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Solution of Lie' nard Equations using Modified Initial Guess Variational Iterative Method (MIGVIM)

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Abstract

In this work, we obtained approximate solutions for Lie' nard equations using modified initial guess variational iteration method (MIGVIM). This method proves to be very promising for obtaining approximate solutions for this kind of equations. We also demonstrate the superiority of MIGVIM over the decomposition method and the variational iteration method for this type of equations by providing numerical comparisons.

Keywords: Variational Iteration, Lagrange multiplier, Lie' nard equations, Adomian decomposition, Modified initial guess variational iteration.

1.0 Introduction

A variational iteration method (VIM) has been proposed by He [1-7]. The advantages of using Modified Variational Iteration method over Decomposition method for solving non- linear problems have been shown in [8]. In Olayiwola et al [9], a modification of the VIM was proposed by introducing initial guess in the correction functional of VIM. Ahmadabadi et al [10] applied VIM on Lie' nard equations. In this paper, we present a modified initial guess variational iteration method for solving the general form of Lie' nard equations.

2.0 Lie' nard Equations

Let f and g be two continuosly differentiable functions on R, with f an even function and g an odd function, then the non-linear second order ordinary differential equation of the form

$$y'' + f(y)y' + g(y) = 0 \qquad (2.1)$$

$$y(0) = g_0(x) \ , \ y'(0) = g_1(x) \qquad (2.2)$$

is called a Lie' nard equation.

Lie' nard equations are intensively studied during the development of radio and vacuum tubes as they can be used to model oscillating circuits.

3.0 Methodology

3.1 He's Variational Iteration Method

This method is a modification of a general Lagrange multiplier method proposed by Inokuti [11]. In the variational iteration method, a differential equation

$$L[y(x)] + N[y(x)] = g(x) \qquad (3.0)$$

was considered, where L and N are linear and nonlinear operators, respectively and $g(x)$ is a known function. According to VIM, we can construct a correction functional

$$y_{n+1}(x) = y_n(x) + \int_0^x \lambda\left[L\left(y_n(t)\right) + N\left(\tilde{y}_n(t)\right) - g(t)\right]dt \qquad (3.1)$$

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where λ is a general Lagrange multiplier, y_n is the nth approximate solution and $\tilde{y}_n$ is a restricted function i.e $\delta\tilde{y}_n = 0$ [2-4]. The successive approximation $y_{n+1}, n \geq 0$ of the solution y will be obtained upon using the determined Lagrange multiplier λ and any selective initial function y_0. Consequently,

$$y = \lim_{n\to\infty} y_n \qquad (3.2)$$

3.2 Modified Initial Guess Variational Iterative Method (MIGVIM)

In modified initial guess variational iteration method equation (3.1) is as

$$y_0(x) = g_0(x) + tg_1(x) + t^2 g_2(x) \qquad (3.3a)$$

$$y_{n+1}(x) = y_n(x) + \int_0^x \lambda\left[y'''_n(t) + f\left(\widetilde{y_n}(t)\right) + g(\widetilde{y_n}(t))\right]dt \qquad (3.3b)$$

where $g_2(x)$ is obtained by substituting for $y_0(x)$ in equation (2.1) at $t = 0$.
Therefore, making the the above correction functional stationary we have

$$\delta y_{n+1}(x) = (1 - \lambda'|_{t=x})\delta y_n + \lambda|_{t=x}\delta y_n + \int_0^x \lambda''\delta y_n dt$$

which yields the following stationary conditions:

$$\begin{cases} \frac{\partial^2\lambda(x,t)}{\partial t^2} = 0, \\ 1 - \frac{\partial\lambda(x,x)}{\partial t} = 0, \\ \lambda(x,x) = 0 \end{cases}$$

It is evident that $\lambda(x,t) = t - x$. By substituting $\lambda(x,t)$ in equation $(3.3b)$ we obtained our iteration formular.

4.0 Application

We now apply the proposed method on some examples to show the ability and efficiency of the proposed methods. Computations were done using Maple 12. We shall test the MIGVIM on the problem solved by [10] using VIM and [12] using the Decomposition method and the results will be compared.

4.1 Example 4.1

Consider the example as presented in [10] as follows

$$y'' - y + 4y^3 - 3y^5 = 0 \qquad (4.1)$$

$$y(0) = \frac{\sqrt{2}}{2}, \quad y'(0) = \frac{\sqrt{2}}{4} \qquad (4.2)$$

The theoretical solution is $y(x) = \sqrt{\frac{1+\tanh(x)}{2}}$ (4.3)

By choosing our selective $y_0(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4}t - \frac{\sqrt{2}}{2}t^2,\ \lambda(x,t) = t - x$ and using equations $(3.3a)$ and $(3.3b)$ the desired results are obtained.

In Table 1 and Table 2 we compare the absolute errors obtained by modified initial guess variational iteration method, variational iteration method and decomposition method as reported in [10].

The results are displayed in Table 1 and Table 2:

Table 1: Comparison between modified initial guess variational iteration method, variational iteration method and decomposition method for example 4.1 at the second iteration i.e at $\boldsymbol{n} = \mathbf{2}$

X	Exact solution	MIGVIM y_2 $at\ n = 2$	Absolute error for MIGVIM y_2 $at\ n = 2$	Absolute error for Decomposition $at\ n = 2$	Absolute error for VIM $at\ n = 2$
0.1	*0.7415079213*	0.7415079208	0.0000000005	0.00000000053784	0.00000000033588
0.2	0.7737490938	0.7737490782	0.0000000156	0.000000064118	0.000000017492
0.3	0.8035274147	0.8035272902	0.0000001245	0.0000011344	0.00000014733
0.4	0.8306470256	0.8306465858	0.0000004398	0.0000089650	0.00000053415
0.5	0.8550196364	0.8550187772	0.0000008592	0.000045008	0.0000010563
0.6	0.8766554531	0.8766547067	0.0000007464	0.00016821	0.0000010877
0.7	0.8956471898	0.8956481886	0.0000009988	0.00050950	0.00000076947
0.8	0.9121504181	0.9121552581	0.00000484	0.0013155	0.0000066035
0.9	0.9263632846	0.9263711516	0.000007867	0.0029894	0.000045759
1.0	**0.9385078998**	**0.9385069356**	**0.0000009642**	**0.0061037**	**0.00019575**

Table 2: Comparison between modified initial guess variational iteration method, variational iteration method and decomposition method for example 4.1 at the second iteration i.e at $\boldsymbol{n} = 3$

x	Exact solution	MIGVIM y_3 $at\ n = 3$	Absolute error for MIGVIM y_3 $at\ n = 3$	Absolute error for Decomposition $at\ n = 3$	Absolute error for VIM $at\ n = 3$
0.1	*0.7415079213*	0.7415079211	0.0000000002	0.0000000000028155	0.000000000000068994
0.2	0.7737490938	0.7737490935	0.0000000003	0.00000000094426	0.000000000012856
0.3	0.8035274147	0.8035274146	0.0000000001	0.000000029895	0.00000000020991
0.4	0.8306470256	0.8306470262	0.0000000006	0.00000034870	0.0000000011004
0.5	0.8550196364	0.8550196378	0.0000000014	0.0000022889	0.0000000024336
0.6	0.8766554531	0.8766554539	0.0000000008	0.0000010139	0.0000000010557
0.7	0.8956471898	0.8956471862	0.0000000036	0.000029054	0.0000000067182
0.8	0.9121504181	0.9121504239	0.0000000058	0.000079427	0.000000027743
0.9	0.9263632846	0.9263633698	0.0000000852	0.00012890	0.00000018790
1.0	**0.9385078998**	**0.9385081848**	**0.000000285**	**0.000040137**	**0.0000013473**

5.0 Conclusion

In this paper, we extended the modified initial guess variational iteration method (MIGVIM) method to solve Lie' nard equations. In the considered example we compared the results obtained by modified initial guess variational iteration method with the results obtained by variational iteration method and decomposition method. Table 1 and Table 2 show the obvious superiority of modified initial guess variational iteration method over the decomposition method and variational iteration method for this kind of equations.

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