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Radiative Fluid Flow Between Fixed Vertical Plates With Suction/Blowing And Mass Transfer In The Presence Of Chemical Reaction

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Abstract

Radiating MHD free convective slip flow with mass transfer and chemical reaction is presented. The governing particles are solved by perturbation method. The temperature, velocity and concentration profiles are presented graphically. The effects of magnetic, Prandtl, Schmidt, radiation, chemical, wave numbers are discussed.

Keywords: Radiative, MHD, convective slip flow, mass transfer, chemical reaction

1.0 Introduction

The free convection flows with mass transfer in the presence magnetic field are important in liquid metals, electrolytes and ionized gases. Other applications are in geophysics, oceanography, drying processes and solidification of binary alloy. When free convection flows occur at high temperature, radiation effects on the flow become significant. Such information is used for the design of nuclear power plants, gas turbines, and the various propulsion devices for aircraft, missiles, and space vehicles [1]. Also, [2] investigated the effect of radiation on free convection from a porous vertical plate. Similarly, [3] studied the transient free convective flow of a viscous incompressible fluid between two parallel vertical walls when the change in the temperature of the walls is compared to the fluid temperature.

The transient free convection flow of a viscous and incompressible fluid between two infinite vertical parallel plates in the presence of constant temperature and mass diffusion is investigated analytically by [4]. Beltrami flow of viscous dusty fluid between two parallel plates due to influence of movement of the plates has been studied [5] and the exact solutions are obtained through use of Laplace transform. The effects of thermal radiation and magnetic field on the hydromagnetic Couette flow through a porous channel are considered by [6]. The solutions are obtained using finite difference technique. From [7], the problem of an unsteady MHD non-Newtonian flow between two parallel fixed porous plates is solved numerically. The presence of the slip parameter that reduces the fluid velocity had been examined by [8]. Clearly, the effects of combined heat and mass transfer on unsteady free convective, viscous incompressible flow past a vertical flat plate in slip-flow regime, when suction velocity oscillate in time about a constant mean has been examined in [9]. It has been shown by [10] that an analytical solution for slip flow between two parallel plates in the micro-scale flow regime is obtained using homotopy analysis. From [11], the Magnetohydrodynamic and chemical reaction effects on unsteady flow, heat and mass transfer characteristics in a viscous incompressible and electrically conduction fluid over a semi-infinite vertical porous plate in a slip-flow regime is investigated. The results obtained showed that chemical reaction has retardation effect on the fluid flow and concentration.

This paper examines the effect of radiation, slip parameter, suction and blowing between vertical plates with chemical reaction and constant applied magnetic field at a fixed interval rather than at infinite interval. Perturbation methods have been used in order to study the effects of the various parameters. Graphs for the fluid flow are presented.

2. Problem Formulation

We consider unsteady, free convection two-dimensional flow of an incompressible and electrically conducting viscous fluid along fixed non-conducting vertical flat plates. The x' axis is taken along the plates in the vertically upward direction and y' axis is taken normal to the plates. A magnetic field of uniform strength B_0 is applied in the direction of flow. Initially, the plates and the fluid are at same temperature T_d while concentration level is C_d. At time $t' > 0$ the plate temperature is raised to arts oscillating in its own plane with a velocity T_w and the concentration C_w. It is assumed

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that the magnetic Reynolds number is very small and the induced magnetic field is negligible in comparison to the transverse magnetic field. It is also assumed that the effect of viscous dissipation is negligible in the energy equation. It is assumed that the thermal diffusion and diffusion thermal effects are negligible**.** Using the Boussinesq approximation, the governing equations in this case see [9], [11] for the flow are the momentum, mass concentration and energy respectively. These are

$$\frac{\partial u'}{\partial t'}+v_0\frac{\partial u'}{\partial y'}=\nu\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma B_0^2}{\rho}u'+g\beta\left(T'-T_d\right)+g\beta^*\left(C'-C_d\right) \tag{1}$$

$$\frac{\partial C'}{\partial t'}+v_0\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}-K^{\bullet}C' \tag{2}$$

$$\frac{\partial T'}{\partial t'}+v_0\frac{\partial T'}{\partial y'}=\frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'} \tag{3}$$

where u' is the velocity of the fluid, C' is the mass concentration, and T' is the temperature of the fluid, t' is time, y' is distance, ν is the kinematic viscosity, g is the gravitational constant, β is the thermal conductivity, β^* is modified thermal conductivity, k is thermal conductivity, ρ is density, C_p is heat capacity at constant pressure, q_r is the radiative heat flux, q is the constant heat flux, B_0 is a constant magnetic field intensity, σ is the electrical conductivity of the fluid, D is diffusion term, ω' is a frequency parameter.

The boundary conditions are:

$$\left.\begin{array}{l} t'\le 0, u'=0, T'=T_d, C'=C_d \quad for\ 0\le y'\le d \\ t'>0, u'-\eta\dfrac{\partial u'}{\partial y'}=0, T'=T_w, C'=C_w \quad at\ y'=0 \\ u'=0, T'=T_d, C'=C_d \ \ at\ y'=d \end{array}\right\} \tag{4}$$

where η is the slip parameter. Assuming the radiative heat flux from the Rosseland approximation to have the form

$$\frac{\partial q_r}{\partial y'}=-4\sigma a^*(T_d{}^4-T'^4) \tag{5}$$

σ is the Stefan Boltzmann, a^* is the mean absorption effect for thermal radiation constant. We assume that the temperature differences within the flow are sufficiently small such that T'^4 can be expanded in a Taylor series about T'_∞ and neglecting higher order terms give

$$T'^4=4T'T_d'^3-3T_d'^4 \tag{6}$$

Substituting in (6) into (5), the equation (3) gives

$$\frac{\partial T'}{\partial t'}+v_0\frac{\partial T'}{\partial y'}=\frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}-\frac{16\sigma a^* T_d'^3}{\rho C_p}(T'-T_d) \tag{7}$$

Introducing the following dimensionless variables

$$\left.\begin{array}{l} u=\dfrac{u'd}{\nu}, t=\dfrac{\nu t'}{d^2}, y=\dfrac{y'}{d}, \Pr=\dfrac{\mu c_p}{\kappa}, G=\dfrac{g\beta d^3(T_w-T_d)}{\nu^2}, \\ Gc=\dfrac{g\beta^* d^3(C_w-C_d)}{\nu^2}, Sc=\dfrac{\nu}{D}, R=\dfrac{16\sigma a^* d^2 T_\infty^3(T_w-T_d)}{\mu C_P}, M=\dfrac{\sigma B_0^2 d^2}{\rho\nu}, \\ \theta=\dfrac{T'-T_d}{T_w-T_d}, C=\dfrac{C'-C_d}{C_w-C_d}, \lambda=\dfrac{dv_0}{\nu}, k=\dfrac{K^* d^2}{\nu}, h=\dfrac{\eta}{d} \end{array}\right\} \tag{8}$$

Substituting (8) into (1), (2), (7) reduce to

$$\frac{\partial u}{\partial t}+\lambda\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-Mu+G\theta+GcC \tag{9}$$

$$\frac{\partial\theta}{\partial t}+\lambda\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}-R\theta \tag{10}$$

$$\frac{\partial c}{\partial t}+\lambda\frac{\partial c}{\partial y}=\frac{1}{Sc}\frac{\partial^2 c}{\partial y^2}-kC \tag{11}$$

$$\left.\begin{aligned} &u-h\frac{\partial u}{\partial y}=0,\theta=1,C=1,y=0\\ &u=0,\theta=0,C=0,y=1\end{aligned}\right\} \tag{12}$$

where G ,is the Grashof number, *Gc* is the concentration number, *Sc* is the Schmidt number, Pr is the Prandtl number, and, *R* and *k* are the radiation and chemical reactive parameters respectively. M is magnetic parameter, *h* is the slip parameter and λ is the suction parameter.
In order to solve (9)-(12), the assume solutions are written in terms of steady and unsteady terms as

$$\left.\begin{aligned} &u(y)=u_0(y)+\lambda u_1(y)e^{i\omega t}\\ &\theta(y)=\theta_0(y)+\lambda\theta_1(y)e^{i\omega t}\\ &C(y)=C_0(y)+\lambda C_1(y)e^{i\omega t}\end{aligned}\right\} \tag{13}$$

Substituting (13) into (9) - (12), we obtain the following equations and boundary conditions:

$$\frac{d^2u_0}{dy^2}-Mu_0=-G\theta_0-GcC_0 \tag{14}$$

$$\frac{d^2\theta_0}{dy^2}-R\Pr\theta_0=0 \tag{15}$$

$$\frac{d^2C_0}{dy^2}-kScC_0=0 \tag{16}$$

$$\left.\begin{aligned} &u_0-h\frac{du_0}{dy}=0,\theta_0=1,C_0=1,y=0\\ &u_0=0,\theta_0=0,C_0=0,y=1\end{aligned}\right\} \tag{17}$$

and

$$\frac{d^2u_1}{dy^2}-(M+i\omega)u_1=-G\theta_1-GcC_1-\frac{du_0}{dy} \tag{18}$$

$$\frac{d^2\theta_1}{dy^2}-(R+i\omega)\Pr\theta_1=-\Pr\frac{d\theta_0}{dy} \tag{19}$$

$$\frac{d^2C_1}{dy^2}-(k+i\omega)ScC_1=-Sc\frac{dC_0}{dy} \tag{20}$$

$$\left.\begin{aligned} &u_1-h\frac{du_1}{dy}=0,\theta_1=1,C_1=1,y=0\\ &u_1=0,\theta_1=0,C_1=0,y=1\end{aligned}\right\} \tag{21}$$

Solving (14)-(21), the solutions become

$$\theta_0(y)=\frac{\sinh\sqrt{R\Pr}\,y}{\sinh\sqrt{R\Pr}} \tag{22}$$

$$C_0(y)=\frac{\sinh\sqrt{kSc}\,y}{\sinh\sqrt{kSc}} \tag{23}$$

$$u_0(y) = A_1 e^{\sqrt{M}y} + A_2 e^{-\sqrt{M}y} - A_3 \sinh\sqrt{R\Pr}y - A_4 \sinh\sqrt{kSc}y \tag{24}$$

$$\theta_1 = B_1 e^{\sqrt{(R+i\omega)\Pr}y} + B_2 e^{-\sqrt{(R+i\omega)\Pr}y} + A^* \cosh\sqrt{R\Pr}y \tag{25}$$

$$C_1(y) = B_3 e^{\sqrt{(k+i\omega)Sc}y} + B_4 e^{-\sqrt{(k+i\omega)Sc}y} + B^* \cosh\sqrt{kSc}y \tag{26}$$

$$\begin{aligned} u_1(y) &= D_1 e^{\sqrt{M+i\omega}y} + D_2 e^{-\sqrt{M+i\omega}y} + D_3 e^{\sqrt{(R+i\omega)\Pr}y} + D_4 e^{-\sqrt{(R+i\omega)\Pr}y} + D_5 e^{\sqrt{(k+i\omega)Sc}y} \\ &+ D_6 e^{-\sqrt{(k+i\omega)Sc}y} + D_7 e^{\sqrt{M}y} + D_8 e^{-\sqrt{M}y} + D_9 \cosh\sqrt{R\Pr}y + D_{10} \cosh\sqrt{kSc}y \end{aligned} \tag{27}$$

The velocity, temperature and concentration (13) respectively become

$$\begin{aligned} u(y,t) &= A_1 e^{\sqrt{M}y} + A_2 e^{-\sqrt{M}y} - A_3 \sinh\sqrt{R\Pr}y - A_4 \sinh\sqrt{kSc}y + \\ &\lambda e^{i\omega t}\{ D_1 e^{\sqrt{M+i\omega}y} + D_2 e^{-\sqrt{M+i\omega}y} + D_3 e^{\sqrt{(R+i\omega)\Pr}y} + D_4 e^{-\sqrt{(R+i\omega)\Pr}y} + D_5 e^{\sqrt{(k+i\omega)Sc}y} \\ &+ D_6 e^{-\sqrt{(k+i\omega)Sc}y} + D_7 e^{\sqrt{M}y} + D_8 e^{-\sqrt{M}y} + D_9 \cosh\sqrt{R\Pr}y + D_{10} \cosh\sqrt{kSc}y \ \} \end{aligned} \tag{28}$$

$$\theta(y,t) = \frac{\sinh\sqrt{R\Pr}y}{\sinh\sqrt{R\Pr}} + \lambda e^{i\omega t}\{ B_1 e^{\sqrt{(R+i\omega)\Pr}y} + B_2 e^{-\sqrt{(R+i\omega)\Pr}y} + A^* \cosh\sqrt{R\Pr}y \ \} \tag{29}$$

$$C(y,t) = \frac{\sinh\sqrt{kSc}y}{\sinh\sqrt{kSc}} + \lambda e^{i\omega t}\{ B_3 e^{\sqrt{(k+i\omega)Sc}y} + B_4 e^{-\sqrt{(k+i\omega)Sc}y} + B^* \cosh\sqrt{kSc}y \ \} \tag{30}$$

The constants appearing in the solutions are

$$A_3 = \frac{G}{(R\Pr - M)\sinh\sqrt{R\Pr}}, \quad A_4 = \frac{Gc}{(kSc - M)\sinh\sqrt{kSc}}, \quad A_5 = \frac{h}{1-h\sqrt{M}}\left\{A_3\sqrt{R\Pr} + A_4\sqrt{kSc}\right\},$$

$$a_1 = \frac{1+h\sqrt{M}}{1-h\sqrt{M}}, \ a_2 = e^{-2\sqrt{M}}, \ A_6 = e^{-\sqrt{M}}\left\{\frac{G}{R\Pr - M} + \frac{Gc}{kSc - M}\right\}, \ A_1 = -A_5 - a_1 A_2,$$

$$A_2 = \frac{A_5 + A_6}{a_2 - a_1}, \ A^* = \frac{\sqrt{R\Pr}}{i\omega\sinh\sqrt{R\Pr}}, \ B_1 = -\left(B_2 + A^*\right), \ B_2 = \frac{A^*}{2\sinh\sqrt{(R+i\omega)\Pr}}\left\{\cosh\sqrt{R\Pr} - e^{\sqrt{(R+i\omega)\Pr}}\right\}$$

$$B^* = \frac{\sqrt{kSc}}{i\omega\sinh\sqrt{kSc}}, \ B_4 = \frac{B^*}{2\sinh\sqrt{(k+i\omega)Sc}}\left\{\cosh\sqrt{kSc} - e^{\sqrt{(k+i\omega)Sc}}\right\}, \ B_3 = -\left(B_4 + B^*\right)$$

$$a_3 = -R\Pr + M + i\omega(1-\Pr), \ a_4 = M - kSc + i\omega(1-Sc) \ \ D_3 = \frac{B_1 G}{a_3}, \ D_4 = \frac{B_2 G}{a_3}, \ D_5 = \frac{B_3 Gc}{a_4},$$

$$D_6 = \frac{B_4 Gc}{a_4}, \ D_9 = \frac{GA^* + A_3\sqrt{R\Pr}}{M - R\Pr + i\omega}, D_{10} = \frac{ScB^* + A_4\sqrt{kSc}}{M - kSc + i\omega}, \ D_7 = \frac{A_1\sqrt{M}}{i\omega}, \ D_8 = -\frac{A_2\sqrt{M}}{i\omega},$$

$$\begin{aligned} D_{11} = \ & D_3 + D_4 + D_5 + D_6 + D_7 + D_8 + D_9 + D_{10} - h\left\{\sqrt{(R+i\omega)\Pr}\,(D_3 - D_4)\right., \\ & \left. + \sqrt{(k+i\omega)Sc}\,(D_5 - D_6) + \sqrt{M}\,(D_7 - D_8)\right) \end{aligned}$$

$$\begin{aligned} D_{12} = \ & D_3 e^{\sqrt{(R+i\omega)\Pr}} + D_4 e^{-\sqrt{(R+i\omega)\Pr}} + D_5 e^{\sqrt{(k+i\omega)Sc}} + D_6 e^{-\sqrt{(k+i\omega)Sc}} \\ & + D_7 e^{\sqrt{M}} + D_8 e^{-\sqrt{M}} + D_9 \cosh\sqrt{R\Pr} + D_{10} \cosh\sqrt{kSc} \end{aligned}$$

$$\left.\frac{\partial\theta}{\partial y}\right|_{y=0} = \frac{\sqrt{R\,\mathrm{Pr}}}{\sinh\sqrt{R\,\mathrm{Pr}}} + \lambda\left\{\sqrt{(R+i\omega)\,\mathrm{Pr}}\,(B_1 - B_2)\right\}e^{i\omega t}$$

$$D_1 = -\left(D_2 a_5 + \frac{D_{11}}{1-h\sqrt{M+i\omega}}\right),\ a_5 = \frac{1+h\sqrt{M+i\omega}}{1-h\sqrt{M+i\omega}},$$

$$D_2 = \frac{1}{e^{-2\sqrt{M+i\omega}} - a_5}\left\{\frac{D_{11}}{1-h\sqrt{M+i\omega}} - D_{12}e^{-\sqrt{M+i\omega}}\right\}$$

3 Results and Discussion

The computation of velocity (real and imaginary parts) for different parameters of angular speed, Grashof number (G), modified Grashof number (Gc), radiation parameter (R), chemical reaction parameter (K), magnetic number (M), and Schmidt number are presented. The results obtained in the present paper are in agreements with [9] if M = 0 = R = K = ε and $\lambda = 1$, and [11] if R = 0 = ε , $\lambda = 1$. The graphs for the velocity are presented in figures 1-14 where the real and imaginary parts of the velocity are denoted by U_r and U_i respectively. From figures 1 and 2, the effects of ωt are shown. From figure 1, the velocity is increasing with increasing ωt , while in figure 2, the reverse is the case. The fluid flow is more pronounced and amplified in figure 2 than figure 1.

In figures 3 and 4, the effects of Grashof number are illustrated. From figure 3, the velocity profiles are increasing with increasing Grashof number. However, from figure 4, though increasing in the first loop is oscillating forming three loops of profiles. The shape of the flow figure 4 is smaller than that of figure 3.

The effect of mass Grashof numbers are shown in figures 5 and 6. As mass Grashof numbers increase, the velocity profiles increase and reach maximum point but figure 6 exhibits oscillation.

Figures 7 and 8 demonstrate the variation of radiation parameter R on the velocity. The real part of the velocity is increasing with increasing radiation parameter. However, in figure 8 the velocity is decreasing as radiation parameter increase. Figures 9 and 10 give the plots of the variation of chemical reaction parameter K on the velocity. The real part of the velocity is increasing with increasing chemical reaction parameter. However, in figure 10 the velocity is decreasing as chemical reaction parameter increase. The flow patterns are very close to each other.

From figures 11 and 12 illustrate the effects of magnetic parameter. It is observed from both figures, as magnet number increases, the velocity decreases.

Figures 13 and 14 represent velocity profiles for different Schmidt numbers. From figure 13, the velocity is increasing as Schmidt number increases while in figure 14, the velocity decreases as Schmidt number increases.

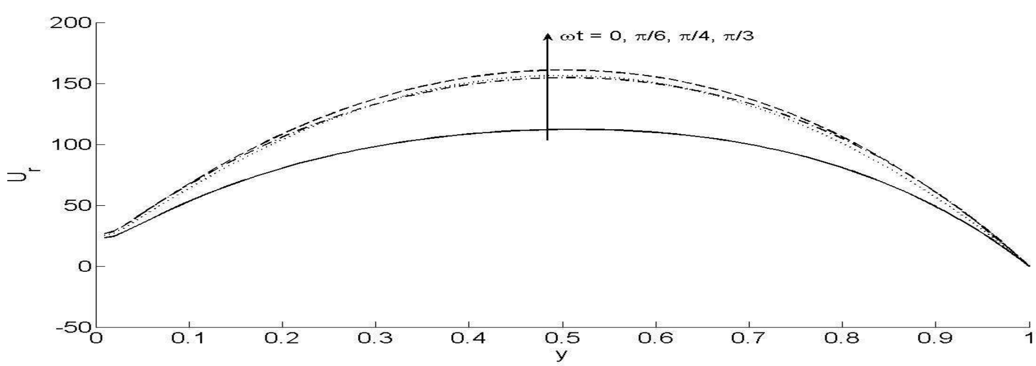

Figure 1: Velocity distribution with varying ωt

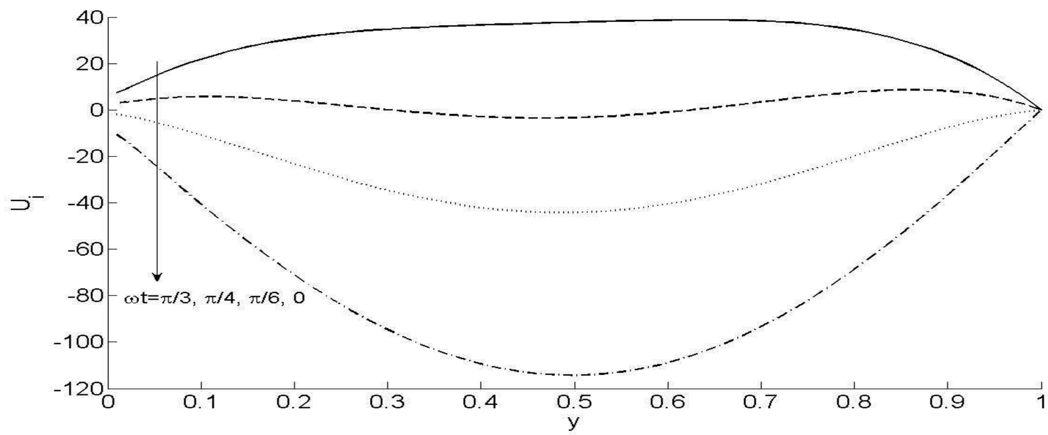

Figure 2: Velocity distribution with varying ωt a

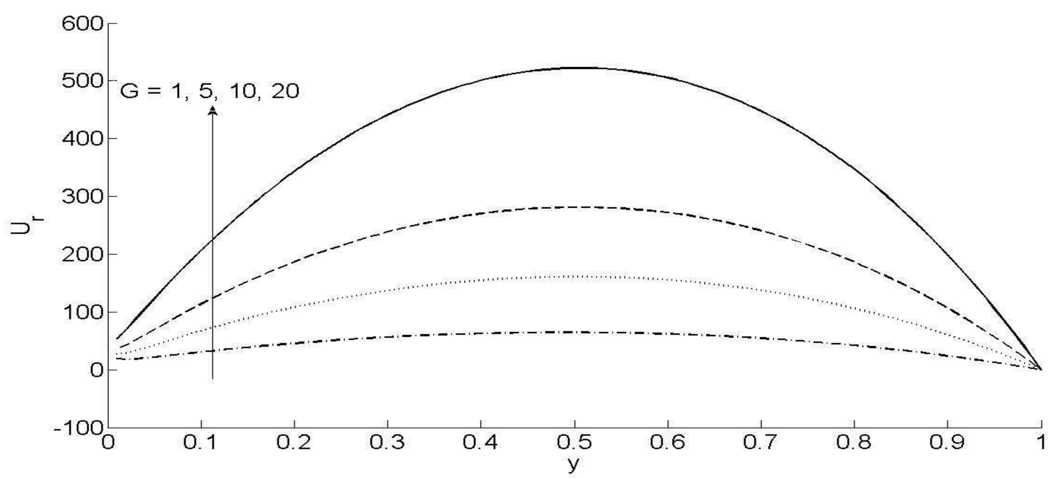

Figure 3: Velocity distribution with varying *G*

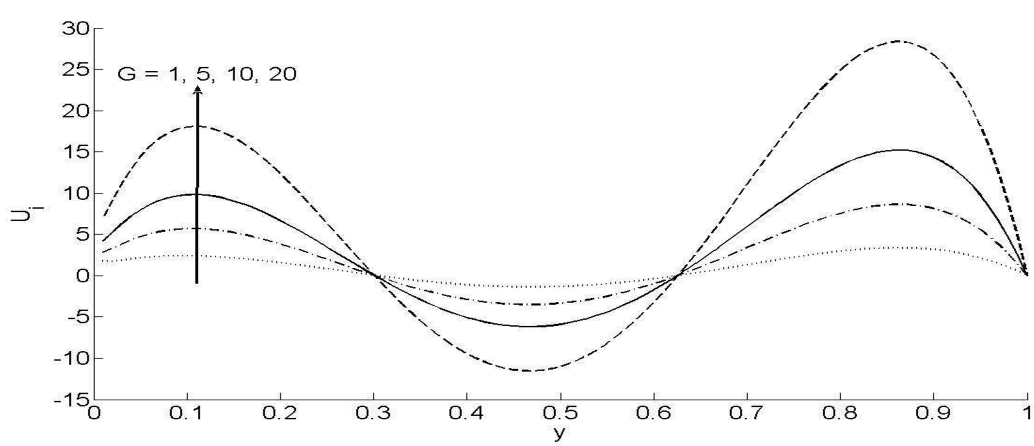

Figure 4: Velocity distribution with varying *G*

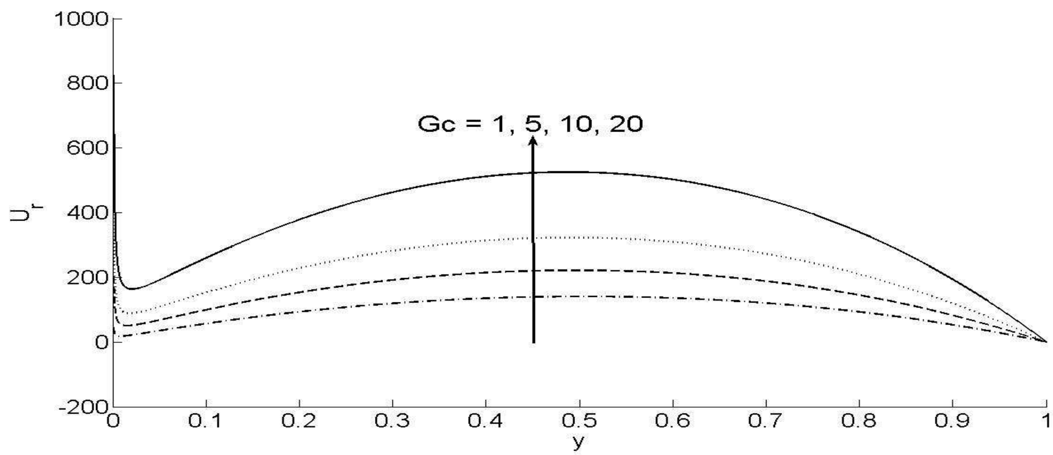

Figure 5: Velocity distribution with varying *Gc*

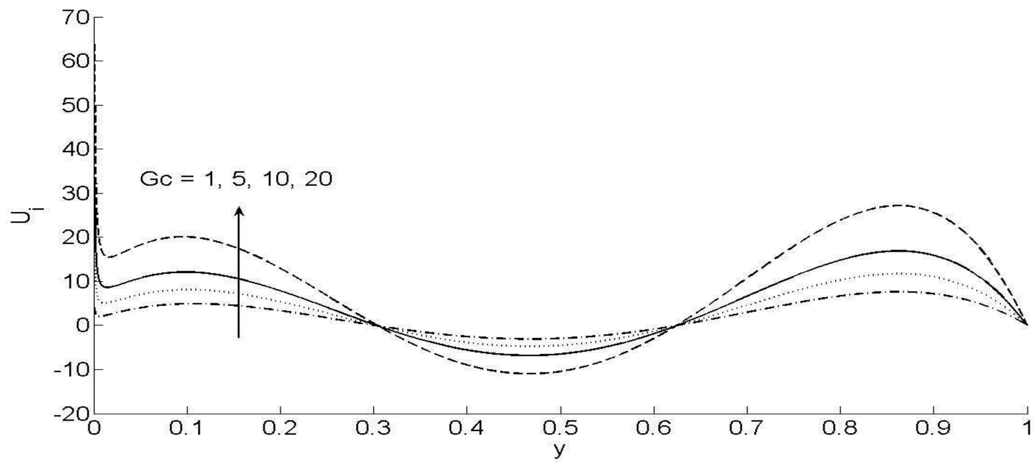

Figure 6: Velocity distribution with varying *Gc*

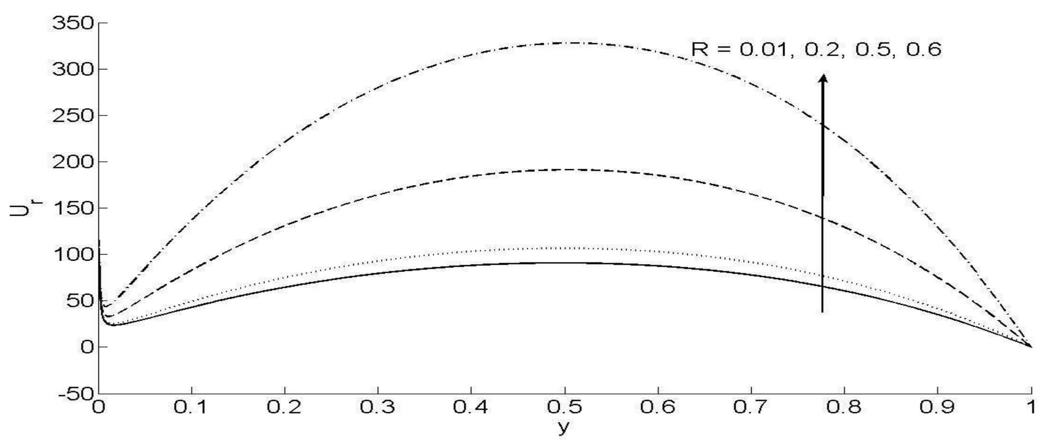

Figure 7: Velocity distribution with varying *R*

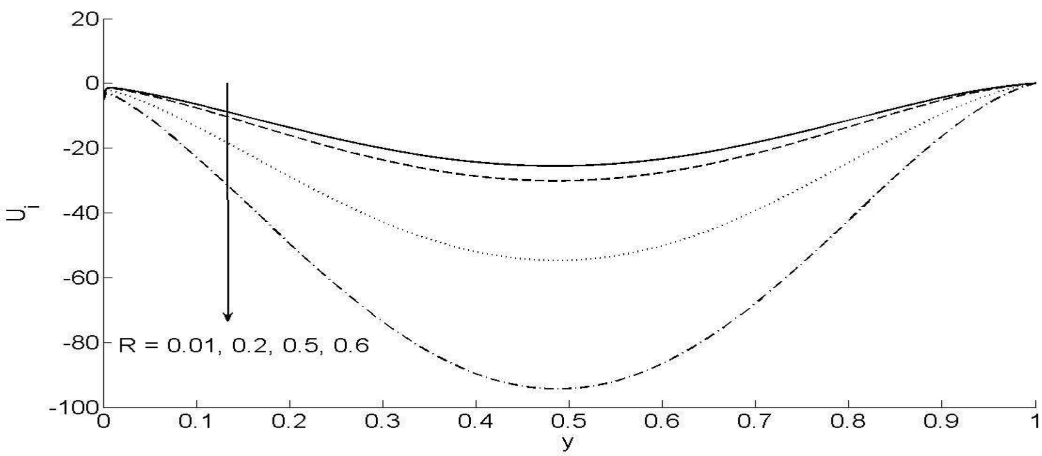

Figure 8: Velocity distribution with varying *R*

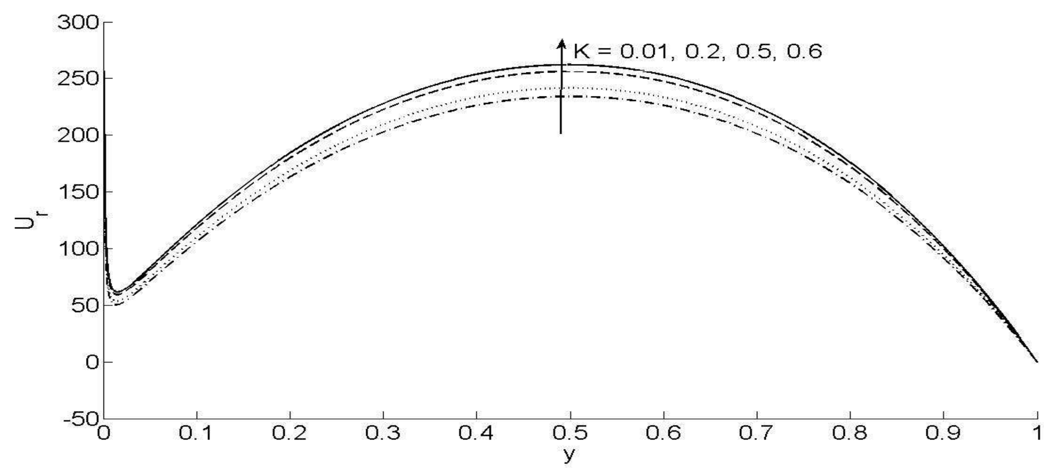

Figure 9: Velocity distribution with varying *K*

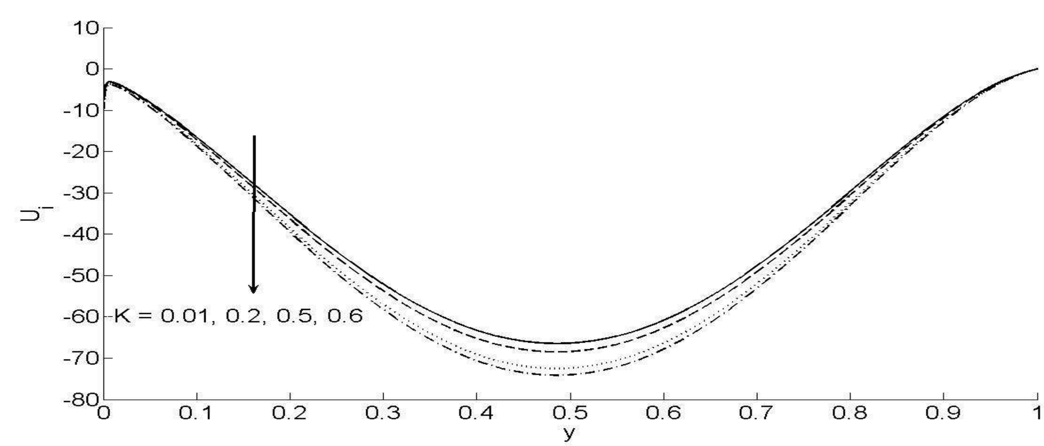

Figure 10: Velocity distribution with varying *K*

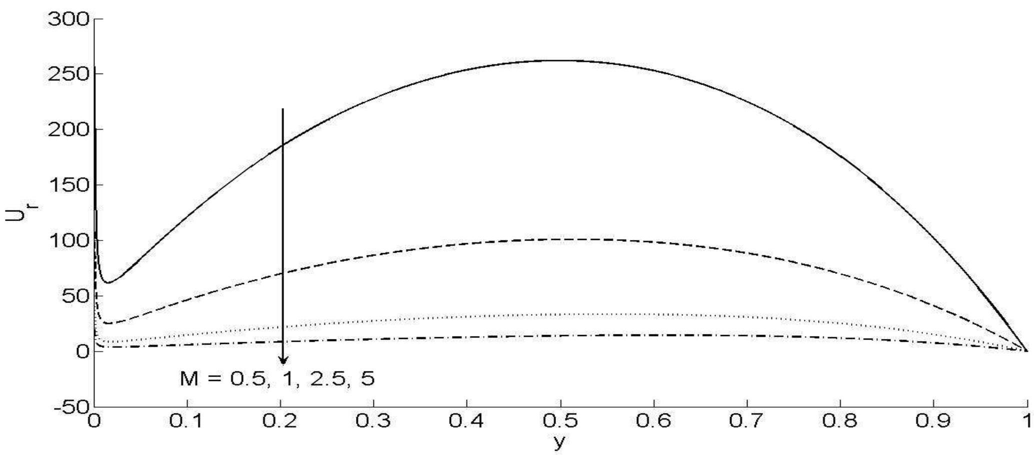

Figure 11: Velocity distribution with varying *M*

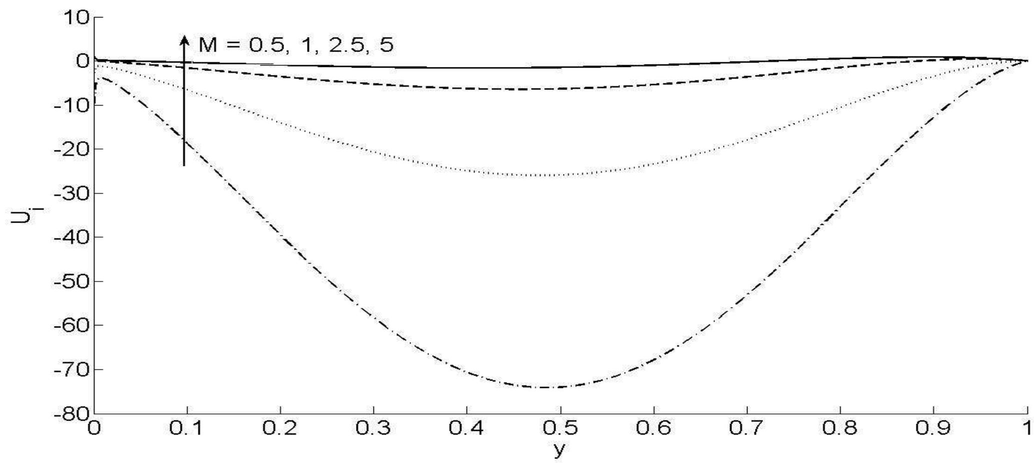

Figure 12: Velocity distribution with varying *M*

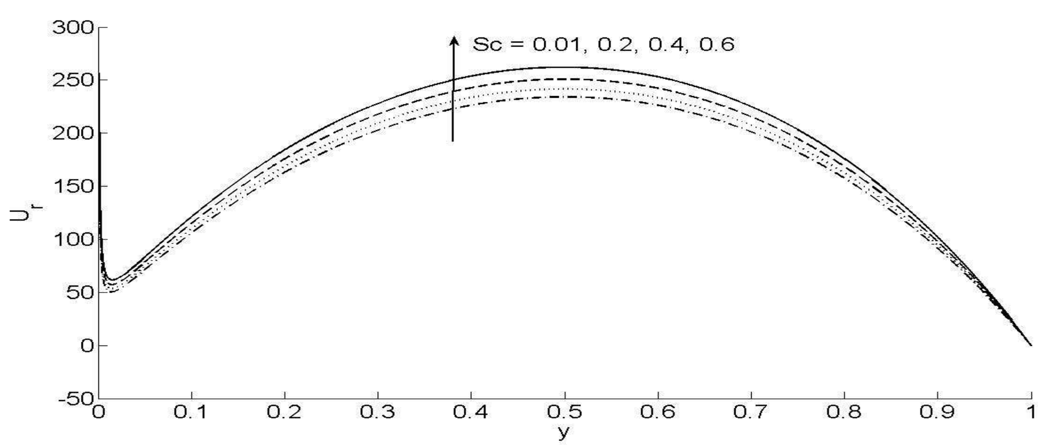

Figure 13: Velocity distribution with varying *Sc*

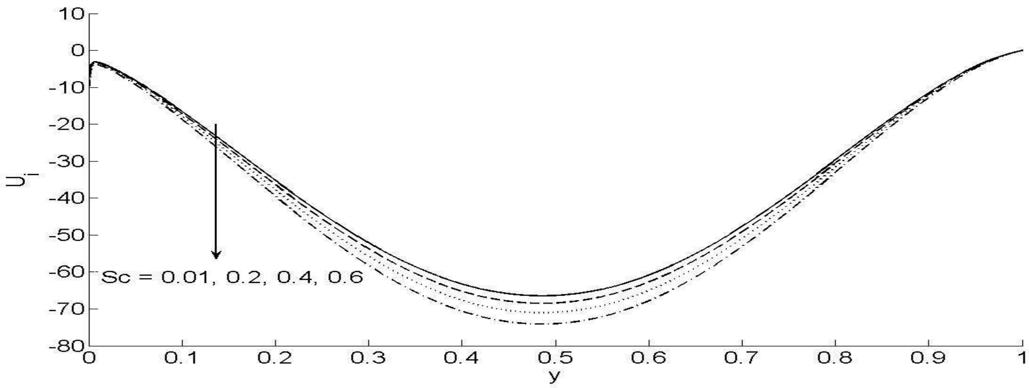

Figure 14: Velocity distribution with varying *Sc*

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