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An Approximate Analytical Solution To Heat And Mass Transfer In The Boundary Layers On An Exponentially Stretching Surface(Weighted Residual Approach)

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Abstract

An approximation to the exact solution of heat and mass transfer problem was obtained through the method of weighted residual via partition. Gauss Legendre formula and Shifted Laguerre formula of integration were used for integration in finite and semi infinite region. Results obtained were compared with the previous method values to know the effectiveness of our method.

1.0 Introduction

Most physical problems rarely have exact solution. The problems of heat and mass transfer on an exponentially stretching continuous surface is therefore not an exception. Numerical methods such as shooting have been applied to solve this problem in [1]. An approximation to the exact solution of this type of problem can also be obtained using methods like differential transform [2], Adomian decomposition[3]. In [1], an approximation formula was obtained and applied to the problem and the results were compared with that of shooting.

In this paper, weighted residual method will be applied using partition method to minimise the residual, with the roots of Laguerre polynomials used as partition points and Legendre formular used to evaluate integral in finite domain and shifted laguerre for semi infinite domain.

2.0 Basic Equations

The laminar flow-boundary and thermal boundary layers on an impermeable plane wall stretching with velocity $u_w = u(x)$ and a given temperature distribution $T_w = T_w(x)$ moving through a quiescent incompressible fluid of a constant temperature T_∞ are given by[1] as

$$\begin{aligned} &U_x + V_y = 0 \\ &UU_x + VU_y = \upsilon U_{yy} \\ &UT_x + VT_y = \alpha T_{yy} \end{aligned} \tag{2.1}$$

with boundary conditions

$$\begin{aligned} &U(x,0) = U_w(x), U(x,\infty) = 0, V(x,0) = 0 \\ &T(x,0) = T_w(x), T(x,\infty) = T_\infty. \end{aligned} \tag{2.2}$$

After using the similarity transformation in [1], the dimensionless function $f(\eta)$ and $\vartheta(\eta)$ satisfy the differential equations

$$\begin{aligned} &f''' + ff'' - \beta f' = 0, \beta \equiv 2 \\ &\vartheta'' + p(f\vartheta' - af'\vartheta) = 0 \\ &f'(0) = 1,\ f'(\infty) = 0,\ f(0) = 0 \\ &\vartheta(0) = 1, \vartheta(\infty) = 0 \end{aligned} \tag{2.3}$$

where $p = \dfrac{\upsilon}{\alpha}$ is the prandtl number.

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3.0 Method of solution

We seek an approximate solution in the form of a polynomial to the differential equation of the form

$$L[u(x)] = f \quad \text{in the domain } \Omega \tag{3.1}$$

$$B_\mu[u] = \Omega \quad \text{On } \partial\Omega \tag{3.2}$$

Where L[u] denotes a general differential operator (linear or non linear) involving spatial derivatives of dependent variable u. F is a known function of position, $B_\mu[u]$ represents the appropriate number of boundary conditions and Ω is the domain with the boundary $\partial\Omega$.

A trial function of the form

$$\phi = \phi_0 + \sum_{i=1}^{n} c_i \phi_i \tag{3.3}$$

is assumed, where c_i are constants to be determined which satisfy the given boundary condition(3.2). The trial function is chosen in such a way that it satisfies all the given boundary conditions including those at infinity. Substitution of equation (3.3) into equation(3.1) gives the residual function R(x). The idea is to minimise the residual function as small as possible. To minimise the residual R(x) were expressed

$$\int_0^\infty R(x)dx = \int_0^a R(x)dx + \int_a^b R(x)dx + ... + \int_b^\infty R(x)dx \approx 0$$

The values a, b...are roots of the Laguerre polynomials. Finite integration of the form

$\int_a^b R(x)dx$ is evaluated using Gauss Legendre formula

$\int_{-1}^{1} R(x)dx = \sum_{k=1}^{n} A_k R(x_k)$ with a transformation equation

$$z = \frac{2x}{b-a} - \frac{b+a}{b-a} \quad \text{and}$$

$A_k = \dfrac{2(1-x_k{}^2)}{n^2(p_{n-1}(x_k))^2}$ and the arguments x_k are the zeros of the nth Legendre polynomial

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n, P_0(x) = 1$$

Table 3.1 shows the roots of Legendre polynomial x_k and the corresponding weight function A_k for n =6.

Table3.1 (Roots of Legendre Polynomial and the corresponding weight functions)

x_i	A_i
±0.93246951	0.17132449
±0.66120939	0.36076157
±0.23861919	0.46791393

Also, infinite integration of the form

$$\int_b^\infty R(x)dx = e^{-b}\sum_{k=1}^{n} w_k e^{x_k}.e^b R(x_k + b)$$

Is evaluated, where $w_k = \dfrac{(n!)^2}{x_k(L_n{}'(x_k))^2}$ and the arguments x_k are the zeros of the nth Laguerre polynomial

$$L_n(x) = e^x \frac{d^n}{dx^n}(e^{-x}x^n)$$

Table 3.2 shows the roots of Laguerre polynomial x_k and the corresponding weight function w_k for n=6.

Table 3.2 (Roots of Laguerre Polynomials and their corresponding weight functions)

x_k	W_k
0.22284660	0.45896467
1.18893210	0.41700083
2.99273633	0.11337338
5.77514357	0.1039920
9.83746742	0.00026102
15.98287398	0.00000090

4.0 Discussion and Results

We assumed the trial function

$$f=\sum_{i=0}^{n} c_i e^{\frac{-i\eta}{4}}$$

With n=8, imposing the boundary condition f(0)=0, we have

$$c_0+c_1+c_2+c_3+c_4+c_5+c_6+c_7+c_8=0 \qquad (4.1)$$

with $f\,'(0)=1$ we have

$$\frac{-1}{4}c_1-\frac{1}{2}c_2-\frac{3}{4}c_3-c_4-\frac{5}{4}c_5-\frac{3}{4}c_6-\frac{7}{4}c_7-2c_8-1=0 \qquad (4.2)$$

The third condition $f\,'(\infty)=0$ is satisfied automatically. The residual R is

$$R=\frac{-1}{64}c_1e^{\frac{-1}{4}\eta}-+\frac{1}{8}c_2e^{\frac{-1}{2}\eta}-\frac{27}{64}c_3e^{\frac{-3}{4}\eta}+-c_4e^{-\eta}-\frac{125}{64}c_5e^{\frac{-5}{4}\eta}-\frac{27}{8}c_6e^{\frac{-3}{2}\eta}-\frac{343}{64}c_7e^{\frac{-7}{4}\eta}-$$
$$8c_8e^{-2\eta}(c_0+c_1e^{\frac{-1}{4}\eta}+c_2e^{\frac{-1}{2}\eta}+c_3e^{\frac{-3}{4}\eta}+c_4e^{-\eta}+c_5e^{\frac{-5}{4}\eta}+c_6e^{\frac{-3}{2}\eta}+c_7e^{\frac{-7}{4}\eta}+8c_8e^{-2\eta})$$
$$(\frac{1}{16}c_1e^{\frac{-1}{4}\eta}+\frac{1}{4}c_2e^{\frac{-1}{2}\eta}-\frac{9}{16}c_3e^{\frac{-3}{4}\eta}+c_4e^{-\eta}+\frac{25}{16}c_5e^{\frac{-5}{4}\eta}+\frac{9}{4}c_6e^{\frac{-3}{2}\eta}\ \frac{49}{16}c_7e^{\frac{-7}{4}\eta}+8c_8e^{-2\eta})$$
$$-2(\frac{-1}{4}c_1e^{\frac{-1}{4}\eta}-\frac{1}{2}c_2e^{\frac{-1}{2}\eta}-\frac{1}{4}c_3e^{\frac{-3}{4}\eta}--c_4e^{-\eta}-\frac{5}{4}c_5e^{\frac{-5}{4}\eta}-\frac{3}{2}c_6e^{\frac{-3}{2}\eta}-\frac{7}{4}c_7e^{\frac{-7}{4}\eta}-2c_8e^{-2\eta})^2$$

Now use the root of Laguerre polynomial with n=6 as the partition points, that is $[0-0.22284660],[0.22284660-1.18893210],[1.18893210-2.99273633],[2.99273633-5.77514357],$ $[5.77514357-9.83746742],[9.83746742-15.98287398],[15.98287398-\infty]$. The interval was minimized by using Gauss Legendre quadrature in six points on each finite domain, while the remaining semi-infinite domain was minimized by using shifted Laguerre formular in six points to obtain seven non-linear equations. The seven equations alongside equations (4.1) and (4.2) were solved to obtain the constants.

$c_0=0.9056512245, c_1=0.00008229897895, c_2=-0.003919350564, c_3=0.007527522198,$
$c_4=-1.774288397, c_5=3.175750965, c_6=-4.513411468, c_7=3.336816229, _8=-1.134209025$

Substituting the constants into the trial function, we obtain

$$f_{wrm}(\infty)=0.9056512243 \quad and \quad f''_{wrm}(0)=-1.281930184$$

Which compares favourably with the exact solution $f_{exact}(\infty)=0.90505639$ and $f''_{exact}(0)=-1.281808$ which are the universal constants of the problem in consideration according to [1].

The value of f can then be substituted into the second differential equation in $\vartheta(\eta)$. Now apply the same procedure to the second differential equation in (2.3) for various values of a and p.

The value of $\vartheta''(\eta)$ =H, which is the wall temperature gradient for various values of prandtl number p and a are presented in Table (4.1), also the graph of various parameters were plotted in Fig1 and Fig2 for comparison purpose

Table 4.1 (Results of H for various value of a and p)

a\p	0.5	1.0	3.0	5.0	8.0	10.0
-1.5	0.20396817	0.37735719	0.9323479	1.4048967	2.0379030	2.4092578
-0.5	-0.17576919	-0.29989218	-0.63620698	-0.8756983	-1.1508632	-1.2990980
0	-0.33042150	-0.549667043	-1.12393651	-1.5233522	-1.9828232	-2.2358252
1	-0.59425102	-0.954819210	-1.86945065	-2.4977261	-3.2319948	-3.64695772
3	-1.00841159	-1.56064839	-2.94041348	-3.8900831	-5.0066756	-5.6339980

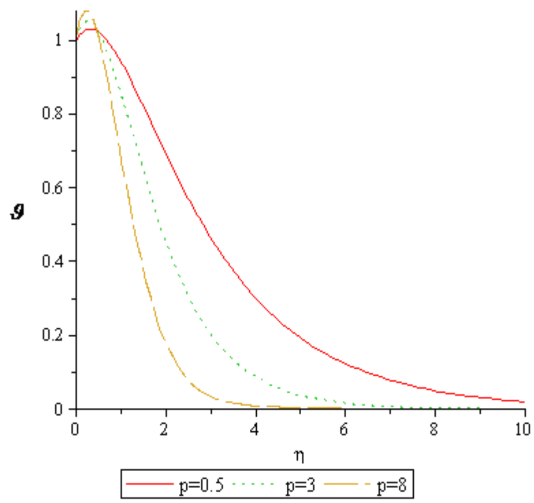

Fig.1:The dimentionless wall temperature g plotted against η for a= -1.5 and p=0.5,3 and 8

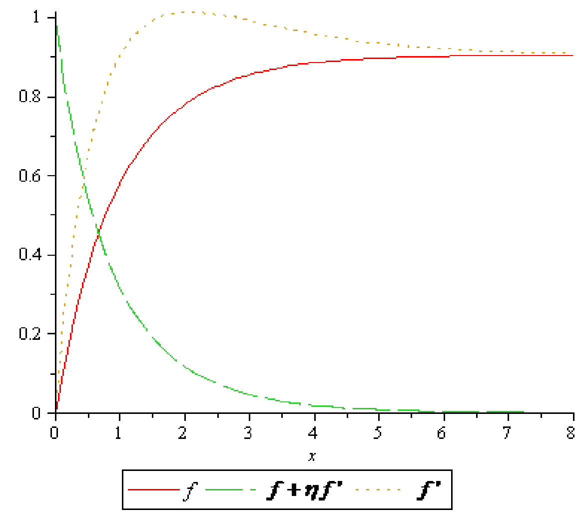

Fig2: plots of f, f' and $f+\eta f'$ against η

5.0 Conclusion

The method of weighted residual for heat and mass transfer in the boundary layers of an exponentially stretching continuous surface was presented. The method expresses the results in analytical form depending on the trial function assumed. Results obtained with this method were compared with that of previous method[1] and it was observed that the method converges quickly in graphical solution presented. Our results also compares favourably with problems whose exact solutions are known, the exact solutions are the universal constants of the problems in consideration. In general the method is simple and efficient with high accuracy of results.

6.0 References

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