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Asymptotic expansion of unsteady gravity flow of a power-law fluid through a porous medium.

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Abstract

We present a paper on the asymptotic expansion of unsteady non-linear rheological effects of a power-law fluid under gravity. The fluid flows through a porous medium. The asymptotic expansion is employed to obtain solution of the non-linear problem. The results show the existence of traveling waves. It is assumed that the viscosity is temperature dependent. We investigate the effects of velocity on the temperature field. We investigate the power-law viscosity exponent on the flow, the Darcy parameter on the temperature profiles and the results obtained are discussed.

Keywords: Unsteady gravity flows; Porous media; Non – Newtonian power- law fluid and Asymptotic expansion.

1.0 Introduction

There have been several studies on the gravity flow of a power-law fluid through a porous medium. Peter and Ayeni[1] studied a note on unsteady temperature equation for gravity flow of a power-law fluid through a porous medium. Cortell [2] studied the unsteady gravity flows of a power-law fluid through a porous medium. Olajuwon and Ayeni [3] treated the flow of a power-law fluid with memory past an infinite plate. Pascal and Pascal [4] studied the similarity solution to some gravity flows of non-Newtonian fluids through porous media. Peter and Ayeni [5] investigated the analytical solution of unsteady gravity flow of a power-law fluid through a porous medium. Zueco [6] also investigated the numerical solutions for unsteady rotating high-porosity medium.

In this paper ,we are interested in the asymptotic behavior of the flow in order to see what the solution will look like in this asymptotic limit.

2.0. Mathematical Formulation

The Mathematical formulation of the flow relevant to the problem is governed by the set of equations proposed by[2].

$$v_r = -\left(\frac{k\rho}{\mu_{ef}}\right)^{\frac{1}{n}} \frac{\partial h}{\partial r}\left|\frac{\partial h}{\partial r}\right|^{\frac{1-n}{n}}$$

$$\frac{\partial (h v_r)}{\partial r} = -\Phi \frac{\partial h}{\partial t}$$

$$\frac{1}{R}\frac{\partial}{\partial R}\frac{\partial h}{\partial R}\left(Rh\left|\frac{\partial h}{\partial R}\right|^{\frac{1-N}{N}}\right) = -\Phi\left(\frac{\mu_{ef}}{k\rho}\right)^{\frac{1}{n}}\frac{\partial h}{\partial t}$$

$$\frac{d}{d\eta}\left(\eta f f^1 \left|f^1\right|^{\frac{1-n}{n}}\right) = a^2\eta\left(\alpha f - \frac{n+\alpha}{n+1}\eta\frac{df}{d\eta}\right) \tag{2.1}$$

Where $a^2 = \Phi\left(\frac{\mu_{ef}}{k\rho}\right)^{\frac{1}{n}}$

k is the permeability, ρ is the density and μ_{ef} is the effective viscosity.

$$f(\eta_1) = 1 \tag{2.2}$$

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$$\left(\frac{df}{d\eta}\right)\eta_1 = 1 \tag{2.3}$$

3.0. Method of Solution

In order to solve the problem and keep it tractable, the set of non-linear ordinary differential equations (2.1) with boundary conditions have been solved analytically by using asymptotic expansion.

Case 1

From equation (2.1) diving through by a^2 we get

$$\frac{1}{a^2}\frac{d}{d\eta}\left[\eta f f'(f')^{\frac{1-n}{n}}\right] = \eta\left[\alpha f - \frac{\alpha+n}{n+1}\eta f'\right] \tag{3.1}$$

From Eq.(3.1) let $a^2 \to 0$ to get

$$\eta\left(\alpha f - \frac{\alpha+n}{n+1}\eta f'\right) = 0 \tag{3.2}$$

Since $\eta \neq 0$ Eq.(3.2) implies,

$$\left(\frac{\alpha(n+1}{n+\alpha}\frac{f}{\eta}\right) = f' \tag{3.3}$$

Let $f = f_0 + a^2 f_1 + a^4 f_2 + ...$ (3.4)

$$f' = f_0' + a^2 f_1' + a^4 f_2' + ... \tag{3.5}$$

$$f_0(1) = 1 \tag{3.6}$$

Putting equation (3.4)-(3.5) into (3.3) we obtain

$$\frac{\alpha(n+1)}{\alpha+n}\eta^{-1}(f_0 + a^2 f_1 + a^4 f_2 + ...) = f_0' + a^2 f_1' + a^4 f_2' + ... \tag{3.7}$$

$$a^0 : \frac{\alpha(n+1)}{\alpha+n}\eta^{-1} f_0 = f_0' \tag{3.8}$$

Let $f_0 = p,\ f_0' = \dfrac{dp}{d\eta}$

Integrating we obtain

$$f_0 = \eta^{\frac{\alpha(n+1)}{\alpha+n}} \tag{3.9}$$

Case 2

From equation (2.1)

Let $n = 1$

Then, we obtain

$$ff' = a^2\eta\left(\alpha f - \frac{\alpha+n}{n+1}\eta f'\right) \tag{3.10}$$

Putting equation (3.4)-(3.5) into (3.10) we obtain

$$\left(a^2\right)^0 : f_0 f_0' = 0 \tag{3.11}$$

$$\left(a^2\right)^1 : f_0 f_1' + f_1 f_0' = \eta\left(\alpha f_0 - \frac{\alpha+1}{2}\eta f_0'\right) \tag{3.12}$$

$$\left(a^2\right)^2 : f_0 f_2' + f_1 f_1' + f_2 f_0' = \eta\left(\alpha f_1 - \frac{\alpha+1}{2}\eta f_1'\right) \tag{3.13}$$

$$\left(a^2\right)^3 : f_1 f_2' + f_2 f_1' = \eta\left(\alpha f_2 - \frac{\alpha+1}{2}\eta f_2'\right) \qquad (3.14)$$

$$\left(a^2\right)^4 : f_2 f_2' = 0 \qquad (3.15)$$

Integrating (3.11)-(3.15) together with the boundary condition (3.6) to get

$$f_0 = 1 \qquad (3.16)$$

$$f_0' = 0 \qquad (3.17)$$

Substitute equation(3.16)-(3.17) into (3.12) to get

$$f_1' = \alpha\eta \qquad (3.18)$$

$$f_1(1) = 0 \qquad (3.19)$$

Integrating (3.18) subject to the condition(3.19) to get

$$f_1 = \alpha\frac{\eta^2}{2} - \frac{\alpha}{2} \qquad (3.20)$$

Differentiating Eq.(3.18) to get

$$f_1' = 2\eta\frac{\alpha}{2} = \alpha\eta \qquad (3.21)$$

Substitute equations (3.16)-(3.21) into (3.13) to get

$$f_2' = \frac{-\alpha\eta^3 - \alpha^2\eta^3}{2} \qquad (3.22)$$

$$f_2(1) = 0 \qquad (3.23)$$

Integrating (3.22) subject to the condition(3.23) to get

$$f_2 = \frac{-\alpha(\eta^4 - 1) - \alpha^2(\eta^4 - 1)}{8} \qquad (3.24)$$

Case 3

From equation (2.1)

Let

$$f(\eta) = a + \frac{b}{1+\eta} + \frac{c}{\left(1+\eta^2\right)} \qquad (3.25)$$

$$f(0) = 10 \qquad (3.26)$$

$$f^1(0) = -0.3 \qquad (3.27)$$

$$f(1) = 2 \qquad (3.28)$$

We obtain

$$f(\eta) = \frac{0.3}{1+\eta} + \frac{15.7}{1+\eta^2} - 6 \qquad (3.29)$$

From equation (2.1) together with the boundary conditions

$$f(0) = 10 \qquad (3.30)$$

$$f(1) = 1 \qquad (3.31)$$

$$f^1(0) = -0.3 \qquad (3.32)$$

We obtain

$$f(\eta) = \frac{0.3}{1+\eta} + \frac{17.7}{1+\eta^2} - 8 \qquad (3.33)$$

Case 4

Integrating equation (3.2)

let $a^2 \to \infty$ to get

$$f(\eta) = 2\eta^{\frac{\alpha(n+1)}{\alpha+n}} \qquad (3,34)$$

From equation (2.1)

assume $a^2 = 0$

When $n = \dfrac{1}{3}$

Integrating to get

$$f(\eta) = \left[-3.333\eta^{1/2} - 0.6\right]^{3/4} \qquad (3.35)$$

4.0 Results

The analytical solutions of equation (3.6), (3.10) and (3.11) were provided for various values of parameter α and n.

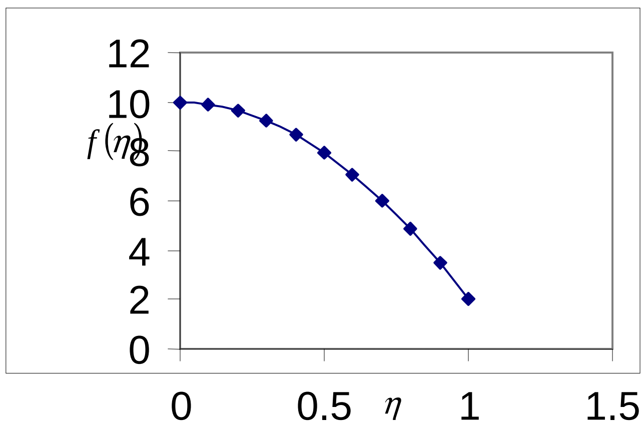

Fig4.1: Graph of the velocity function f against the similarity variable η when α = -0.3, β = -7.7 and n= ½.

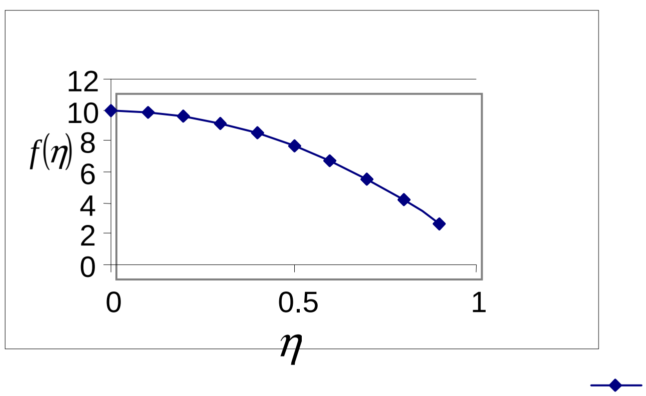

Fig4.2: Graph of the velocity function f against the similarity variable η when α = -0.3, β = -8.7 and n= ½.

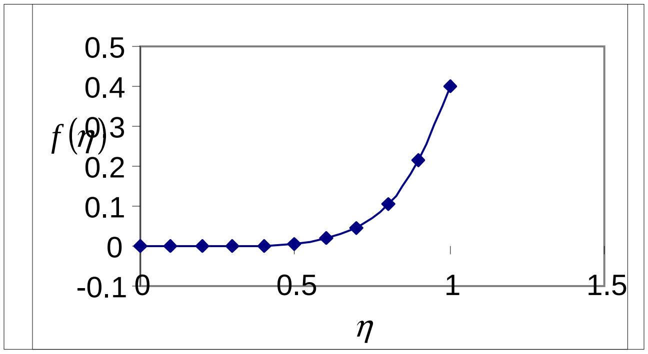

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Fig4.3: Graph of the velocity function f against the similarity variable η when α =2 and n= ½.

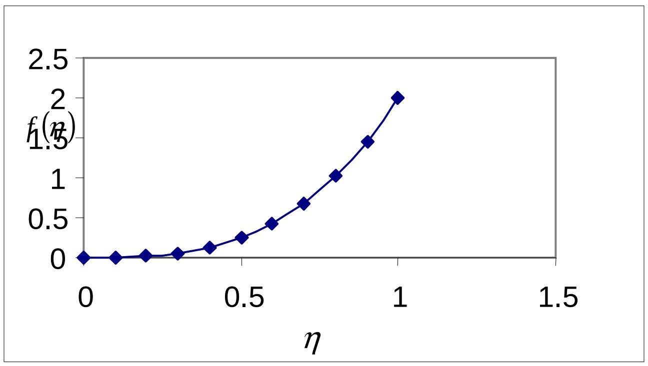

Fig4.4: Graph of the velocity function f against the similarity variable η when α =-1 and n= ½.

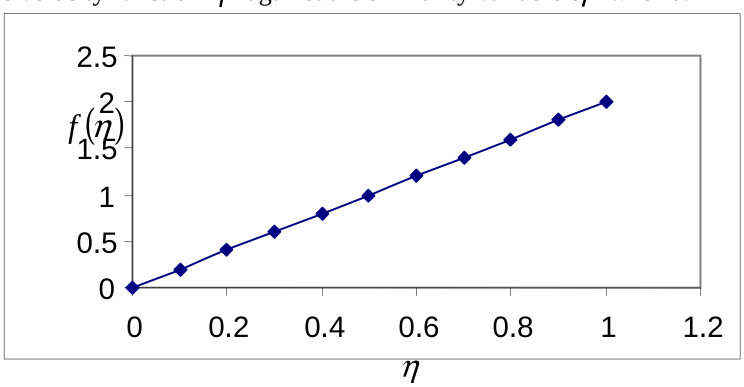

Fig4.5: Graph of the velocity function f against the similarity variable η when α =1 and n= ½.

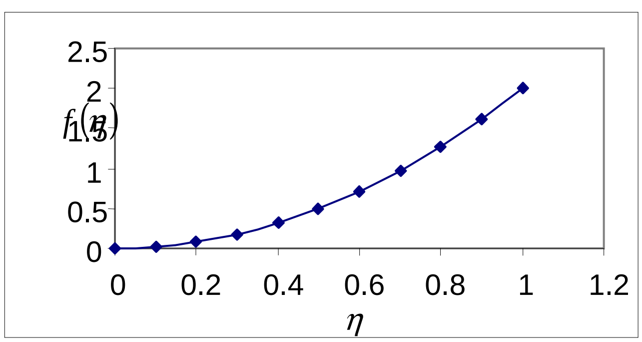

Fig4.6: Graph of the velocity function f against the similarity variable η when α =-2 and n= ½.

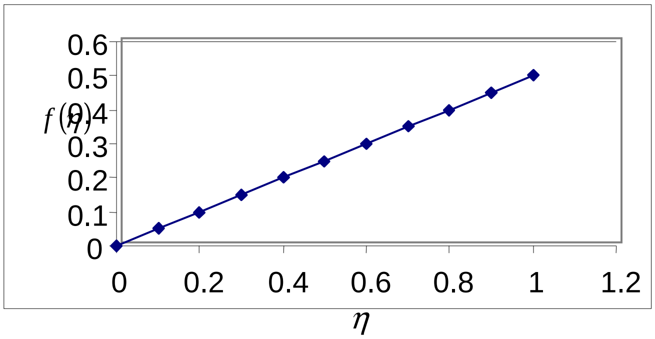

Fig4.7:Graph of the velocity function f against the similarity variable η when $\alpha = {}^{1}/_{10}$ and n= ½.

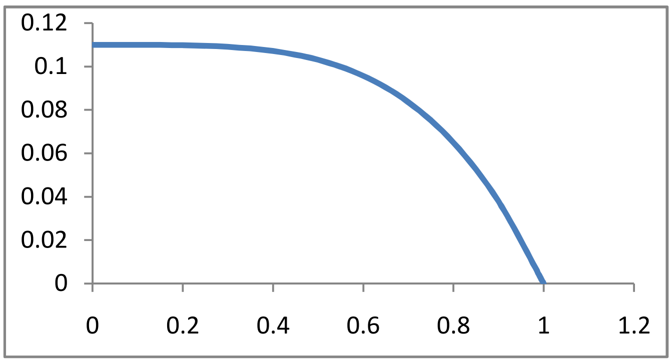

Fig4.8:Graphofthevelocityfunction f againstthesimilarityvariable η when $\alpha = {}^{1}/_{10}$

Conclusion

We obtained a suitable expression for unsteady gravity flow of a power-law fluid through a porous medium. The unsteady profile and the velocity profile were studied for various values of α and power-law viscosity index n. The fluid is Pseudo-plastic for n<1.We plot the graph of velocity function f against the similarity variable η. It is seen from Figs 4.1-4.2 that the effect of the power-law index n is to decrease the flow characteristics. It is seen from Figs 4.3-4.8that the effect of the power-law index n is to increase the flow characteristics. On the other hand, from analytical calculations we also see that the parameter n affects both the flow characteristics and the accuracy of the approximate solutions significantly.

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