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Solution of IVP of Second Order ODE with Oscillatory Solutions using Variational Iterative Method (VIM)

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Abstract

A Numerical method for solution of IVP of second order with oscillatory solutions using VIM is developed. The method is applied to solve some initial value problems of second order ODE with oscillatory solutions. The results are compared with some existing methods and found to compete favourably with existing methods. In fact the VIM method gives the same result as the exact solutions of the problems considered.

Keywords: Variational Iteration Method, Lagrange multiplier, oscillatory solutions, ODE.

1.0 Introduction

The solution of second order ordinary differential equation of the type

$$\left.\begin{array}{c} y'' = f(t, y(t), y'(t)') \,, \\ y(t_0) = y_0 \\ such\ that \\ f(t+T, y(t+T), y'(t+T)) = f(t, y(t), y'(t)) \end{array}\right\} \qquad (1.0)$$

is a function that possesses at least two derivatives in some interval $[a, b]$ and satisfied the equation and associated initial conditions.The commonest numerical method for solving a periodic IVP problem of equation (1.0) is by reduction of the problem into a system of first order ordinary differential equations of the form

$$\left.\begin{array}{c} y' = f(x, y) \\ y(x_0) = y_0 \end{array}\right\} \qquad (1.1)$$

One then adopts any appropriate numerical method for solving the resulting first order system of ordinary differential equations [1-4]. Fatunla [5] claimed that the solution of the initial value problem in equation (1.0) is highly oscillatory in nature and thus severely restricts the mesh size of the conventional linear multi-step method and results in the adoption of intolerable small mesh size in the integration interval (a, b). A class of methods for the solution of periodic initial value problems based on one-step trigonometric polynomial functions was developed by Gautshi [6] as reported in [1]. Similarly, Ademiluyi et al [7] developed a class of methods for the solution of periodic initial value problems based on two-step trigonometric polynomial functions.

The aim of this paper is to use VIM to solve second order ordinary differential equations with oscillatory solutions following the method developed by [8].

2.0 Mathematical formulation

In this paper, we shall consider an ordinary differential equation of the form

$$\left.\begin{array}{c} y'' = f(t, y(t), y'(t)) \\ y(a) = y_0, y(a) = y_1, a \le x \le b \end{array}\right\} \qquad (2.0)$$

whose oscillatory solutions satisfy $f\big(t, y(t)\big) = f(t+T, y(t+T))$

3.0 Methodology

3.1 He's Variational Iteration Method

This method is a modification of a general Lagrange multiplier method proposed by Inokuti [9]. In the variational iteration method, a differential equation

$$L[y(x)] + N[y(x)] = g(x) \qquad (3.0)$$

was considered, where L and N are linear and nonlinear operators, respectively and $g(x)$ is a known function. According to VIM, we can construct a correction functional

$$y_{n+1}(x) = y_n(x) + \int_0^x \lambda\big[L\big(y_n(\tau)\big) + N\big(\tilde{y}_n(\tau)\big) - g(\tau)\big]d\tau \qquad (3.1)$$

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where λ is a general Lagrange multiplier, y_n is the nth approximate solution and $\tilde{y}_n$ is a restricted function i.e $\delta\tilde{y}_n = 0$ [10-12]. The successive approximation $y_{n+1}, n \geq 0$ of the solution y will be obtained upon using the determined Lagrange multiplier λ and any selective initial function y_0. Consequently,

$$y = \lim_{n\to\infty} y_n$$

4.0 Sampled Problems

The performance of the proposed method on periodic initial value problems of second order ordinary differential equations was tested on the following sampled problems and the result compared with that obtained in [8].

4.1 Example 4.1

$$y'' = y \qquad (4.0)$$

$$y'(0) = 1, y(0) = 0$$

The theoretical solution to equation (4.0) is $y(x) = \sin x$.

Using VIM, it can readily be seen that the Lagrange multiplier $\lambda = \sin(\tau - x)$. By choosing a selective function $y_0 = x$ and using equation (3.1)

$$y_1(x) = y_0(x) + \int_0^x (\sin(\tau - x))\left[\frac{d^2 y_0(\tau)}{d\tau^2} + y_0(\tau)\right] d\tau \qquad (4.1)$$

This gives;

$$y_1(x) = \sin x \qquad (4.2)$$

Equation (4.2) is the first numerical approximation which gives a result equaling the theoretical solution.

4.2 Example 4.2

$$y'' = y \qquad (4.3)$$

$$y'(0) = 1, y(0) = 0$$

The theoretical solution is $y(x) = \sin x + \cos x$.

Here, the Lagrange multiplier $\lambda = \sin(\tau - x) + \cos(\tau - x)$. The selective function is chosen to be

$y_0 = 1 + x - \frac{x^2}{2}$ and using equation (3.1), the successive iteration are:

$$y_1 = 1 - x + 2\sin x$$

$$y_2 = 2\sin x + 2\cos x - 1$$

$$y_3 = \sin x + \cos x$$

It is evident that y_3 coincides with the theoretical solution.

5.0 Conclusion

In this paper, VIM method was applied to solve periodic initial value problems of second order ordinary differential equations. The ability of the method to give an efficient solution which equals the exact solution with just few iteration makes it remarkable when compared with the method used in [8]. This method can be extended to periodic initial value problems greater than order two.

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