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Effects of Unsteady Flow Past An Infinite Vertical Plate With Variable Velocity, Temperature and Constant Mass Flux

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Abstract

The effects of unsteady flow past an infinite vertical plate with variable temperature and constant mass flux are investigated. Laplace transform technique is used to obtain velocity and concentration fields. The computation of the results indicates that the velocity profiles increase with increase in Grashof numbers, mass Grashof numbers and time. For combined parameters, the velocity field is increasing with increase of mass Grashof number and decreases with increasing Grashof, Schmidt and Prandtl numbers. Similarly, concentration profile is decreasing with increasing Schmidt number.

Keywords: unsteady flow, infinite vertical plate, variable temperature, constant mass flux.

1.0 Introduction

The importance of laminar free convection and radiation effects and mass transfer finds applications in technological and engineering fields and manufacturing industries such as rocket propulsion systems, spacecraft reentry, aerodynamics, plasma physics, glass production, furnace engineering, design of reliable equipment, nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering applications [1], [2].

The nonlinear fluid flow in [3] with the effect of magnetic field on the transient free convection flow of an electrically conducting fluid over an impulsively started isothermal plate has been discussed. The resulting nonlinear equations are solved numerically. The velocity and concentration decreases with increasing magnetic parameter. Similarly, the chemical parameter increases the velocity of the fluid for a generative chemical reaction while for destructive chemical reaction it is decreased.

Recently, [4] studied the MHD flow of a viscous incompressible fluid past an oscillating vertical plate with radiation and variable mass diffusion. Applying Laplace transfer method, the velocity and temperature fields as well as the concentration were obtained.

Also [5] investigated the thermal radiation effects on the flow past an impulsively started infinite vertical plate with variable temperature and mass flux. Applying Laplace transforms, the temperature, velocity and concentration were obtained. The results obtained indicate velocity increases with decreasing radiation parameter. Similarly, velocity increases with increasing thermal Grashof number. In a similar case, [6] examined the interaction of free convection with thermal radiation of a viscous incompressible flow past an impulsively started vertical plate with heat and mass transfer. The numerical results for temperature, concentration are shown graphically. It is observed that temperature, velocity decrease in the boundary layer while skin friction increase with the increase in radiation parameter.

Similarly, [7] considered free convection and mass transfer effect on oscillating vertical plate in the presence of a homogenous first order chemical reaction. Velocity increases with decreasing radiation parameter or chemical reaction parameter.

In their investigation, [8] gave an exact solution to the flow of a viscous incompressible unsteady flow past an infinite vertical oscillating plate with variable temperature and mass diffusion. Applying Laplace transform method, the effects of velocity and concentration are studied for different parameters like phase angle, chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number and time. From [9], the radiation effect of Couette flow with uniform temperature and constant mass flux has been studied leading to exact solutions. The effects of linearly accelerated flow have been examined extensively [10], [11], [12]. Hence the present study discussed the analytical

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solution to the boundary layer flow of an electrically conducting fluid in an infinite vertical plate with heat and mass transfer, with prescribed linear velocity, variable temperature and mass flux without viscous dissipation and chemical reaction.

2 FORMULATION

We consider the mass effects on unsteady flow of a viscous incompressible fluid past a vertical infinite plate with variable velocity, temperature and constant mass flux is studied. It is assumed no heat dissipation and chemical reaction exit. The *x*-axis is taken along the plate in the vertically upward direction. Initially, the fluid is at rest. For time $t > 0$, the temperature and concentration is raised. The governing equations in non-dimensionless form following []9], [10],[11], [12] are given by

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T_\infty\right) + g\beta^*\left(C' - C_\infty\right) \tag{1}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{2}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} \tag{3}$$

where u' is the velocity of the fluid, C' is the mass concentration, and T' is the temperature of the fluid, t' is time, y' is distance, ν is the kinematic viscosity, g is the gravitational constant, β is the thermal conductivity, β^* is modified thermal conductivity, k is thermal conductivity, ρ is density, C_p is heat capacity at constant pressure, D is the diffusivity..

The initial and boundary conditions are:

$$\left.\begin{array}{l} t \le 0, u' = 0, T' = T_\infty, C' = C_\infty \quad for\ all\ y' \\ t > 0, \quad u' = u_0 t, T' = T_\infty + t(T_0 - T_\infty), \dfrac{\partial C'}{\partial y'} = -\dfrac{q^*}{D}\, at\ y' = 0 \\ u' \to 0, T' \to T_\infty, C' \to C_\infty\ as\ y' \to \infty \end{array}\right\} \tag{4}$$

Introducing the following dimensionless variables

$$\left.\begin{array}{l} u = \dfrac{u'}{u_0}, t = \dfrac{v_0^2 t'}{\nu}, y = \dfrac{y' v_0}{\nu}, \Pr = \dfrac{\nu\rho c_p}{k}, Gr = \dfrac{g\beta\nu(T_0 - T_\infty)}{v_0^2 u_0}, \\ Gc = \dfrac{g\beta^*\nu(C_0 - C_\infty)}{v_0^2 u_0}, Sc = \dfrac{\nu}{D}, \theta = \dfrac{T' - T_\infty}{T_0 - T_\infty}, C = \dfrac{C' - C_\infty}{C_0 - C_\infty} \end{array}\right\} \tag{5}$$

Equations (1) to (4) reduce to

$$\frac{\partial u}{\partial t} = G\theta + \frac{\partial^2 u}{\partial y^2} + GcC \tag{6}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{7}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{8}$$

The initial and boundary conditions become

$$\left.\begin{array}{l} t \le 0: \quad u = 0, \quad \theta = 0, \; C = 0 \quad for\ all\ y \\ t > 0: \quad u = t, \quad \theta = t, \; \dfrac{\partial C}{\partial y} = -1 \text{ at } y = 0 \\ \qquad u \to 0, \; \theta \to 0, \; C \to 0 \text{ as } y \to \infty \end{array}\right\} \tag{9}$$

where *u* is the velocity of the fluid, *C* is the mass concentration, and θ is the temperature of the fluid, then *G* is the Grashof number, *Gc* is the concentration number, *Sc* is the Schmidt number and Pr is the Prandtl number.

3 METHOD OF SOLUTION

The solutions of (6) – (8) by Laplace transform subject to the boundary conditions (9) are

$$\bar{\theta} = \frac{1}{s^2} e^{-\sqrt{s\Pr}y} \tag{10}$$

$$\bar{C} = \frac{1}{s^{\frac{3}{2}}\sqrt{Sc}} e^{-\sqrt{sSc}y} \tag{11}$$

$$\bar{u} = \frac{1}{s^2} e^{-\sqrt{s}y} + \frac{Gr}{(\Pr-1)s^3}\left\{e^{-\sqrt{s}y} - e^{-\sqrt{s\Pr}y}\right\} + \frac{Gc}{(Sc-1)\sqrt{Sc}s^{\frac{5}{2}}}\left\{e^{-\sqrt{s}y} - e^{\sqrt{sSc}y}\right\} \tag{12}$$

Inverting equations (10) to (12), we obtain

$$\theta(\xi,t) = F_1(\xi,\Pr,t) \tag{13}$$

$$C(\xi,t) = \frac{2\sqrt{t}}{\sqrt{Sc}}\left\{\frac{1}{\sqrt{\pi}}e^{-\xi^2 Sc} - \xi\sqrt{Sc}\,\mathrm{erfc}\left(\xi\sqrt{Sc}\right)\right\} \tag{14}$$

$$u(\xi,t) = F_1(\xi,1,t) + \frac{Gr}{(\Pr-1)}\left\{F_2(\xi,1,t) - F_2(\xi,\Pr,t)\right\}$$
$$+\frac{Gc}{(Sc-1)\sqrt{Sc}}\left\{F_3(\xi,1,t) - F_3(\xi,Sc,t)\right\} \tag{15}$$

where

$$\xi = \frac{y}{2\sqrt{t}}$$

$$F_1(\xi,\Pr,t) = t\left(1+2\xi^2\Pr\right)\mathrm{erfc}\left(\xi\sqrt{\Pr}\right) - 2t\xi\sqrt{\frac{\Pr}{\pi}}e^{-\xi^2\Pr}$$

$$F_2(\xi,\Pr,t) = \frac{t^2}{6}\left\{\left(3+12\xi^2\Pr+4\xi^4\Pr^2\right)\mathrm{erfc}\left(\xi\sqrt{\Pr}\right) - \xi\sqrt{\Pr}\left(10+4\xi^2\Pr\right)\frac{e^{-\xi^2\Pr}}{\sqrt{\pi}}\right\}$$

$$F_3(\xi,Sc,t) = \frac{4}{3}\sqrt{t^3}\left\{\left(1+\xi^2 Sc\right)\frac{e^{-\xi^2 Sc}}{\sqrt{\pi}} - 0.5\left(3+2\xi^2 Sc\right)\xi\sqrt{Sc}\;\mathrm{erfc}\left(\xi\sqrt{Sc}\right)\right\}$$

4 RESULTS AND DISCUSSION

The effects of various parameters and time for the fluid fields will be assessed. To have clear understanding into the nature of the flow distributions, we computed the velocity and concentration fields. The graphs plotted are shown in Figures 1 to 11. The Prandtl number for water is chosen as 7, the Schmidt number Sc = 0.2, time t = 0.4 and other values where used are stated. The results obtained with linear velocity are in agreements with existing works [10], [11], [12]. For a fix time, the velocity profile emanates from there.

Figures 1 to 10 represent the velocity profiles for the fluid flow. Figure 1 gives the velocity profile for various Grashof numbers. It is seen that velocity is increasing as Grashof number increases. Similarly, Figure 2 depicts the velocity profile for mass Grashof number. As Gc increases, velocity profile increases. It is clear that the maximum velocity of figure 2 is higher than that of Figure 1.

Figures 3 to 4 represent the velocity profiles for various with different Gr , Gc and Gr , Gc , Sc, Pr respectively. From Figure 3, it is observed that while other parameters are fixed, velocity is decreasing with increasing Gr and increases with increase of Gc. Figure 4, the velocity is reducing for increasing Gr, Sc and Pr while it is increasing for increasing Gc. The profiles of the two figures differ slightly.

Figures 5 to 7 show the velocity profiles for various Schmidt numbers with different Gr and Gc. From Figure 5, as Gr and Gc are equal, the velocity profile is highest over 2.2 while in Figure 6, as Gr is greater than Gc, the velocity drops to about 1.3. Similarly, in Figure 7, as Gr is less than Gc, the velocity profile decreases to over 1.8 but higher than that of Figure 6.

Figures 8 to 10 represent the velocity profiles for varying time with different values of Gr and Gc. From Figure 8, as Gr > Gc, the velocity profile increases and reaches a maximum of 5.5. Figure 9 shows the variation of time on the velocity profiles. For Gr = Gc, the velocity profile increases but drop to about a maximum of 4.3. From Figure 10, as Gc > Gr, the velocity profile is highest reaching a maximum of 7.5, compared with Figures 8 and 9.

Figure 11 gives the concentration profile with variation of Schmidt numbers. It is seen that the concentration is decreasing with increasing Schmidt numbers.

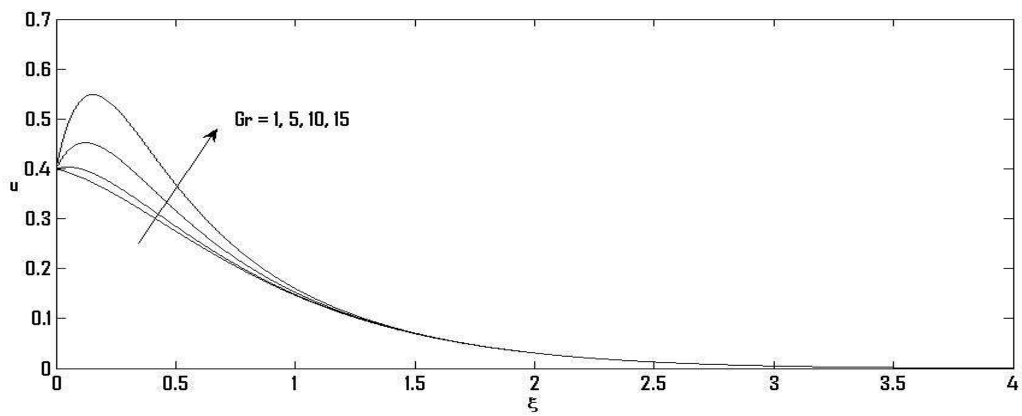

Figure 1: Velocity distributions with different Grashof numbers

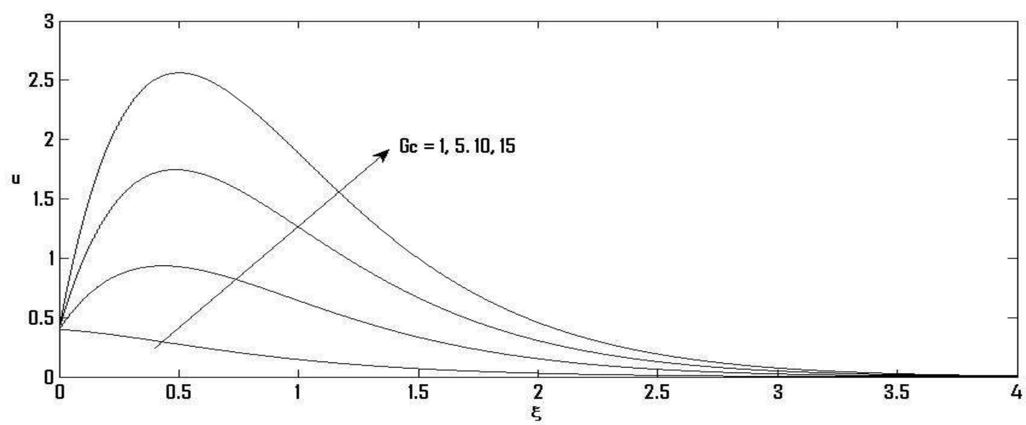

Figure 2: Velocity distributions with different mass Grashof numbers

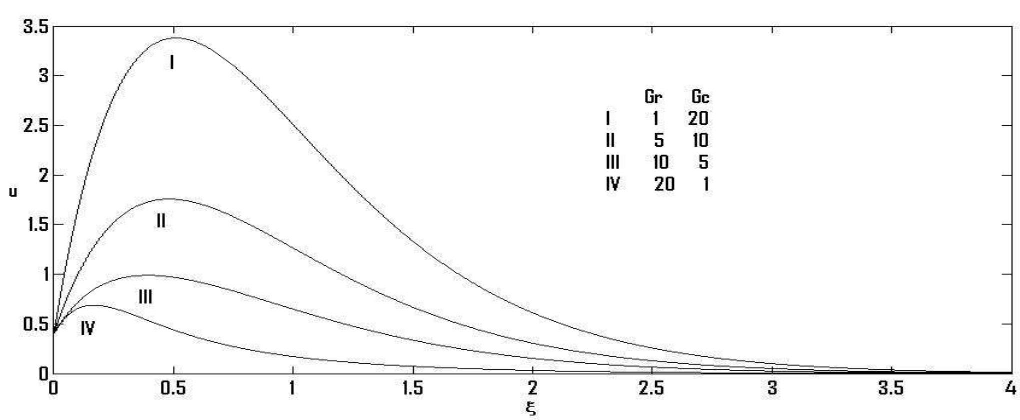

Figure 3: Velocity profiles with varying Grashof and mass Grashof numbers.

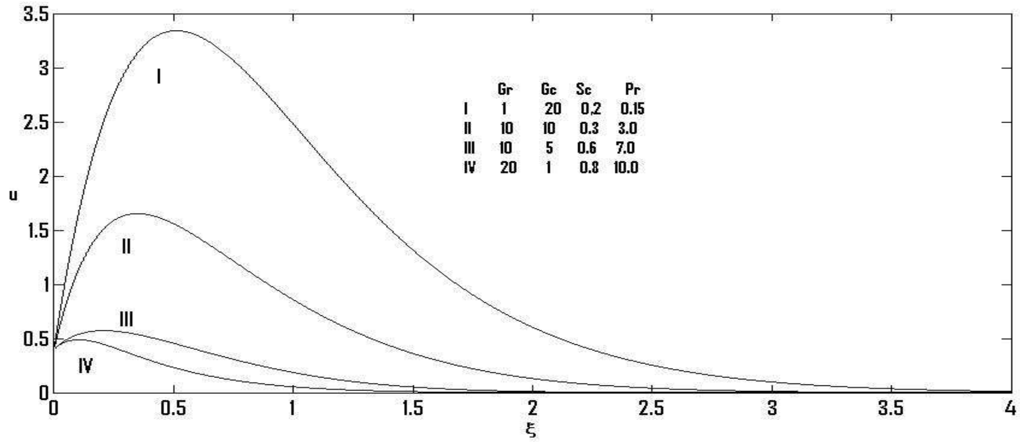

Figure 4: Velocity distributions with varying Gr, Gc, Sc and $\Pr$.

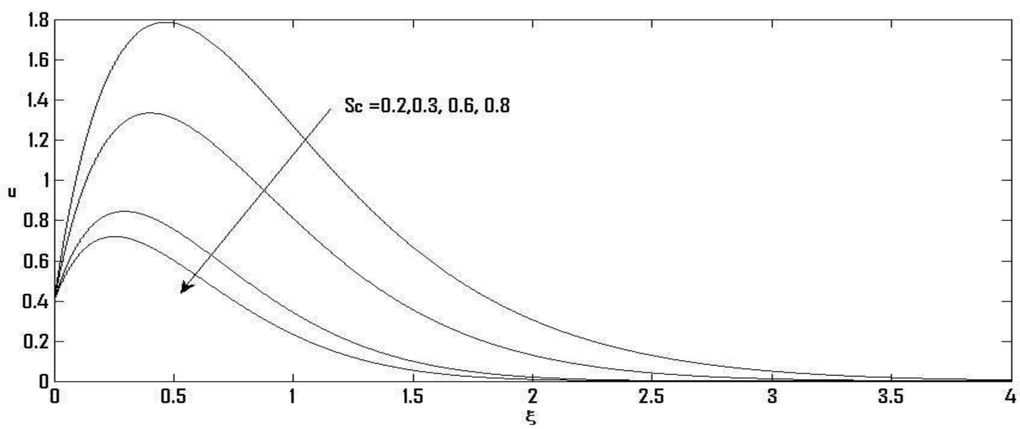

Figure 5: Velocity distributions with Schmidt numbers as $Gr = Gc = 10$.

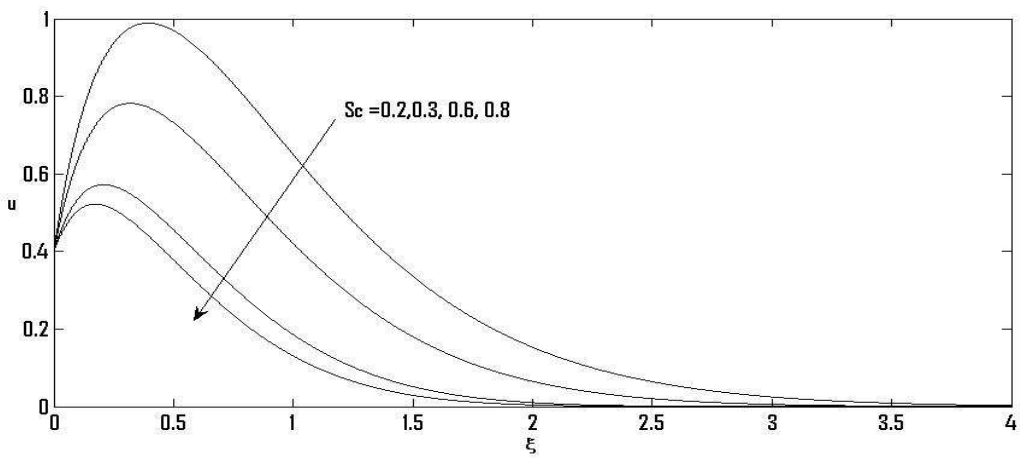

Figure 6: Velocity distributions with different Schmidt numbers as $Gr = 10, Gc = 5$.

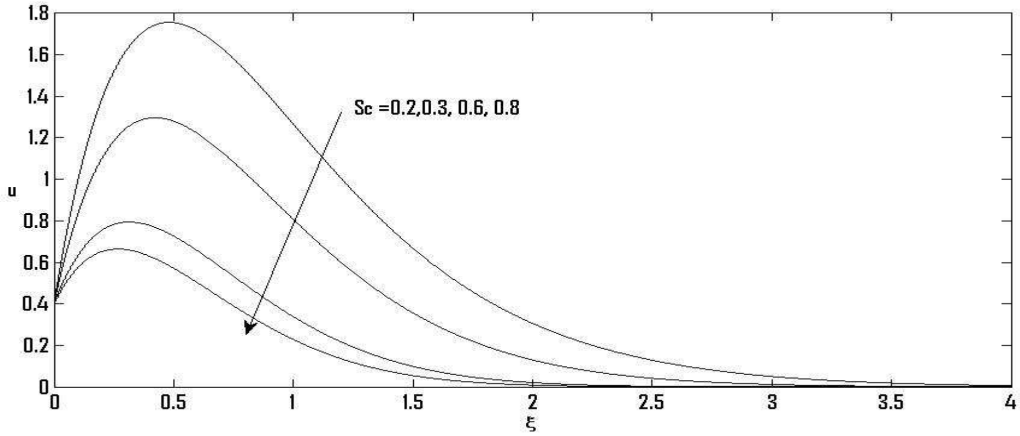

Figure 7: Velocity distributions with different Schmidt numbers as $Gr = 5, Gc = 10$.

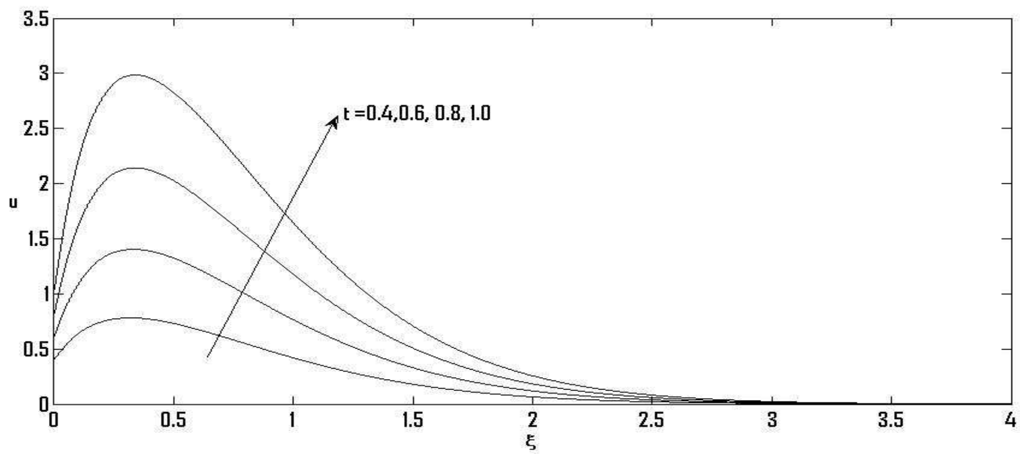

Figure 8: Velocity distributions with different time as $Gr = 10, Gc = 5$.

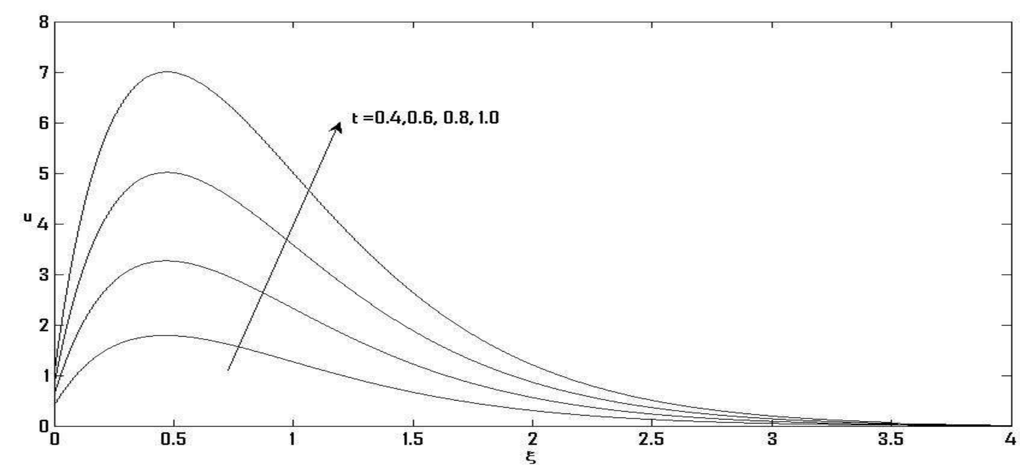

Figure 9: Velocity distributions with different time as $Gr = Gc = 5$.

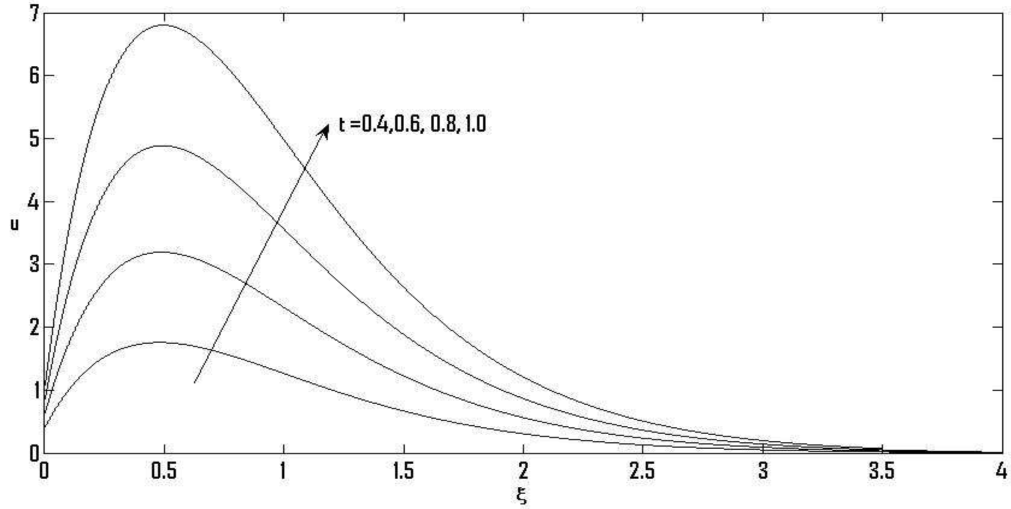

Figure 10: Velocity distributions with different time as $Gr = 5, Gc = 10.$

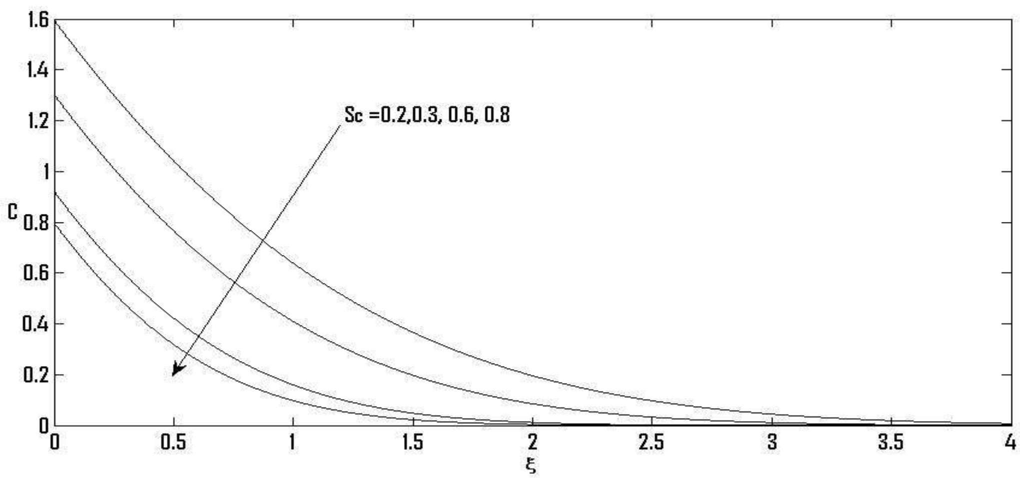

Figure 11: Concentration distributions with different Schmidt numbers

5 CONCLUSIONS

Effects of unsteady flow past an infinite vertical plate with variable temperature and constant mass flux are studied. The velocity profiles increase with increase in Grashof numbers, mass Grashof numbers and time for fixed parameters. With combined parameters, the velocity field decreases with increasing Grashof, Schmidt and Prandtl numbers while it increases for increasing Gc. It is observed that concentration profile is decreasing with increasing Schmidt number.

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