) of the joints in the kinematic chain of the manipulator, when the links are assumed to be rigid bodies. The assembly of sequential links and joints make up a kinematic chain [3]. An assemblage of links and joints was defined by Joseph Duffy [4] as a kinematic chain that may be opened or closed. It can be an opened or closed loop, or it can be a combination of opened and closed loops. A kinematic chain can be of open architecture, when referring to serial connected manipulators, or closed architecture, when referring to parallel manipulators as in the examples shown in Figure 1. These skeletal forms are essentially geometrical models which can be labelled conveniently with the joint variables and the link lengths [4].

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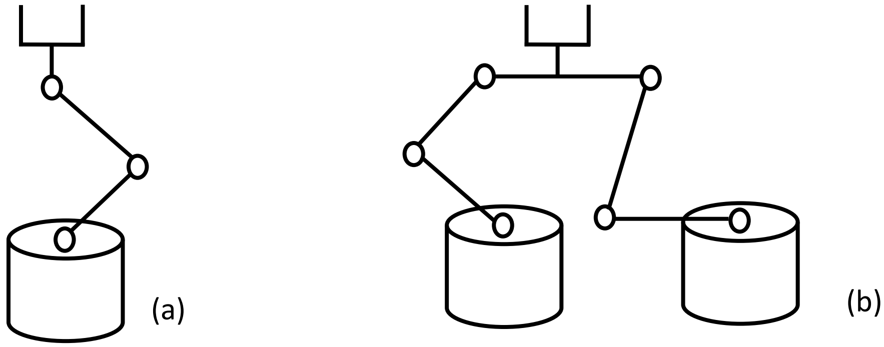
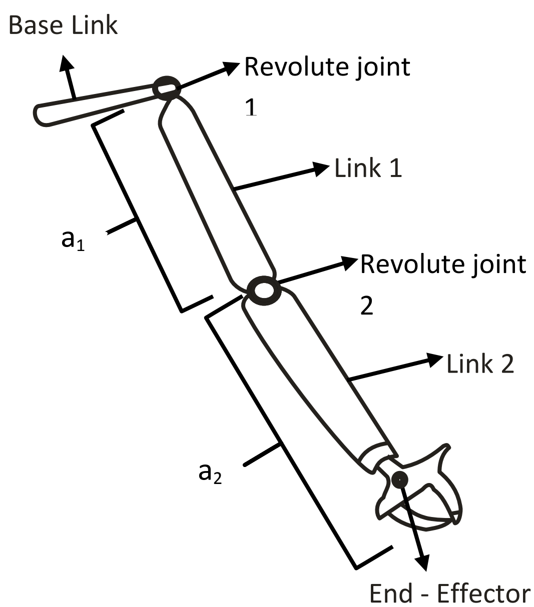


Fig. 1: Planar examples of kinematic chains of manipulators: (a) Serial chain as open type (b) Parallel chain as closed type

The type of manipulator arm described in this paper is basically a series of rigid bodies in a kinematic structure commonly referred to as open kinematic chain; this is shown in Figure 2. Manipulator arm is subdivided into various parts; this includes: the links, the joints and the end – effector. This paper discusses mainly the geometrical property of the motion.



a_1 is the upper arm link length while a_2 is the end – effector link length

Fig. 2: Parts of Manipulator Arm showing the Revolute Joint – Link Structure in an Open Kinematic Chain

1.1 The Links

The links are the linearly rigid bodies which lie in between two joints. The link that is held fixed is called the base link which serves as the frame of reference. Each link member is numbered from 0 to n; the base link is numbered 0 while the most distal link is numbered n.

The links of the R.M.S are made from plastic pipes and each of the links can be likened to human arms connected by joints; a typical example of this is shown in Figure 2.

1.2 The Joints

The joints are the part of the manipulator which are usually instrumented with position sensors which allow relative position or motion of neighbouring links to be measured. The joint types in robots are usually related to revolute and prismatic pairs with one degree of freedom (D.O.F). According to Philippe and Michael **[5]**, degrees of freedom have to do with the number of independent position variables that have to be specified in order to locate all parts of the mechanism. The Revolute or rotary joint takes the form of a hinge in which adjacent links rotate with respect to each other about the joint axis; a good example of the revolute joint is the hinge joint in man while the prismatic or sliding joint is a type of joint in which the relative displacement between links is a translation. They can be modeled as shown in Figure 3.

Fig.3: Schemes for joints in robots: (a) Revolute (b) Prismatic joint

The manipulator arm described in this work has 2 major joints at the arm which are revolute and thus, it has two degrees of freedom. It also possesses the base link (link 0), upper arm link (link 1) and the fore arm link (link 2). For these reasons, it is therefore a two – link revolute planar manipulator. It is a planar manipulator because all the joints lie on the same axis; it also has two co-ordinates (x, y).

1.3 The End – Effector

In robotics, an end – effector is a device or tool connected to the end of a robot arm. The structure of an end – effector, and the nature of the programming and hardware that drives it depends on the intended task. If a robot is designed to set a table and serve a meal, then robotic hands, more commonly called grippers are the most functional end – effectors [1]. The same or similar gripper might be used, with greater force, as a plier or wrench for tightening nuts or crimping wire. In a robot designed to tighten screws, however, a driver–head end – effector is more appropriate; a gripper will be a hindrance in that application. The driver–head can be attached directly to the robot arm and it can also be easily removed and replaced with a device that operates with similar motion, such as a bit for drilling or an emery disk for sanding. Also, for a robot designed to pick up a piece of iron, the end – effector may be an electromagnet.

A robot arm can accommodate only certain end – effector task modes without changes to the auxiliary hardware and/or programming. It is not possible to directly replace a gripper with a screw driver head, for example, and expect a favourable result. It is necessary to change the programming of the robot controller and use a different set of end – effector motors to facilitate torque rather than gripping force [6]. Once this is done, the gripper can then be replaced with a driver – head. The end – effector motion is caused by motions of the intermediate links between the base link and the last link. The relative motion of adjacent links is caused by motion of the joint connecting the two links. There is an actuator in each of the joints that will carry out the actual movement; also, there are sensors to move the actuators in a desired angle. Thus, the end – effector location can be determined by investigating the position and orientation of each link member in series from the base to the end – effector [7]. To describe the position and orientation of each link member, frames are usually used; these frames are represented using square brackets

The end – efffector designed for the R.M.S is a two – finger gripper. In order to represent the position and orientation of end – effector, a co-ordinate frame say [P] is attached to the last link. The location of this frame is now described relative to another [0], that is, frame attached to the base link.

1.4 Descriptions of Positions and Orientation

As earlier explained, frames are usually represented using square brackets. For example, Let [A] be the co-ordinate frame A with X_A, Y_A and Z_A denoting the unit vectors in the direction of the three principal axes. A point P in space may be represented with respect to [A] by the 3x1 position vectors shown in equation (1).

$$ {}^{A}P = \begin{bmatrix} P_X \\ P_Y \\ P_Z \end{bmatrix} \tag{1} $$

The 3×1 position vector stated in equation (1) can be represented in pictorial form as shown in Figure 4.

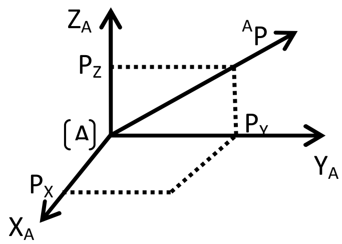


Fig. 4: Pictorial representation of the 3×1 position vector P reference A (${}^{A}P$)

Let point P be a point on a manipulator end – effector or any rigid body. P reference A (${}^{A}P$) represents the position of the end – effector with respect to [A]. To find the orientation of the end – effector, a co-ordinate [B] is attached to the body. This is shown in Figure 5.

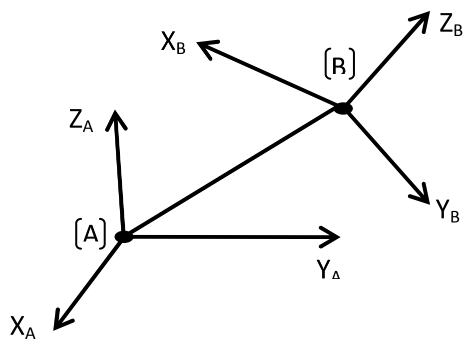


Fig. 5: Diagram showing how the orientation of the end – effector can be found

According to Francis and Andras **[3],** [B] is described relative to [A] with the matrix expression shown in equation (2).

$$ {}^{A}_{B}R = {}^{A}X_{B}\, {}^{A}Y_{B}\, {}^{A}Z_{B} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \tag{2} $$

Where ${}^{A}X_{B}$, ${}^{A}Y_{B}$ and ${}^{A}Z_{B}$ are unit vectors in co-ordinate frame B expressed in terms of frame A. ${}^{A}_{B}R$ is the rotation matrix describing Frame B relative to Frame A (reference point). Since the components of any vector are simply the projections of that vector onto the unit directions of its reference frame, hence,

$$ {}^{A}_{B}R = {}^{A}X_{B}\ {}^{A}Y_{B}\ {}^{A}Z_{B} = \begin{pmatrix} X_B.X_A & Y_B.X_A & Z_B.X_A \\ X_B.Y_A & Y_B.Y_A & Z_B.Y_A \\ X_B.Z_A & Y_B.Z_A & Z_B.Z_A \end{pmatrix} \quad (3) $$

2.0 Theoretical Analysis

2.1 Mapping of Frames

The term mapping is sometimes used when changing descriptions from frame to frame. ${}^{A}_{B}R$, for example, means mapping frame B onto reference frame A or orientation of [B] with reference to [A] as shown in Figure 6.

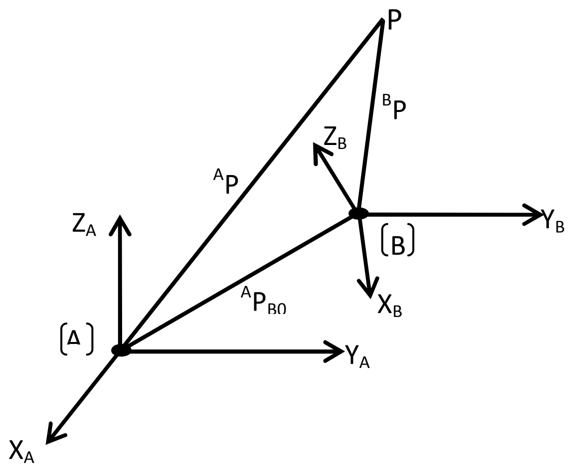


Fig. 6: Diagram illustrating frame mapping using [A] and [B].

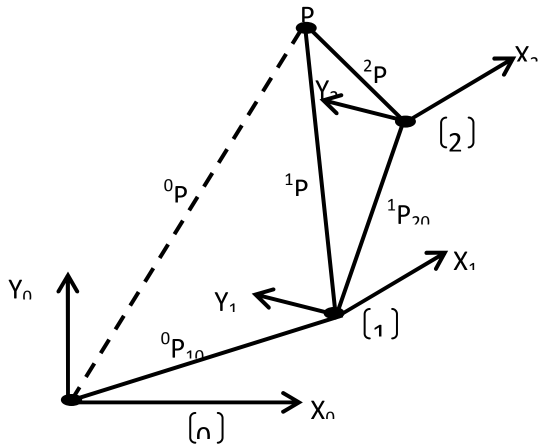
P is a point in ... equation (4) where $^AP_{B0}$ is the position vector of origin of [B] with reference to [A]. ^BP is another position vector with reference to [B].

Therefore,

$$ {}^{A}P = {}^{B}P + {}^{A}P_{B0} \ldots\ldots\ldots\ldots \quad (4) $$

2.2 Frame Mapping in the Two Degrees of Freedom Planar Manipulator Arm

The designed manipulator arm in this work has 3 frames and two co-ordinates (x, y). This is illustrated in Figure 7.



[0] is the base or reference frame cited at the shoulder joint
[1] is the upper arm frame cited at the elbow joint
[2] is the end – effector frame
P is a point in the co-ordinate frame 2
${}^{0}P_{10}$ is the position vector of origin of [1] with reference to [0]
${}^{1}P_{20}$ is the position vector of origin of [2] with reference to [1]
${}^{0}P$ is the position vector of point P in [2] relative to [0]
${}^{1}P$ is the position vector of point P in [2] relative to [1]
${}^{2}P$ is the position vector of point P with reference to [2]

Fig. 7: Diagram of frame mapping in the two – D.O.F planar manipulator arm

The position vector of point P in [2] relative to [1] can be described in terms of the rotation matrix of [2] with reference to [1] { ${}^{1}_{2}R$ } by equation (5).

$$ {}^{1}P = {}^{1}_{2}R\, {}^{2}P + {}^{1}P_{20} \quad (5) $$

Likewise, the position vector of point P in [2] relative to [1] can also be described in terms of the transformation matrix of [2] with reference to [1] by equation (6). The transformation matrix of [2] with reference to [1] is represented in matrix form as shown in equation (7).

$$ {}^{1}P = {}^{1}_{2}T\, {}^{2}P \quad (6) $$

$$ {}^{1}_{2}T = \left(\begin{array}{c|c} R_{1,2} & {}^{1}P_{20} \\ \hline 0\ 0\ 0 & 1 \end{array}\right) \quad (7) $$

Also, $${}^{0}P = {}^{0}_{1}T\, {}^{1}P \quad (8)$$

In general, ${}^{0}P = {}^{0}_{k}T\, {}^{k}P$

Where, ${}^{0}_{k}T$ is the homogeneous transformation matrix relating K – frame with 0 – frame

Substituting equation (6) into (8) gives

$$ {}^{0}P = {}^{0}_{1}T\, {}^{1}_{2}T\, {}^{2}P \quad (9) $$

From equation (9),

$$ {}^{0}_{2}T = T_{0,2} = {}^{0}_{1}T\, {}^{1}_{2}T \quad (10) $$

Where $T_{0,2}$ is the transformation matrix of [2] relative to [0]

$T_{0,1}$ is the position and orientation or transformation of [1] relative to [0].
$T_{1,2}$ is the transformation of [2] relative to [1].
Equation (10) thus shows that the transformation of end – effector frame with respect to the base frame can also be found by finding the transformation of one frame with respect to its preceding one and then finally multiplying them all together to get the transformation of end – effector with reference to base frame [8].

In terms of the known description of frame 1 and frame 2, the expression for $T_{0,2}$ can be expressed in terms of rotation matrix and position vector as

$$ {}^{0}_{2}T = \left(\begin{array}{c|c} {}^{0}_{1}R\,{}^{1}_{2}R & {}^{0}_{1}R\,{}^{1}P_{20} + {}^{0}P_{10} \\ \hline 0\,0\,0 & 1 \end{array}\right) \tag{11} $$

3.0 Experimental Work

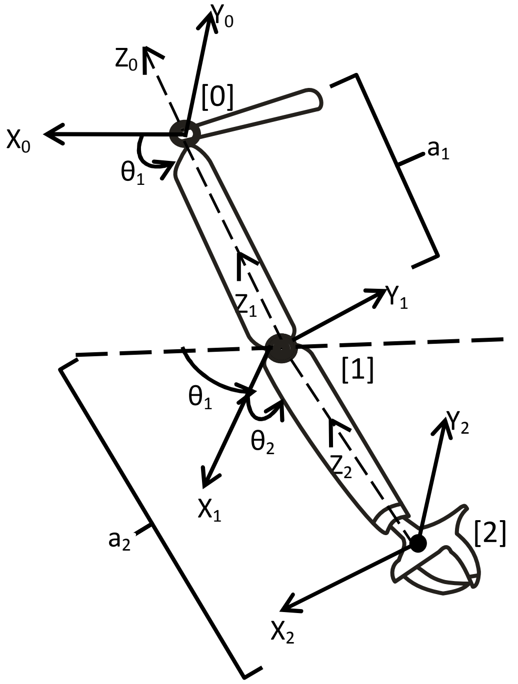
3.1 Assigning Link Frames

The Manipulator arm explained in this paper and as illustrated in Figure 8 is described with respect to three frames namely:

1) The base or reference frame cited at the shoulder joint. This is the frame attached to link 0 (that is, the base of the manipulator) **[9].**
2) The upper arm frame cited at the elbow joint. This is the frame attached to link 1.
3) The fore arm frame. This is the frame attached to link 2 or the end – effector link.

To define a co-ordinate frame attached to each link, the common normal between the two joint axes must be determined for the link **[3]**. However, no such link exists for the base and the last links since each of these links has only one joint axis. For these two links, the co-ordinate frames are defined as follows.

- For the last link, the origin of the co-ordinate frame can be chosen at any convenient point of the end – effector. The orientation of the co-ordinate frame must be determined so that X_2 axis intersects the last joint axis at a right angle. The choice of Y_2 is arbitrary **[10].**
- For the base link, the origin of the co-ordinate frame can be chosen at any arbitrary point on the joint axis; the Z_o axis must be parallel to the joint axis while the orientation of X_o and Y_o axis about the joint axis is arbitrary.
- For the first link, the origin of the co-ordinate [1] is located at the intersection of the joint axis 1 and the common normal between joint axis 2 and 1. The X_1 axis is directed along the extension line of the common normal while the Z_1 axis is along the joint axis 1. The Y_1 axis is chosen such that the resultant [Y] forms a right hand co-ordinate system. Since all the joints are on the same plane for the 2 D.O.F planar manipulator arm, the common normal between the two joint axes cannot be obtained; hence, the choice of X_1 axis is arbitrary. The Y_1 axis is still chosen such that the resultant [Y] forms a right hand co-ordinate system.



$a_1 = 33.5$cm; $a_2 = 39.0$cm; $\theta_1 = 35^0$; $\theta_2 = 15^0$

Where [0] = Reference or Base frame cited at shoulder joint

[1] = Frame 1 cited at elbow joint

[2] = Frame 2 cited at end – effector

X_0 & Y_0 are respectively the X and Y co-ordinates of [0]

X_1 & Y_1 are respectively the X and Y co-ordinates of [1]

X_2 & Y_2 are respectively the X and Y co-ordinates of [2]

It should be noted that Z_0, Z_1, and Z_2 co-ordinates are not taken into consideration for [0], [1], and [2] respectively because all the joints are on the same plane.

Fig. 8: Assigning Frames to the Two – Links Revolute Planar Manipulator Arm

4.0 Results and Discussion

4.1 The Denavit – Hartenberg (D-H) Model

In a general form, the transformation of one frame can be obtained with reference to the other frame by using the Denavit – Hartenberg general formula shown in equation (12). [1], [11], [12].

$$
{}^{i-1}_{\;\;i}T = \begin{pmatrix} \mathrm{Cos}\theta_i & -\mathrm{Sin}\theta_i\,\mathrm{Cos}\alpha_i & \mathrm{Sin}\theta_i\,\mathrm{Sin}\alpha_i & a_i\,\mathrm{Cos}\theta_i \\ \mathrm{Sin}\theta_í & \mathrm{Cos}\theta_i\,\mathrm{Cos}\alpha_í & -\mathrm{Cos}\theta_i\,\mathrm{Sin}\alpha_i & a_i\,\mathrm{Sin}\theta_i \\ 0 & \mathrm{Sin}\alpha_i & \mathrm{Cos}\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

The variables employed in finding the transformation of one frame with reference to other frames are known as Denavit – Hartenberg Parameters. They are: θ_i, α_i, a_i and d_i, with the following definitions:

- í: Number of links
- θ_i: This is the angle between $X_{í-i}$ and $X_í$ measured about $Z_{í-i}$; this is often called the joint angle or joint displacement.
- α_i: This is the angle between joint axis í and joint axis í+i in the right hand sense. That is, the angle between $Z_{í-1}$ and $Z_í$ measured along $X_í$ .It is also called link twist.
- a_i: This is the length of the common normal. That is, the distance from $Z_{í-i}$ and $Z_í$ measured along $X_í$. It is also called link length.
- d_i: This is the distance from $X_{í-i}$ to $X_í$ measured along $Z_{í-i}$. It is at times called link offset.

The θí, α_i, a_i and d_i are collectively referred to as the Link or Denavit – Hartenberg parameters.

4.2 Calculating the Denavit – Hartenberg Parameters

The designed manipulator is a two – link revolute planar arm in which the two joint axes of the two – link

Manipulator arms are parallel; according to Francis and Andras [3], the link offset d_i for parallel joint axis is zero. Since the two axes are parallel, no angle deviation exist between them; hence, their link twist $\alpha_i = 0$. Before the link parameters can effectively be derived, frames must be assigned to the various links as shown in Figure 8. The link or rigid lamina upon which the shoulder joint is firmly mounted on forms the base link or the reference frame/link. The D-H parameters for each links tabulated in Table 1 were obtained when the manipulator arm was at its folding position and the mathematics of the transformation matrix for the two – D.O.F revolute planar manipulator arm was carried out using the data in Table 1.

Table1: The D-H Parameters for the Two Links

Link no í	a_i	α_i	d_i	θ_i
1	33.5 cm	0	0	35^0
2	39.0 cm	0	0	15^0

4.3 Finding the Transformation Matrix for the two – D.O.F Revolute Planar Manipulator Arms.

Direct Kinematics is a mechanics which deals with the determination of transformation (position and orientation) of any point on the manipulator with respect to any other frame, given the joint displacements of the manipulator [13]. For a manipulator, the overall homogeneous transformation matrix can be derived using Denavit – Hartenberg matrix which is given in equation (12); thus, the position and orientation of one frame with reference to the other can be obtained using the parameters in Table 1. From equation (12),

Let $\text{Cos}\theta_i = c_1$

$\text{Sin}\theta_i = s_1$

$\theta_1 = 35^0$

$\theta_2 = 15^0$

Recall the transformation of end – effector with reference to base frame stated in equation (10)

$$ {}^0_1T(\theta_1) = \begin{pmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{13} $$

$$ {}^0_2T = T_{0,2} = {}^0_1T\,{}^1_2T $$

Substituting the parameters into equation (13), gives

$$ {}^0_1T(\theta_1) = \begin{pmatrix} 0.82 & -0.57 & 0 & 27.47 \\ 0.57 & 0.82 & 0 & 19.10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{14} $$

$$ {}^1_2T(\theta_2) = \begin{pmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{15} $$

$$ {}^{1}_{2}T(\theta_2) = \begin{pmatrix} 0.97 & -0.26 & 0 & 37.83 \\ 0.26 & 0.97 & 0 & 10.04 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{16} $$

$$ {}^{0}_{2}T = \begin{pmatrix} 0.82 & -0.57 & 0 & 27.47 \\ 0.57 & 0.82 & 0 & 19.10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.97 & -0.26 & 0 & 37.83 \\ 0.26 & 0.97 & 0 & 10.04 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{0}_{2}T = \begin{pmatrix} 0.6472 & -0.7661 & 0 & 52.7108 \\ 0.7661 & 0.6472 & 0 & 48.9779 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{17} $$

Equation (17) is the transformation matrix of end – effector frame with respect to the base frame given the joint vectors.

4.4 Inverse Kinematics

Inverse kinematics is a mechanics that deals with the determination of joint displacements given a position and orientation (known as transformation) of the end – effector **[14]**. The joint angles are obtained from the end – effector transformation matrix given in equation (17).

Let T (equation (18)) be the position and orientation of the end – effector.

$$ T = \begin{pmatrix} n_x & s_x & a_x & P_x \\ n_y & s_y & a_y & P_y \\ n_z & s_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{18} $$

From equation (18), the total number of objective functions is 12, comprising of three vectors (each with three elements) and a position vector (with three elements). $[n_x, n_y, n_z]^T$ is normal vector (x – axis) of end – effector frame, $[s_x, s_y, s_z]^T$ is sliding vector (y – axis) of end – effector frame and $[a_x, a_y, a_z]^T$ is approach vector (z – axis) of end – effector frame all with respect to reference frame **[7]**. Here, sliding vector represents the vector normal to the surface of distal link of end – effector. Thus, this vector has to be collinear with normal vector of the surface of the object at the point of contact. Hence, the objective function is formulated using the normal and position vectors (six components) for achieving the desired position with proper alignment of object and end – effector surfaces normal. That is, a single objective function is formed from six objective functions by weighted summation.

Recall that $T = {}^{0}_{2}T = T_{0,2} = {}^{0}_{1}T\,{}^{1}_{2}T$

From equations (13) and (15),

$$ {}_2^0T = T = \begin{pmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}_2^0T = T = \begin{pmatrix} c_1\,c_2 - s_1s_2 & -c_1\,s_2 - s_1c_2 & 0 & a_2c_1c_2 - a_2s_1s_2 + a_1c_1 \\ s_1\,c_2 + c_1s_2 & -s_1\,s_2 + c_1c_2 & 0 & a_2s_1c_2 - a_2c_1s_2 + a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (19) $$

Since equations (18) and (19) are equal, the elements of the normal vector can be compared with that of the position vector. Comparing the element of row1 and column 1 of equation (18) with that of equation (19) gives

$$ n_x = c_1c_2 - s_1s_2 \qquad (20) $$

From the trigonometric ratio

$$ Cos(\theta_1 + \theta_2) = Cos\theta_1 Cos\theta_2 - Sin\theta_1 Sin\theta_2 \;=\; c_1c_2 - s_1s_2 $$

$$ \therefore Cos(\theta_1 + \theta_2) = c_1c_2 - s_1s_2 = n_x $$

$$ \theta_1 + \theta_2 = Cos^{-1} n_x \qquad (21) $$

Substitute the value of n_x in equation (17) to (21)

$$ \theta_1 + \theta_2 = Cos^{-1} 0.6472 = 49.669 \approx 50^0 \qquad (22) $$

Comparing the element of row1 and column 4 of equation (18) with that of equation (19) gives

$$ P_x = a_2(c_1c_2 - s_1s_2) + a_1c_1 \qquad (23) $$

Substitute equation (20) into equation (23) to get

$$ P_x = a_2n_x + a_1c_1 \qquad (24) $$

Substitute the values of P_x and n_x in equation (17) and a_1 and a_2 in Table 1 into equation (24) to get

$$ 52.7108 = 39 \times 0.6472 + 33.5 \times C_1 $$

$$ 52.7108 = 25.2408 + 33.5C_1 $$

$$ 52.7108 - 25.2408 = 33.5C_1 $$

$$ 27.47 = 33.5C_1 $$

$$ C_1 = \frac{27.47}{33.5} = 0.82 $$

$$ Cos\theta_1 = 0.82 $$

$$ \theta_1 = Cos^{-1} 0.82 = 34.92^0 $$

$$ \theta_1 \approx 35^0 $$

From equation (22),

$$\theta_1 + \theta_2 = 50^0$$

$$35 + \theta_2 = 50^0$$

$$\theta_2 = 50^0 - 35^0 = 15^0$$

The joint displacements are:

$$\theta_1 = 35^0$$

$$\theta_2 = 15^0$$

5.0 Conclusion

In this paper, a two – degree of freedom manipulator joint which is revolute is presented. The link parameters and the individual joint transformation matrices with respect to the reference frame have been derived using the Denavit – Hartenberg model. Detailed study was also carried out for the solution of forward and inverse kinematics of a two – link planar manipulator arm with emphasis on the joint angles (θ_1 and θ_2) and the result thus showed that a good correlation exist between the transformation of any point on the manipulator end – effector with respect to the base frame, given the joint displacements of the manipulator and the determination of joint displacements given a position and orientation of the end – effector.

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